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# 中国核科技报告

# CHINA NUCLEAR SCIENCE AND TECHNOLOGY REPORT

## 用多变量频域控制理论分析 HTR-10 蒸汽发生器中两回路耦合密度波不稳定性

## FLOW INSTABILITY ANALYSIS OF TWO-CIRCUIT-COUPLED DENSITY-WAVE OSCILLATIONS IN HTR-10 SG USING MULTI-VARIABLE FREQUENCY-DOMAIN CONTROL THEORY

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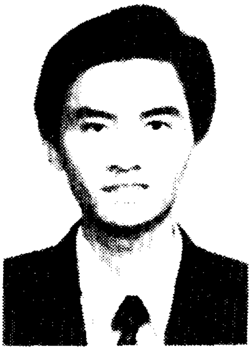

牛风雷:清华大学核能技术设计研究院硕士,1994年毕业于清华大学工程力学系流体力学专业。

NIU Fenglei: Master of Institute of Nuclear Energy Technology, Tsinghua University. Graduated from Department of Engineering Mechanics, Tsinghua University in 1994, majoring in hydrodynamics.

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# 用多变量频域控制理论分析 HTR-10 蒸汽发生器中两回路耦合密度波不稳定性

**牛风雷　厉日竹**

（清华大学核能技术设计研究院，北京）

## 摘　要

10 MW 高温气冷堆（HTR-10）的蒸汽发生器（SG）是立式、直流、螺旋管状蒸汽发生器。其工作在中压参数下，必须考虑两相流不稳定性。采用了 Zuber-Findlay 漂移流模型，借助于多变量频域法分析两相流密度波不稳定性。分析了一、二回路之间的反馈耦合作用，提出了多输入、输出传递矩阵的数学表达式，推导了多通道反馈系统模型。编制了程序 AFIHTR，并计算出 HTR-10 SG 中两回路耦合密度波振荡的稳定条件。

# Flow Instability Analysis of Two-circuit-coupled Density-wave Oscillations in HTR-10 SG Using Multi-variable Frequency-domain Control Theory

NIU Fenglei LI Rizhu

(Institute of Nuclear Energy Technology, Tsinghua University, Beijing)

## ABSTRACT

10 MW high temperature gas cooled reactor steam generator (HTR-10 SG) is once-through-vertical helical-tubing steam generator, and operates at middle pressure parameters. Two-phase flow instabilities must be paid attention in the SG. Two-phase flow density-wave instabilities are analyzed by means of multi-variable frequency-domain method based on Zuber-Findlay type of drift-flux model. The feedback coupling effect between the primary and the secondary circuit is analyzed. The mathematical expressions of multi-input to multi-output transfer matrix of the system are proposed, and the multi-channel feedback system model is deduced. A computer code AFIHTR is derived on above bases. The stability condition of two-circuit-coupled density-wave oscillations in HTR-10 SG is obtained by using this code.

# INTRODUCTION

10 MW high temperature gas cooled reactor (HTR-10) steam generator (SG), shown in Fig. 1, is the key device which connects and separates the primary and the secondary circuit of HTR-10. It consists of 30 helical tube bundle modules, each of which is composed of heat transfer tube, central pipe, fixed support structure and outer case. The coolant of HTR-10, high temperature helium, enters the SG with temperature of 700℃ and flows down along the flow path between the central pipes and the outer cases of the helical tube bundle modules. In SG the helium is cooled to the temperature of 250℃ and then flows into the helium blower. The helium is pressurized in the blower and then returns to the reactor. The water in the secondary circuit, with the temperature of 104℃ and the pressure of 6.0 MPa, enters SG from the bottom inlet of feed water connected tubes and then flows up through heat transfer tubes, which are composed of some small diameter tubes coiled in the form of a helix. In SG the water is heated into the superheated steam with the temperature of 440℃ and the pressure of 4.0 MPa. The steam finally leaves SG from the bottom outlet of steam connected tubes and goes into turbogenerator (Li Weihua. Thermal hydraulic calculation report of 10 MW high temperature gas-cooled test reactor steam generator. INET, Tsinghua University, 1996).

