Paper presented at Beam Instrumentation Worskhop
Santa Fe, October 20-24

TRI-PP 93-97
Nov 1993

IMPROVING THE EFFICIENCY OF THE DESIGN OF CYLINDRICAL STRIPLINE BEAM MONITORS

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ABSTRACT

A basic stripline beam position monitor consists of four strips 90° apart inside a circular beam pipe. To avoid signal distortion the strip to wall spacing must be selected to make the strip impedance match that of the end connections. The problem is treated as two dimensional, quasi-TEM and reduces to an electrostatic case. The impedance is first calculated using a finite element relaxation technique to solve the Laplace equation. The energy in the field is then used to obtain the distributed strip capacitance and impedance. This method requires a very fine grid and converges slowly. In the second method the strip is assumed to be thin and is replaced by a set of charge pipes. This method is applicable to the stripline geometry because the cylinder can be replaced by a set of image charge pipes. A modified Green's function is integrated over the charge pipes using a Gauss-Chebyshev quadrature. The result is a set of simultaneous linear equations which can be solved very quickly. A monitor had been constructed and the strip impedance and cross coupling coefficients had been measured.

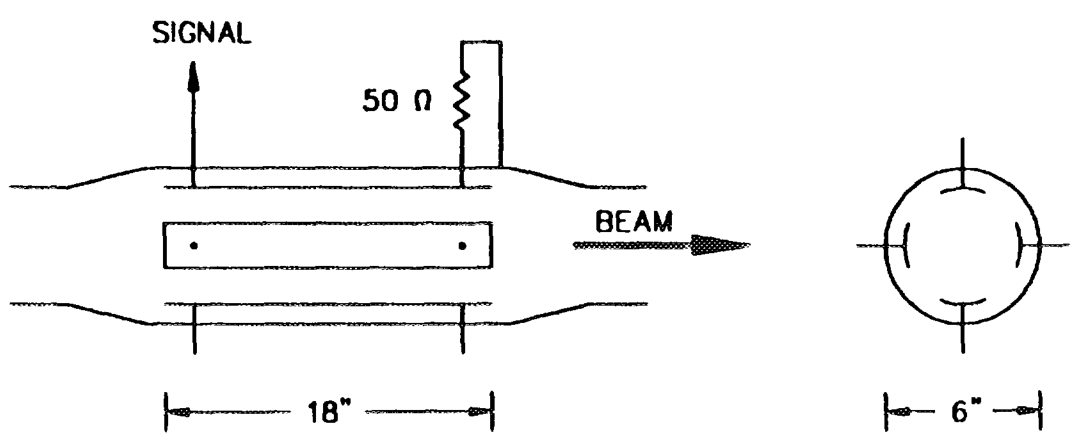

Figure 1. The stripline to wall spacing must be set accurately to match the strip impedance to the end terminations.

INTRODUCTION

The position, intensity, and time structure of a proton beam can be measured by using a stripline monitor. The device consists typically of four striplines held inside the walls of a cylindrical beam pipe, figure 1. The device constructed at TRIUMF uses strips 0.062 in thick and 18 in long subtending 45°. The cylinder inner radius is 2.96 in. The beam at TRIUMF consists of a 23 MHz stream of proton bunches from 150 ps to 5 ns in duration, with an intensity of about 1 μA to 140 μA, respectively. As each bunch passes through the monitor, it induces a signal of up to several millivolts on the striplines. Signals are taken from the upstream ends of the strips while the downstream ends are terminated in resistive loads. It is important that the geometry of the striplines be designed so that the impedance matches that of the signal cables and terminating resistors in order to avoid reflections. Though the repetition rate of the beam bunches is 23 MHz, the short beam bunch creates a signal rich in harmonics. Frequencies of up to several GHz are present for the shortest bunches.

