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Isolated Prompt Photon Cross Sections*

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Abstract

We show that the conventionally defined partonic cross section for the production of isolated prompt photons is not an infrared safe quantity. We work out the case of $e^+e^- \to \gamma + X$ and discuss implications for hadron reactions.

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ISOLATED PROMPT PHOTON CROSS SECTIONS

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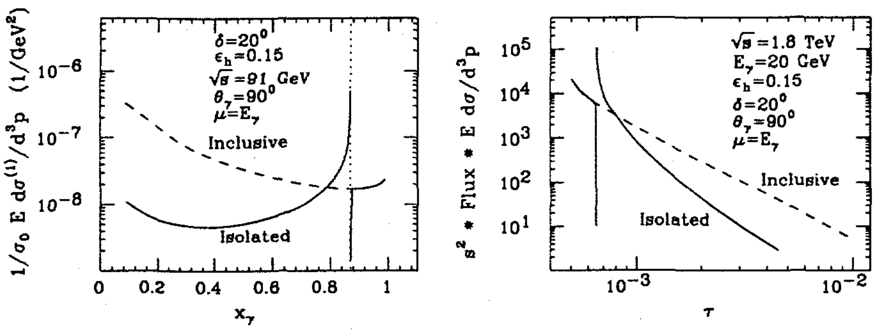
We show that the conventionally defined partonic cross section for the production of isolated prompt photons is not an infrared safe quantity. We work out the case of $e^+e^- \to \gamma + X$, and discuss implications for hadron reactions.

In e^+e^- and in hadron-hadron reactions at collider energies, *prompt* photons are observed and their cross sections are measured only if the photons are relatively isolated in phase space. Isolation is required to reduce various hadronic backgrounds including those from the electromagnetic decay of mesons, e.g., $\pi^o \to 2\gamma$. The essence of isolation is that a cone of half-angle δ is drawn about the direction of the photon's momentum, and the isolated cross section is defined for photons accompanied by less than a specified amount of hadronic energy in the cone, e.g., $E_h^{\text{cone}} \leq E_{\max} = \epsilon_h E_\gamma$; E_γ denotes the energy of the photon. Theoretical predictions will therefore depend upon the additional parameters δ and ϵ_h. Isolation removes backgrounds, but it also reduces the signal. For example, it reduces the contribution from processes in which the photon emerges from the long-distance *fragmentation* of quarks and gluons, themselves produced in short-distance hard collisions.

Hard photons in e^+e^- processes arise as QED bremsstrahlung from the initial beams, radiation that is directed along angles near $\theta_\gamma = 0$ and π, and as final state radiation from *direct* and *fragmentation* processes. The final state radiation populates all angles, with a distribution having both transverse, $1 + \cos^2\theta_\gamma$, and longitudinal components [1].

Much of the predictive power of perturbative QCD derives from factorization theorems [2]. *Conventional* factorization expresses a physical quantity as the convolution of a partonic term with a nonperturbative long-distance matrix element, and it requires that the partonic term, calculated perturbatively order-by-order in the the strong coupling strength α_s, have no infrared singularities. Using $e^+e^- \to \gamma X$ as an example, we demonstrate that the perturbatively calculated partonic part for the isolated photon cross section is not infrared safe. The infrared sensitivity shows up first in the next-to-leading order quark-to-photon fragmentation contribution [3].

For the quark fragmentation contributions, there are two sources of hadronic



(a) (b)

Figure 1: One-loop quark fragmentation contributions to the isolated and inclusive cross sections (a) as a function of $x_\gamma = 2E_\gamma/\sqrt{s}$ in $e^+e^- \to \gamma X$ at $\sqrt{s} = 91$ GeV, and (b) as a function of τ in $p\bar{p} \to \gamma X$ at $\sqrt{s} = 1.8$ TeV.

energy in the isolation cone: a) from fragmentation of the quark itself, E_{frag}, and b) from non-fragmenting final-state partons, $E^{cone}_{partons}$, that enter the cone. When the maximum hadronic energy allowed in the isolation cone is saturated by the fragmentation energy, $E_{\max} = E_{frag}$, there is no allowance for energy in the cone from other final-state partons. In particular, if there is a gluon in the final state, the phase space accessible to this gluon is restricted. By contrast, isolation does not affect the virtual gluon exchange loop contribution. Therefore, for isolated photons, the infrared singularity from the virtual contribution is not canceled completely by the *restricted* gluon emission contribution [3].