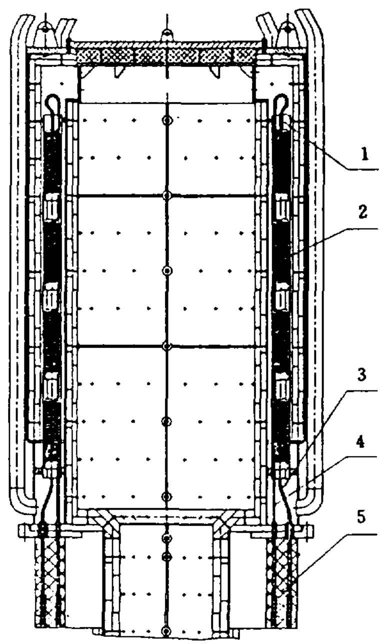


Fig. 1 Profile of the steam generator of HTR-10
1——helium flow path; 2——helical tube bundle module; 3——outlet nozzle; 4——inlet nozzle; 5——heat insulator

The steam generator operates at middle pressure parameters and the two-phase flow instability is often encountered. Density-wave oscillation is a common two-phase instability phenomenon found in nuclear reactors, steam generators as well as other industry heat exchangers. It often occurs in positive slope portion of hydrodynamic curve. It may cause flow excursions or oscillations of flow and thermal parameters. Flow oscillations may cause forced mechanical vibration of components or system control problem. Flow oscillations can also affect the critical heat flux, may induce boiling crisis and wall temperature oscillation which can eventually lead to tube failure due to thermal fatigue. Thus, two-phase flow instability affects the safety of the systems or equipments directly.

Few reports on flow instability of HTR SG have been obtained. The research work for two-phase flow instability almost focus on vertical or horizontal tubes and the research in helical-coiled tubes is much less . Several available reports to date mention a single-variable frequency-domain model (such as NUFREQ) and neglect the influences of other heat transfer tubes and external loop. The system is simplified into single channel model.

This paper discusses the flow instability of two-circuit-coupled density-wave oscillations. It is based on one-dimension thermal-equilibrium conservation equations obtained by using Zuber-Findlay type of drift-flux model. By means of flow resistance and heat transfer characteristics in helical-coiled tubes, the multi-input to multi-output transfer matrix is deduced using linearized perturbation theory and Laplace transformation. System stability is judged by Nyquist stability criterion of multi-variable frequency-domain control theory.

# 1 MATHEMATICAL MODEL

## 1.1 Fundamental conservation equations

For analyzing conveniently, the following assumptions are made in the development of the model[1].

1) The single-phase flow is incompressible and the two-phase flow is under thermodynamic equilibrium.

2) All semi-empirical formulae of flow resistance and heat transfer under steady state condition are still suitable under dynamic state condition.

3) The single-phase fluid is assumed to be ideal fluid.

4) The potential energy and the kinetic energy in energy conservation equation are neglected.

Taking account of the above assumptions, the fundamental conservation equations are expressed as follows[1,2].

1) Single-phase section

Mass equation:

$$\frac{\partial V}{\partial x} = 0$$

Energy equation:

$$\rho\left(\frac{\partial h}{\partial t}+V\frac{\partial h}{\partial x}\right)=\frac{qP_{W}}{A_{x}}$$

Momentum equation:

$$-\,\mathrm{d}p=\rho\left(\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}\right)\mathrm{d}x+\frac{f\rho V^{2}}{2d}\mathrm{d}x+\rho g\mathrm{d}z+\sum_{m}k_{m}\frac{\rho V^{2}}{2}\delta(x-x_{m})$$

2) Two-phase section (Zuber-Findlay type of drift-flux model)

Mass equation:

$$\frac{\partial}{\partial t}[(1-\alpha)\rho_{f}]+\frac{\partial}{\partial x}[(1-\alpha)\rho_{f}V_{f}]=-\Gamma_{g}$$

$$\frac{\partial}{\partial t}(\alpha\rho_{g})+\frac{\partial}{\partial x}(\alpha\rho_{g}V_{g})=\Gamma_{g}$$

