Paper presented at International Conference on High Energy Accelerators
HEACC'92, Hamburg, July 20 - 25

TRI-PP-92-117
Nov 1992

A Multistrip Beam Profile High Order Moment Monitor

Yan Yin
TRIUMF, 4004 Wesbrook Mall
Vancouver, BC, Canada, V6T 2A3

Abstract

A design is given of a multistrip wall current monitor which can be used to measure transverse high order moments of the beam. Bench test results are presented and analyzed. The device can be applied to continuously and non-destructively measure beam properties like emittance or momentum spread.

1 Introduction

Studies have been made at various labs[1][2] to use beam position monitors as non-intercepting emittance or quad--rupole moment measuring devices.

When a particle beam passes through a conducting pipe, an image current will be produced, which is not only determined by the beam position but also by the density distribution of the beam.

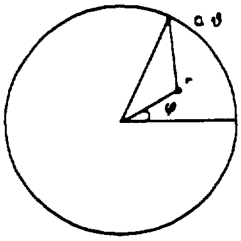

Fig.1: Wall current due to a line current in a conducting cylinder.

For a delta function line current I, at point (r, ϕ), the image current density, J, on a conducting cylinder of radius a at point (a, θ) (see Fig.1) is given as[1]:

$$J_{image}(r, \phi, a, \theta) = \frac{I}{2\pi a}\frac{(a^2 - r^2)}{a^2 + r^2 - 2ar\cos(\theta - \phi)}. \quad (1)$$

Expanding in powers of r/a gives :

$$J_{image}(r, \phi, a, \theta) = \frac{I}{2\pi a}\left[1 + 2\sum_{k=1}^{\infty}(\frac{r}{a})^k \cos k(\theta - \phi)\right]. \quad (2)$$

The above expansion represents a series of azimuthal components. The wavelength of the dipole mode is the circumference of the beam pipe $2\pi a$. If we stretch the circumference of the pipe into a straight line, this expansion can be seen very clearly.

This implies that a delta function line current, when it is off centre, will induce high order components in the wall current distribution.

Since we would like to measure emittance, we are only interested in the moments around the centroid. According to the definition, the quadrupole moment around the electric centroid is:

$$\frac{\int \rho(r, \phi) r^2 \cos 2\phi \, r dr d\phi}{\int \rho(r, \phi) r dr d\phi} \quad (3)$$

When the beam electrical centroid is in the centre of the beam pipe, each component in the wall current distribution has similar expression as the moment of each order, but with a factor of $2/a^k$. When the beam has a transverse distribution, the wall current will be the integral over all the charges. When the beam is off centre, this gives rise to quadrupole, sextupole etc. components in the wall current distribution, which do not simply reflect the beam profile high order moments around the centroid. In order to measure the quadrupole moment, one has to eliminate the dipole and the other high order mode contaminants in the wall current distribution caused by beam centroid position shift. But by simply adding or subtracting the signals from 4 BPM pick-ups, it is not possible to do so.[1].

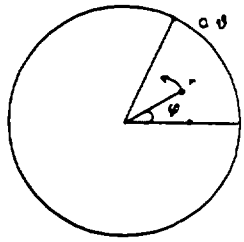

Fig.2: Beam size, centroid position shift and wall current distribution.

However, when the beam size is small enough compared with the size of the beam pipe, one can assume the profile information (i.e the high order moments) can be obtained by subtracting the wall current distribution contributed by a delta function line current with a total charge equal to the beam at position $\overline{r}, \overline{\phi}$, at the electrical centre of the beam. A calculation has been done to estimate the accuracy of this assumption. To simplify the problem, two delta function line charges are considered, see Fig.2. The largest distance between them is 1/5 of the beam pipe diameter. One line charge is placed on the x axis, the other is turned around the origin. By moving the first one along the x axis, both the distance

VOL 28 № 24

between the particles and the distance from the electrical centre to the monitor mechanical centre change, therefore covering the whole area of interest.

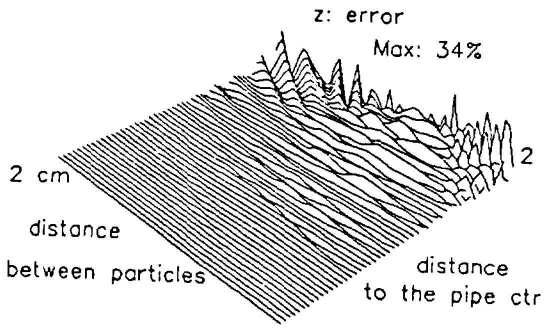

Fig.3 The accuracy of the assumption.

The calculation shows the error is related to the distance between the two particles and the electrical centroid's offset from the mechanical centre. In the extreme case, in a 10cm pipe, with two line charges 2 cm apart, and offset 2 cm, the error is 30%. But when the two charges are 2 cm apart and their centroid is only 5 mm from the origin, the error is only one percent, see Fig.3. The figure also shows that when beam position shift is smaller than the beam size, the wall current high order components can be determined to very good accuracy. The result is valid for a real beam, because it can be considered as a collection of many delta function line charges.

It is obvious to see that components of order equal or higher than quadrupole mode, are contributed to by both how far the profile deviates from circular and the position shift; but the dipole mode can only be induced by the beam electrical centroid shift from the centre of the pipe.

If more strips are used, more information from the wall current distribution can be obtained. By using FFT (Fast Fourier Transform) which deals with the discrete signals, and normalizing each component with the first component, i.e. DC component, each mode will have a simple coefficient of $2(\frac{\bar{r}}{a})^k$. The dipole mode has the coefficient of $2(\frac{\bar{r}}{a})$. Since a is known, $\bar{r}$ can be easily obtained. From the peak position of the dipole mode, $\bar{\phi}$ can also be determined. Therefore, the position of the beam electrical centroid is determined in polar coordinates.