THE FINITE DIFFERENCE METHOD

The impedance, for a given strip height, was calculated by using a computer program called RELAX3D [1]. This FORTRAN program solves Laplace's equation for user defined boundary conditions. In the case of the stripline monitor, the problem was simplified to a two dimensional geometry, a cross section through the strips. The area is divided into a square grid of evenly spaced points and an initial guessed value of the voltage assigned to each point. While holding the voltages on the boundaries fixed, the voltage on the rest of the grid points are allowed to relax. Relaxation consists of replacing the voltage at each point by a weighted average of its neighbouring points. One iteration has been done when the averaging process has been applied to all points. To accelerate the convergence, over-relaxation is used. The calculated change at each point is increased by a factor which is decreased from 2 to 1 over 10 successive iterations, constituting a sweep. Many sweeps must be done before the voltage mesh converges to an acceptable accuracy. A FORTRAN program was then used to calculate the total energy in the field from the voltages on the grid. From the energy, the capacitance and impedance are readily obtained.

The program was run for two cases. In the first case, only one strip had a voltage on it, the others were held at ground. In the second case, all four strips were held at the same non-zero voltage. The former case would approximate excitation by a very misteered beam. The latter case would occur for a well centered beam and is the more common situation. Runs were made for three values of strip to wall spacing. The program was run with mesh sizes of 101 by 101, 201 by 201, and 401 by 401 points. The process is inefficient. Up to 6000 iterations per run were required for convergence. At the highest resolution, the program requires about 6 minutes on a VAXstation 4090. The results are shown in table I.

Table I. The results of the finite difference method.

h (inches)	Number of Active Strips	101x101 Grid C (pF/m)	202x202 Grid C (pF/m)	404x404Grid C (pF/m)
0.169	1	116.6	130.9	140.1
0.469	1	59.1	62.0	63.3
0.769	1	43.2	43.9	44.8
0.169	4	116.6	127.6	136.1
0.469	4	59.1	56.0	57.2
0.769	4	43.2	35.9	36.6

The impedance, Z, can be derived from the capacitance per unit length, C, by where c is the speed of light. For an impedance of 50 Ω the capacitance is 66.7 F/m.

$$Z = \frac{1}{c\,C} \tag{1}$$

It was found that a simple formula could be used to fit the results

$$C = \epsilon_0 \frac{w}{h} + k \tag{2}$$

where ϵ_0 is the permittivity of free space, w and k are parameters, and h is the strip to wall spacing. This formula allowed the results to be interpolated. The value of h for 50 Ω impedance was found to be 0.428 in with one strip charged and 0.391 in with four strips charged.

THE MULTIPIPE METHOD

The multipipe method has been applied to problems involving flat striplines [2], however it can also be applied to the cylindrical geometry of a stripline monitor. The voltage on a strip is the integral of the Green's function times the charge density over all the strips. The integral can be approximated using a Gauss-Chebyshev quadrature method. The continuous charge distribution on each strip is replaced by a set of charge pipes located on the strips. From reference 3, we have

$$V(\overline{x_m}) \approx \sum_{n=1}^{N} q_n\, G_{mn}, \qquad m \neq n \tag{3}$$

where $\bar{x}_m$ is the location of the *m* th pipe, $V(\bar{x}_m)$ is the strip potential, q_n is the charge coefficient of the *n* th pipe, G_{mn} is the Green's function and N is the number of source pipes. This is a set of N equations in N unknowns. The ground plane, in this case a cylinder, is replaced by the source pipe images. The Green's function becomes

$$G_{mn}(\overline{x_m}) = \begin{cases} G_{source}(\overline{x_m}, \overline{x_n'}) + G_{image}(\overline{x_m}, \overline{x_n'}), & m \neq n \\ G_{source}(0, r_m) + G_{image}(\overline{x_m}, \overline{x_m'}), & m = n \end{cases} \qquad (4)$$

where r_m are constants dependent only on the number of pipes and strip width.

As the number of pipes per strip increase, their radii decrease. In this note, the pipes have been approximated by line charges and the charge coefficients are treated as line charge densities. A line charge inside a grounded cylinder has an image line charge of equal density, outside the cylinder, figure 2, where

$$a\, b = r^2 \qquad (5)$$

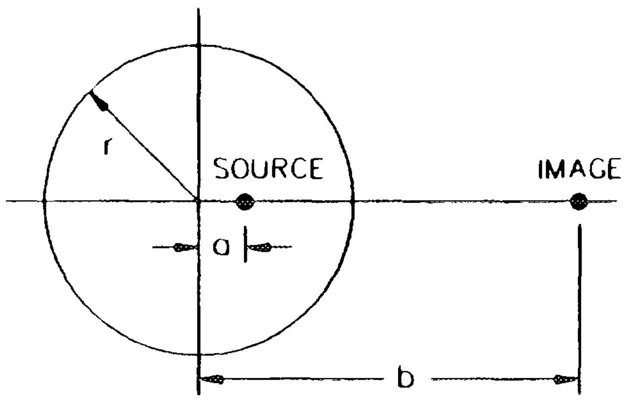

Figure 2. The image of a line charge inside a grounded cylinder is an opposite line charge outside the cylinder.