Energy equation:

$$\frac{\partial}{\partial t}[(1-\alpha)\rho_{f}h_{f}+\alpha\rho_{g}h_{g}]+\frac{\partial}{\partial x}[(1-\alpha)\rho_{f}h_{f}V_{f}+\alpha\rho_{g}h_{g}V_{g}]=\frac{qP_{W}}{A_{x}}$$

Momentum equation:

$$-\,\mathrm{d}p=\left[\frac{\partial}{\partial t}(\rho_{tp}V_{tp})+\frac{\partial}{\partial x}(\rho_{tp}V_{tp}^{2})\right]\mathrm{d}x+f\Phi_{f0}^{2}\frac{(\rho_{tp}V_{tp})^{2}}{2\rho_{f}d}\mathrm{d}x+\rho_{tp}g\mathrm{d}z+$$
$$\frac{\partial}{\partial x}\left\{\left(\frac{\rho_{f}-\rho_{tp}}{\rho_{tp}-\rho_{g}}\right)\frac{\rho_{f}\rho_{g}}{\rho_{tp}}[V_{gj}+(C_{0}-1)j]^{2}\right\}\mathrm{d}x+\sum_{m}k_{m}\frac{(\rho_{tp}V_{tp})^{2}}{2\rho_{f}}\Phi(x)\delta(x-x_{m})$$

Zuber type of void fraction transfer equation:

$$\frac{\partial\alpha}{\partial t}+C_{k}\frac{\partial\alpha}{\partial x}=\left(\frac{\rho_{f}}{\rho_{f}-\rho_{g}}-C_{0}\alpha\right)\Omega \qquad (1)$$

where kinematic-wave velocity

$$C_{k}=C_{0}j+V_{gj}+\alpha\left(\frac{\mathrm{d}V_{gj}}{\mathrm{d}\alpha}+j\frac{\mathrm{d}C_{0}}{\mathrm{d}\alpha}\right)$$

characterististic frequency of phase change

$$\Omega = \frac{\Gamma_g(\rho_f - \rho_g)}{\rho_f\rho_g}$$

steam generation rate $\Gamma_g = \dfrac{qP_W}{A_x h_{fg}}$

drift velocity $V_{gj} = V_g - C_0 j$

$C_0$ is the void distribution parameter; $h$ is the specific enthalpy; $q$ is the heat flux density; $\alpha$ is the void fraction; f is the Darcy-Weisbach friction coefficient; $\Phi_{f0}^2$ is the two-phase friction multiplier; $\Phi$ is the two-phase local resistance multiplier; $V_{tp}$ is the two-phase mixture velocity; the other variables are explained in reference [1].

The two-phase mixture density is

$$\rho_{tp} = (1 - \alpha)\rho_f + \alpha\rho_g \quad (2)$$

According to Eqs. (1) and (2), the following expression is obtained

$$\frac{\partial D_k}{\partial t} + C_k\frac{\partial D_k}{\partial x} = - D_k\Omega_{tp}$$

where $$D_k = C_0\rho_{tp} + (1 - C_0)\rho_f$$

$$\Omega_{tp} = \frac{C_0 q P_W(\rho_f - \rho_g)}{A_x h_{fg}\rho_f\rho_g}$$

### 1.2 Two-circuit-coupled transfer matrix

In the present analysis model, the channel is divided into a number of nodes. The steady-state thermal-hydraulic parameters are considered as being constant in each node. Using linearized perturbation theory and Laplace transformation, the system transfer matrix can be described by[1]

1) Primary circuit

$$\delta X_{1,out}^i(s) = H_1^i(s) \cdot \delta X_{1,in}^i(s) \quad (3)$$

2) Secondary circuit

Single-phase section:

$$\delta X_{2,sp,out}^i(s) = H_{2,sp}^i(s) \cdot \delta X_{2,sp,in}^i(s) \quad (4)$$