If conventional factorization were true, the fragmentation contributions to the physical cross section would be expressed in the factorized form

$$E_\gamma \frac{d\sigma^{iso}_{e^+e^-\to\gamma X}}{d^3\ell} = \sum_c \int_{\max\left[x_\gamma,\frac{1}{1+\epsilon_h}\right]}^{1} \frac{dz}{z}\, E_c \frac{d\hat{\sigma}^{iso}_{e^+e^-\to cX}}{d^3p_c}\left(x_c = \frac{x_\gamma}{z}\right)\frac{D_{c\to\gamma}(z,\delta)}{z}\,; \tag{1}$$

$x_\gamma = 2E_\gamma/\sqrt{s}$ and $x_c = 2E_c/\sqrt{s}$. The sum extends over $c = q, \bar{q}$ and g, and $D_{c\to\gamma}(z,\delta)$ is the nonperturbative function that describes fragmentation of parton "c" into a photon. The lower limit of integration results from the isolation requirement with the assumption that all fragmentation energy is in the isolation cone [3]. Because of the isolation condition, the phase space constraints are different in three regions: a) $x_\gamma < 1/(1+\epsilon_h)$, b) $x_\gamma = 1/(1+\epsilon_h)$, and c) $x_\gamma > 1/(1+\epsilon_h)$. We summarize here the situation in the different regions and show that the next-to-leading order partonic term for quark fragmentation, $E_q d\hat{\sigma}^{iso}_{e^+e^-\to qX}/d^3p_q$, is infrared sensitive [3] *at and below* the point

$x_\gamma = 1/(1+\epsilon_h)$.

When $x_\gamma < 1/(1+\epsilon_h)$, subprocesses with two-body final states do not contribute. Therefore, there is no leading-order quark (or antiquark) fragmentation contribution, and one-loop virtual diagrams do not contribute. The well-known infrared pole singularity of the real gluon emission diagrams, having the form $1/(1-x_q)$ as $x_q = x_\gamma/z \to 1$, *remains uncanceled* in $\hat{\sigma}^{iso}_{e^+e^-\to qX}$. After convolution with $D_{q\to\gamma}(z)$, this inverse power infrared sensitivity yields a logarithmic divergence proportional to $\ell n(1/x_\gamma - (1+\epsilon_h))$ in the cross section $\sigma^{iso}_{e^+e^-\to\gamma X}$. As shown in Fig. 1(a), this means that the isolated cross section would become larger than the inclusive cross section in the vicinity of $x_\gamma \to 1/(1+\epsilon_h)$, a result that is certainly not physical. This infrared sensitivity in $\hat{\sigma}^{iso}_{e^+e^-\to qX}$ signals a clear breakdown of conventional perturbative factorization.

When $x_\gamma = 1/(1+\epsilon_h)$, it is possible that $x_q = x_\gamma/z = 1$. Therefore, the one-loop virtual gluon exchange diagrams, whose contributions are proportional to $\delta(1-x_q)$, do contribute. However, isolation constraints limit the phase space accessible to gluon emission in the real subprocess, $e^+e^- \to q\bar{q}g$. Consequently, the infrared divergences in the real and virtual contributions do not cancel completely in the isolated case, unlike the inclusive case. Working in $n = 4-2\epsilon$ dimensions, we find[3]

$$E_q \frac{d\sigma^{iso}_{e^+e^-\to qX}}{d^3p_q} \sim \left\{ \frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left(\frac{3}{2} - \ell n\frac{\delta^2}{4}\right)\right\} \delta(1-x_q) \ + \text{finite terms} \ . \qquad (2)$$

The presence of the uncanceled $1/\epsilon$ and $1/\epsilon^2$ terms means that the regulator ϵ cannot be set to 0. Therefore, at $x_q = 1$, corresponding to $x_\gamma = 1/(1+\epsilon_h)$, the partonic term for quark fragmentation is infrared divergent, and the perturbative calculation is not well-defined. Conventional perturbative factorization again breaks down.