Together, the two line charges create an equipotential over the whole cylinder radius with a voltage

$$u = -\frac{q}{2\Pi\epsilon_0} \ln \frac{a}{r} \qquad (6)$$

where q is the line charge (C/m). The 2-D Green's function of a line charge in free space is

$$G(\overline{x}, \overline{x'}) = -\frac{1}{2\,\Pi\,\epsilon_0} \ln |\overline{x}-\overline{x'}| \qquad (7)$$

For convenience, the Green's function can be modified to leave zero volts at the cylinder radius. For a given strip to wall spacing, a is constant and the voltage u can be subtracted from the Green's function of the source and image

$$G(\overline{x},\overline{x'}) = G_{source}(\overline{x},\overline{x'}) + G_{image}(\overline{x},\overline{x'}) + \frac{1}{2\Pi\epsilon_0}\ln\frac{a}{r} \quad (8)$$

The technique was applied to the two cases; one strip charged, the other three grounded, then all four strips charged. For each case, calculations were made for 3, 4 and 5 pipes per strip, at three values of strip to wall spacing. The solution for five pipes per strip, involving a set of twenty linear equations in twenty unknowns, takes about ten seconds of computer time using Mathematica for Windows [3] on a 486 PC. Most of the time is used to symbolically set up the equations and calculate the Green's functions. Only about one second is used to solve the simultaneous equations. The results are shown in table II.

Table II. Results of multipipe method.

h (inches)	Number of Active Strips	3 Pipes/Strip C (pF/m)	4 Pipes/Strip C (pF/m)	5 Pipes/Strip C (pF/m)
0.169	1	166.1	152.4	148.6
0.469	1	63.2	63.2	63.2
0.769	1	44.0	44.0	44.0
0.169	4	161.4	148.5	144.8
0.469	4	57.6	57.6	57.7
0.769	4	36.5	36.5	36.5

Figure 3 is a contour plot of the upper right quadrant of the voltage field that would result from the source and image line charges for four charged strips with five pipes per strip. The circular equipotential running between the source and image strips would represent the grounded pipe. The far field approaches that of the real solution whereas the near fields form concentric circles around the line charges. A truer representation of the field could be found by first fitting polynomials to the line charge densities along the strips to give continuous charge distributions

$$q_i = \frac{P(x_i)}{\sqrt{1 - \left(\frac{2\,x_i}{w}\right)^2}} \quad (9)$$

where w is the arc length of the strip, x_i is the distance of the *i* th pipe around the strip from its centre and $P(x_i)$ is a polynomial with an order of one less than the number of pipes per strip.