Two-phase section:

$$\delta X^i_{2,\mathrm{tp,out}}(s) = H^i_{2,\mathrm{tp}}(s) \cdot \delta X^i_{2,\mathrm{tp,in}}(s) \tag{5}$$

where

$$\delta X^i_{1,\mathrm{out}}(s) = [\delta T_{1,i-1}(s), \delta W_{1,i-1}(s), \delta q_i(s)]^T$$

$$\delta X^i_{1,\mathrm{in}}(s) = [\delta T_{1,i}(s), \delta W_{1,i}(s), \delta R_i(s)]^T$$

$$\delta X^i_{2,\mathrm{sp,out}}(s) = [\delta W_{2,i}(s), \delta \Delta p_{2,i}(s), \delta h_{2,i}(s), \delta R_i(s)]^T$$

$$\delta X^i_{2,\mathrm{sp,in}}(s) = [\delta W_{2,i-1}(s), \delta \Delta p_{2,i-1}(s), \delta h_{2,i-1}(s), \delta q_i(s)]^T$$

$$\delta X^i_{2,\mathrm{tp,out}}(s) = [\delta D_{k,i}(s), \delta C_{k,i}(s), \delta W_{2,i}(s), \delta V_{2,i}(s), \delta h_{2,i}(s), \delta \Delta p_{2,i}(s), \delta R_i(s)]^T$$

$$\delta X^i_{2,\mathrm{tp,in}}(s) = [\delta D_{k,i-1}(s), \delta C_{k,i-1}(s), \delta W_{2,i-1}(s), \delta V_{2,i-1}(s), \delta h_{2,i-1}(s), \delta \Delta p_{2,i-1}(s), \delta \Omega_{\mathrm{tp},i}(s)]^T$$

$H^i_1(s)$, $H^i_{2,\mathrm{sp}}(s)$, $H^i_{2,\mathrm{tp}}(s)$ are two-circuit-coupled transfer matrixes; $T$ is the fluid temperature; $W$ is mass velocity; $\Delta p$ is pressure drop; $\delta R_i(s)$ is a function of $\delta W_{2,i}(s)$, $\delta p_{2,i}(s)$, $\delta x_{e,i}(s)$ and $\delta T_{2,i}(s)$.

The two-circuit-coupled transfer matrix model is shown in Fig. 2.

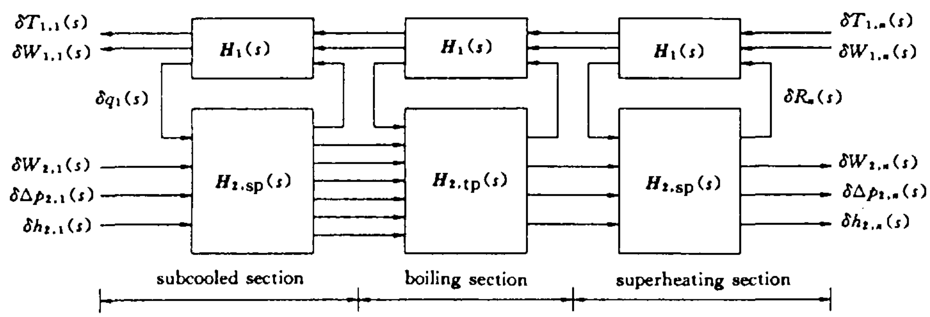


Fig. 2 Block diagram of two-circuit-coupled transfer matrix model

## 2 ANALYSIS METHOD

### 2.1 Multi-channel feedback system model

In the cycle system consisted of No. i helical tube and external loop, its overall pressure drop perturbation is

$$\delta\Delta p_{loop,i}(s) = \delta\Delta p_{l,i}(s) + \delta\Delta p_{tp,i}(s) + \delta\Delta p_{v,i}(s) + \delta\Delta p_{t}(s)$$

where subcooled pressure drop perturbation

$$\delta\Delta p_{l,i}(s) = Q_{l,i}(s) \cdot \delta W_{in,i}(s)$$

boiling pressure drop perturbation

$$\delta\Delta p_{tp,i}(s) = Q_{tp,i}(s) \cdot \delta W_{in,i}(s)$$

superheating pressure drop perturbation

$$\delta\Delta p_{v,i}(s) = Q_{v,i}(s) \cdot \delta W_{in,i}(s)$$

external pressure drop perturbation

$$\delta\Delta p_{t}(s) = Q_{t}(s) \cdot \sum_{j=1}^{n} \delta W_{in,j}(s)$$

$Q_{l,i}(s)$, $Q_{tp,i}(s)$, $Q_{v,i}(s)$ can be derived from Eqs. (4) and (5); $Q_t$ $(s)$ is obtained from experiment.