To recapitulate, in $e^+e^- \to \gamma + X$, the next-to-leading order partonic term associated with the quark fragmentation contribution is infrared sensitive when $x_\gamma \le 1/(1+\epsilon_h)$. Conventional perturbative factorization of the cross section for isolated photon production in e^+e^- annihilation breaks down in the neighborhood of $x_\gamma = 1/(1+\epsilon_h)$. The isolated cross section, as usually defined, is not an infrared safe observable and cannot be calculated reliably in conventional perturbative QCD at and below $x_\gamma = 1/(1+\epsilon_h)$.

In hadron collisions, $A + B \to \gamma X$, we are interested in the production of isolated prompt photons as a function of the photon's transverse momentum, p_T. At next-to-leading order in QCD, one must include fragmentation at next-to-leading order. At this level, problems analogous to those in e^+e^- annihilation are also encountered in the hadronic case. As an example[3], we consider the contribution from a quark-antiquark subprocess in which the flavors of the initial and final quarks differ: $q' + \bar{q}' \rightarrow q + \bar{q} + g$, where q fragments to γ. We take equal values for the incident parton momentum fractions, $x_a = x_b = x = \sqrt{\tau}$. In the translation to the hadronic case, the variable x_γ becomes $\hat{x}_T$ where $\hat{x}_T = 2p_T/\sqrt{\hat{s}} \sim x_T/x$ with $x_T = 2p_T/\sqrt{s}$.

The special one-loop quark fragmentation contribution to the observed cross section takes the form

$$E_\gamma \frac{d\sigma_{AB\rightarrow\gamma X}}{d^3\ell} \sim \int_{x_T^2}^{1} d\tau \; \Phi_{q'\bar{q}'}(\tau)\; E_\gamma \frac{d\sigma_{q'\bar{q}'\rightarrow\gamma X}}{d^3\ell}(\tau) \quad + \quad \text{other subprocesses} \; . \tag{3}$$

In Fig. 1(b), we show the integrand in Eq. (3) obtained after convolution with the parton flux $\Phi(\tau)$. The corresponding contribution to the hadronic cross section is the area under the curve in Fig. 1(b) from x_T^2 to 1. It is evident that the convolution with the parton flux substantially enhances the influence of the region of infrared sensitivity. The divergences above and below the point $\hat{x}_T = 1/(1+\epsilon_h)$ [or $\sqrt{\tau} = x_T(1+\epsilon_h)$] are integrable logarithmic divergences, and thus they yield a finite contribution if an integral is done over all τ. However, we stress that the perturbatively calculated one-loop partonic cross section $E_q\, d\hat{\sigma}^{iso}_{q'\bar{q}'\rightarrow qX}$, has an inverse-power divergence as $x_q \rightarrow 1$ and has uncanceled $1/\epsilon$ poles in dimensional regularization[3]. The pole divergence for $\hat{x}_T < 1/(1+\epsilon_h)$ becomes a logarithmic divergence after convolution with a long-distance fragmentation function. Even though this logarithmic divergence near $\sqrt{\tau} = x_T(1+\epsilon_h)$ is integrable, it is certainly not correct to accept at face value a prediction in which a perturbatively calculated isolated cross section exceeds the inclusive cross section before the integration over τ.

The results in both the e^+e^- and hadronic cases challenge us to find a modified factorization scheme and/or to devise more appropriate infrared safe observables.

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