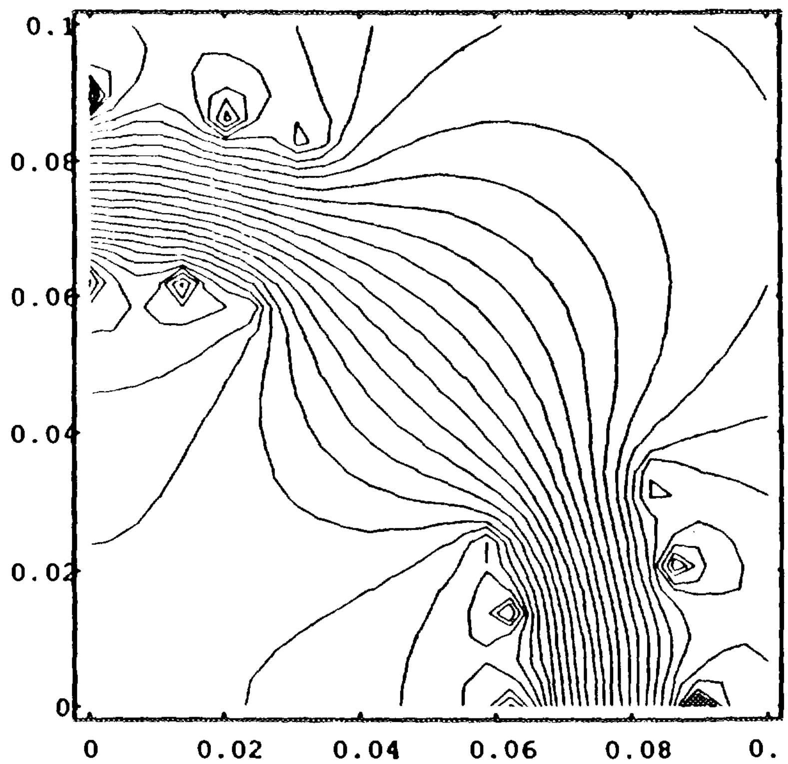

Figure 3. Equipotential lines calculated using the multipipe method. The scales are in meters.

COMPARISON OF CALCULATED RESULTS

Figures 4 and 5 are graphs of the capacitances calculated by the finite difference and multipipe methods. The trends for both one and four charged strips are very similar. The finite difference method converges from below while the multipipe method converges from above. When the strip to wall spacing is small, the former method requires a finer grid while the latter requires more pipes per strip. The latter is numerically faster to compute, however.

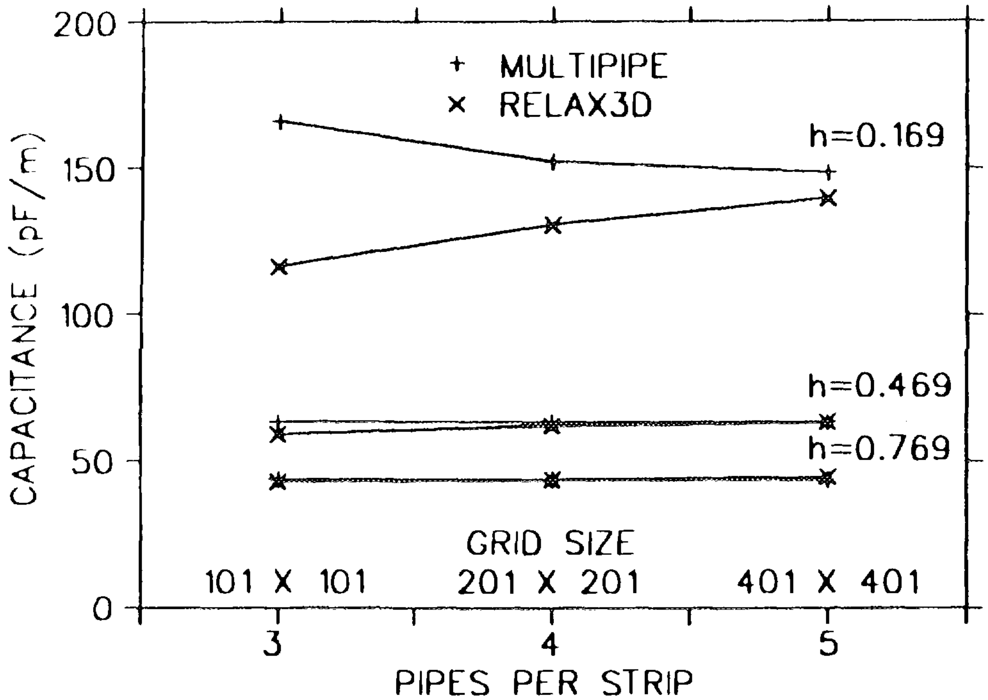

Figure 4 A comparison of the calculated results of the two methods for one strip charged.