Thus,

$$\delta W_{in,i}(s) = Q_{l,i}^{-1}(s) \cdot \delta\Delta p_{l,i}(s) \tag{6}$$

$$\delta\Delta p_{l,i}(s) = \delta\Delta p_{loop,i}(s) - Q_{tp,i}(s) \cdot \delta W_{in,i}(s)$$

$$- Q_{v,i}(s) \cdot \delta W_{in,i}(s) - Q_{t}(s) \cdot \sum_{j=1}^{n} \delta W_{in,j}(s) \tag{7}$$

The matrix form of Eqs. (6) and (7) is

$$\delta \boldsymbol{W}_{in}(s) = \boldsymbol{Q}_{l}^{-1}(s) \cdot \delta\Delta \boldsymbol{P}_{l}(s)$$

$$\delta\Delta \boldsymbol{P}_{l}(s) = \delta\Delta \boldsymbol{P}_{loop}(s) - \boldsymbol{T}(s) \cdot \delta \boldsymbol{W}_{in}(s)$$

where

$$\delta W_{in}(s) = [\delta W_{in,1}(s), \delta W_{in,2}(s), \cdots, \delta W_{in,n}(s)]^T$$

$$\delta\Delta p_{l}(s) = [\delta\Delta p_{l,1}(s), \delta\Delta p_{l,2}(s), \cdots, \delta\Delta p_{l,n}(s)]^T$$

$$\delta\Delta p_{loop}(s) = [\delta\Delta p_{loop,1}(s), \delta\Delta p_{loop,2}(s), \cdots, \delta\Delta p_{loop,n}(s)]^T$$

$$Q_1^{-1}(s) = \begin{bmatrix} Q_{1,1}^{-1}(s) & & & \\ & Q_{1,2}^{-1}(s) & & \\ & & \ddots & \\ & & & Q_{1,n}^{-1}(s) \end{bmatrix}$$

$$T(s) = \begin{bmatrix} Q_{tp,1}(s) + Q_{v,1}(s) + Q_t(s) & Q_t(s) & \cdots & Q_t(s) \\ Q_t(s) & Q_{tp,2}(s) + Q_{v,2}(s) + Q_t(s) & \cdots & Q_t(s) \\ \cdots & \cdots & \cdots & \cdots \\ Q_t(s) & Q_t(s) & \cdots & Q_{tp,n}(s) + Q_{v,n}(s) + Q_t(s) \end{bmatrix}$$

The structure of the closed-loop system is depicted in Fig. 3.

**2.2 Stability criterion**

According to Fig. 3, the return difference matrix of feedback system is[3]

$$D(s) = I + T(s) \cdot Q_1^{-1}(s)$$

and the closed-loop characteristic polynomial is given by

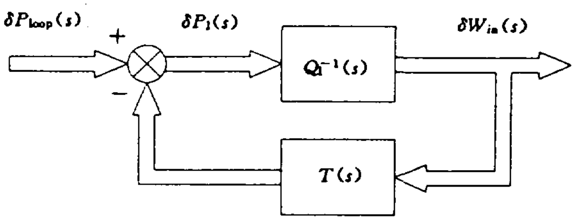

Fig. 3 State feedback control system

$$\rho_c(s) = \rho_o(s) \cdot \det D(s)$$

where $\rho_o(s)$ is the opened-loop characteristic polynomial.

According to the multi-variable frequency-domain control theory, the Nyquist stability criterion may be stated:

The closed-loop system is stable if and only if the net number of clockwise encirclements of the point $s=0+0j$ by the Nyquist diagram of $\det D(s)$ plus the number of zeros of $\rho_o(s)$ in the right half-plane of the complex plane is zero. That is

$$\mathrm{enc} D(s) = - n_o$$

Here, according to Eqs. (3), (4) and (5), $n_o=0$.

## 3 RESULTS AND DISCUSSION

**3.1 Effect of operating pressure on density-wave oscillation**

It is notable that system pressure affects density-wave oscillation, as shown in Fig. 4. Increasing system pressure makes decrease of density difference between vapor phase and liquid phase, and tends to stabilize the system. In the same exit quality, pressure drop perturbation caused by operating parameters at high pressure is less than that at low pressure. So it is not enough to produce sustained flow rate oscillation.

### 3.2 Effect of mass velocity on density-wave oscillation

Fig. 5 illustrates that the increase of mass velocity makes the increase of system stability. For fixed heat flux and inlet subcooling, increasing the mass velocity can shorten the length of the two-phase section, which in turn leads to a lower overall pressure drop for the two-phase portion. Eventually, the increase of mass velocity tends to stabilize the system at a certain heat flux[1,4].

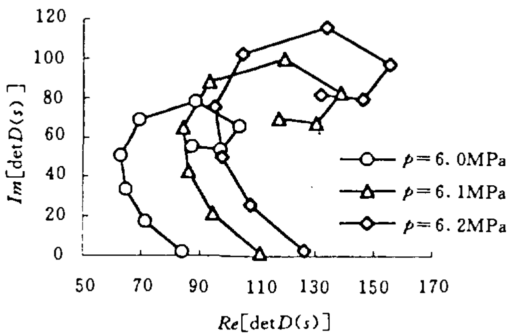


Fig. 4 Effect of operating pressure on density-wave oscillation

### 3.3 Effect of inlet resistance coefficient on density-wave oscillation

The increase of inlet resistance coefficient makes large increase of single-phase resistance and it can produce a damping effect on oscillation. The comparisons are shown in Fig. 6. Therefore, increasing the inlet resistance coefficient can serve as a means of suppressing density-wave instabilities[1,4].

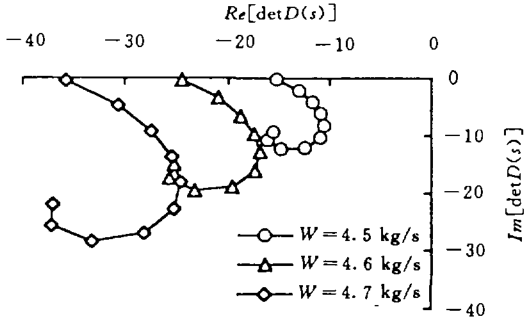


Fig. 5 Effect of mass velocity on density-wave oscillation

## 4 CONCLUSIONS

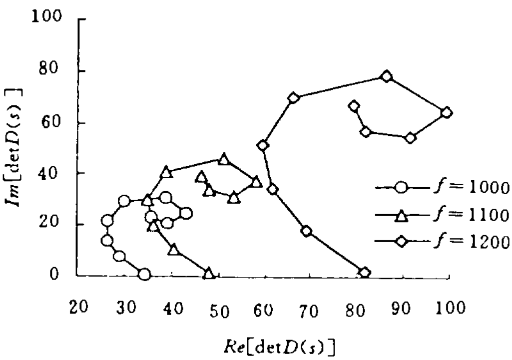


Fig. 6 Effect of inlet resistance coefficient on density-wave oscillation

The multi-variable frequency-domain analytical model has been applied widely in flow instability analysis of density-wave oscillations. Studies by this method have indicated that the density-wave oscillations in HTR-10 SG are strongly dependent on the heat flux variation, mass velocity, inlet flow restriction, subcooling, and system pressure. Calculated results indicate that the flow of HTR-10 SG is stable. Some experimental results that have already been obtained are found to be similar to the results of the theoretical analysis. Further experiments are currently in progress.

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