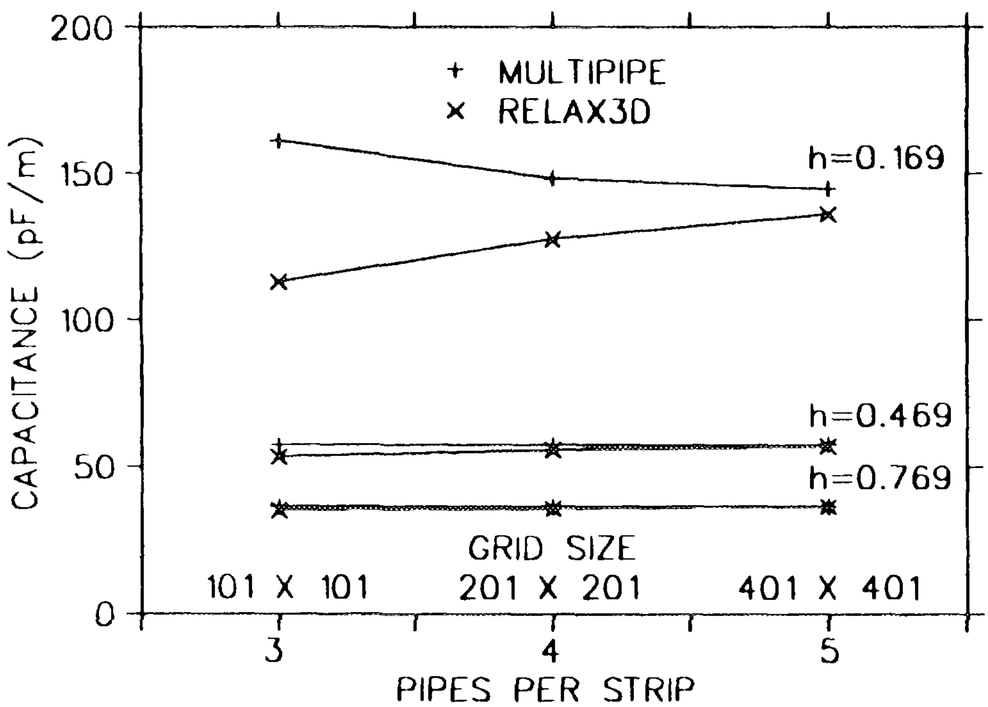

Figure 5 Four strips charged.

IMPEDANCE MEASUREMENT

A stripline monitor was constructed and tested in the lab prior to installation in 1988. An HP8753A network analyzer with an HP85046A test set was used in the time domain reflectometer mode to measure the input reflection coefficient of a strip. The downstream end of the strip and both ends of the other three strips were terminated. The network analyzer performs a frequency sweep and then mathematically transforms the result into the time domain response to a step function. Figure 6 shows the measurement for a strip to wall spacing of 0.405 in. The flat sections at the beginning and end of the trace are the 50Ω connecting cable and the downstream termination, respectively. The nearly flat valley is the stripline. Mismatches are seen at the strip ends due to boron nitride suppport structures and vacuum feedthroughs.

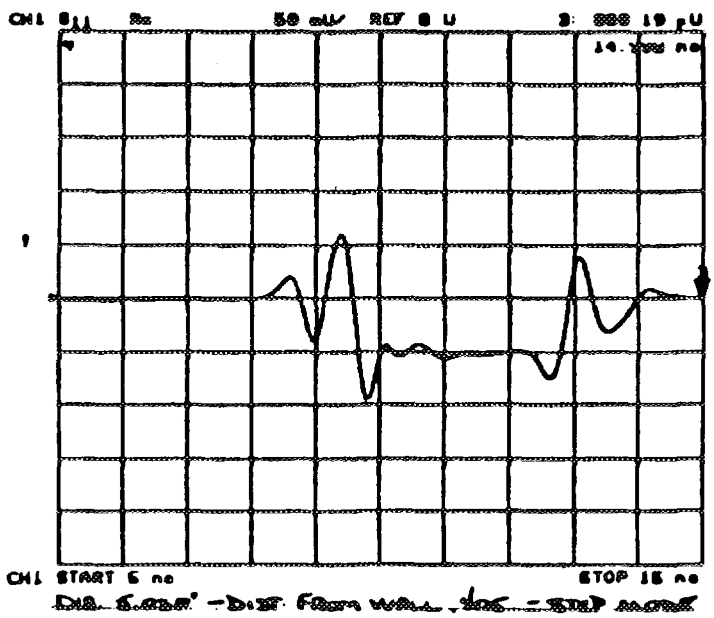

Figure 6. The reflection coefficient of a stripline measured using time domain reflectometry.

The reflection coefficient of the stripline section was recorded for five strip to wall spacings. The results were interpolated to find the spacing which resulted in a strip relection coefficient of zero, which was 0.448 in. To make the tests easier, only one set of strips with a fixed arc length of 2.012 in was used. At a strip to wall spacing of 0.448 in the strips subtended 45.9°. This would imply, to first order, a "corrected" spacing of 0.439 in for a 45° strip. Table III compares the strip spacing found using all three methods. The measured value agrees with both calculated values within the tolerance of the measurement.

Table III. The strip to wall spacing for 50 Ω impedance by various methods.

Method	Number of Strips	Strip to Wall Spacing for 50 Ω (inches)
RELAX3D	1	0.428
Multipipe	1	0.436
Raw Measurement	1	0.448
Corrected Measurement	1	0.439
RELAX3D	4	0.377
Multipipe	4	0.391

CROSS COUPLING COEFFICIENTS

The mutual capacitance C_{ij} between the *i* th and the *j* th strip can be calculated as

$$C_{ij} = \frac{\sum_{m=1}^{N_i} q_m}{V_j} \qquad (10)$$

where N_i is the number of pipes on the *i* th strip, q_m is the charge of the *m* th pipe, V_j is the potential on the *j* th strip, and the rest of the strips are grounded. The capacitances, using five pipes per strip, are shown in table IV.

Table IV. The strip mutual capacitances.

h (inches)	C_{11} (pF/m)	C_{21} (pF/m)	C_{31} (pF/m)
0.169	148.6	1.554	0.677
0.469	63.2	2.341	0.936
0.769	44.0	3.156	1.160
0.409	70.3	2.175	0.885

The coupling, k, between two strips, when they are 1/4 wavelength long and have the same capacitance to ground, can be calculated [4].

$$k = \frac{C_{ij}}{C_{jj} + C_{ij}} \qquad (11)$$

The coupling was measured using the network analyzer for h = 0.409 in., figure 7. The length of the strips is 1/4 wavelength long at the top of the first peak (164 MHz).

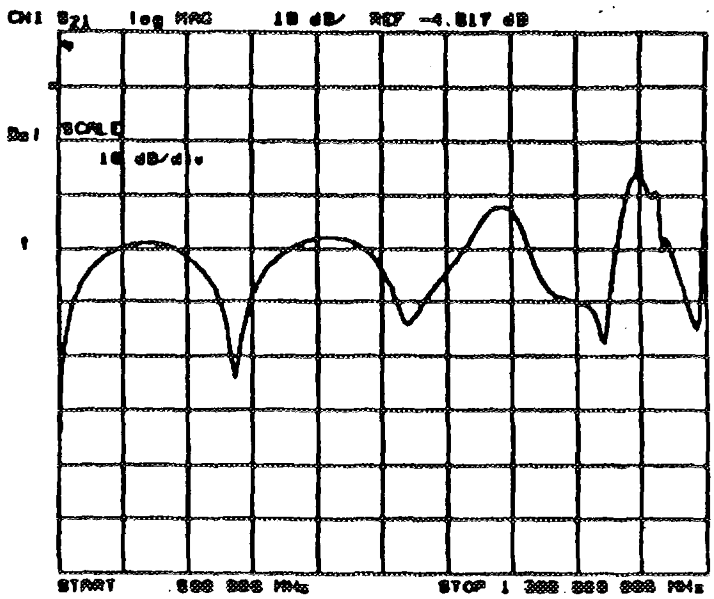

Figure 7. The coupling between adjacent strips for a wall strip to wall spacing of 0.409 in.

The cross coupling coefficients are useful for in-situ testing of the monitor. Table V compares the calculated and measured values.

Table V. The coupling coefficients of the strips.

Strip Configuration	Calculated Coupling (dB)	Measured Coupling (dB)
adjacent	-30.5	-28
opposite	-38.1	-36

ACKNOWLEDGEMENTS

We would like to thank George Mackenzie for inspiring the monitor development, Tom Ries for the mechanical design, and Richard Lee for the RELAX3D and field energy calculations.

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