From equation 1, using the above $\bar{r}$ and $\bar{\phi}$, the high order components of the wall current induced by the delta function line charge can be obtained, and therefore can be subtracted from the measured signals at those pick-ups. The part of the wall current distribution which is purely due to the beam transverse distribution can be deduced from the residual. And with FFT, the amplitudes and spatial phase angle of each high order component can be easily determined.

For a Gaussian beam, the beam distribution is known and the relationship between the amplitudes of the second order component and the quadrupole moment is calculable[1]. For a non-Gaussian beam, the relation between the shape of the beam cross section and the quadrupole moment may not be calculable. It might not be expressed with a simple function. This device will be a very useful tool to solve the problem.

2 A multistrip monitor and its bench test

A monitor with 16 strips has been made and bench tested. All 16 strips are cut into two sections and bridged with 10 ohm resistors. For the bench test the signals were first multiplexed by relays, then amplified and peak detected. Antennas with different shapes were used to simulate beam with different transverse cross sectional shapes. The antenna can be moved precisely by an X-Y table to simulate the beam position change.

Because the strips and the resistors won't be exactly same, a calibration is needed. By placing a round pipe antenna in the centre of the monitor, measuring the signal at all strips, and then normalizing by the average signal value of all strips, calibration coefficients were obtained for each strip.

Fig.4 and Fig.5 show the signals at 16 strips for a round pipe placed 1mm off centre, and an elliptical pipe placed 2 mm off centre. In order to show clearly the modes, the signals are extended into 3 periods. From FFT, the distance from the electrical centre of the antenna can be obtained quite accurately, as $\bar{r} = 1.098$ and 2.012 mm, and the direction, i.e. angle $\bar{\phi}$, can be obtained from the phase of the first order component in the FFT.

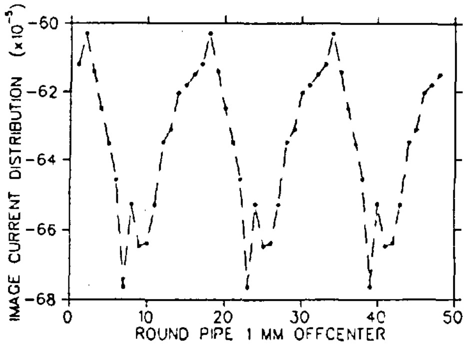

Fig.4 Wall current for a round antenna offset 1 mm.

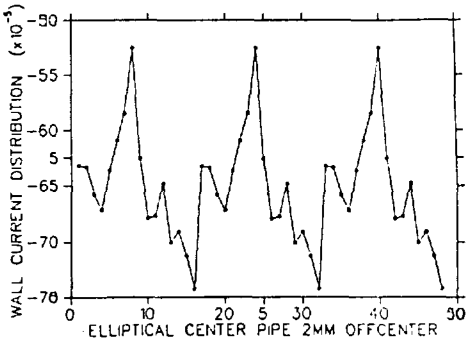

Fig.5 Wall current for an elliptical antenna offset 2 mm.

After subtracting the high order components caused by the beam centroid shift, the spectrum has only quadru pole mode and the other high modes disappear. Fig.6 shows the FFT spectrum for the filtered signals.

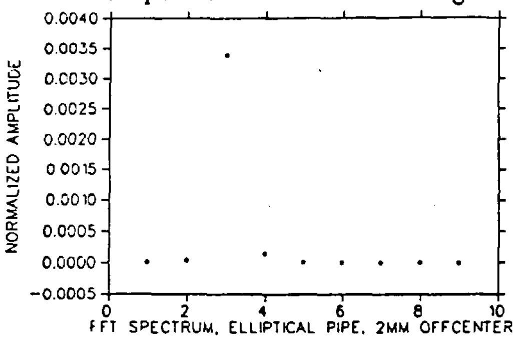

Fig.6 FFT spectrum of the filtered signal.

The quadrupole moment of the elliptical antenna can be calculated with formular 3. Here the size of the antenna is 8.6mm x 2.9mm. Density ρ is constant for our bench test signal. Calculation shows that the quadrupole moment is 4.09, the measured data is 0.0034. Applying the factor of $2/a^2$, it will be 4.25. Considering the antenna is not really elliptical, the result and the theoretical calculation are rather close.

2.1 The number of strips

The multistrip monitor (for the bench test) has 16 strips for the purpose of observing higher moments. The preliminary test only measured an elliptical antenna. The number of strips needed is found from the sampling theorem. Although the sampling theorem states that 2 samples per cycle will be enough to reconstruct the signal, nevertheless at the Nyquist frequency it is possible that the samples coincide with zeros of the sinusoid and the function is incorrectly assigned zero amplitude. Thus we must have more than two samples per cycle for the highest frequency present[5]. To make an FFT 2^n samples will be more convenient. Hence to measure the dipole moment, i.e. position shift, we need 4 pick-ups, to measure quadrupole, we need at least 6 and 8 is preferred. To observe even higher moments, one needs more strips.

2.2 Sensitivity

The sensitivity of the monitor has two meanings: the sensitivity to the beam current and the resolution to the high order moment. The signal that a pick-up can sense not only depends on the portion of the wall current flowing through it, but also on the frequency response of the pick-ups. The multistrip monitor is a wall current monitor, i.e a wide band monitor. By loading with ferrite outside the pipe, the low frequency response can easily be improved down to few kHz. The high frequency response usually extends to above 1 GHz, and by using smaller resistors, it can be much higher[6]. This also gives the possibility to measure the beam transverse moments at a selected cross section within the bunch length, as long as all pick-ups sense the beam simultaneously.

Since the sum of the wall charge is equal to the charge on the beam, for the cylindrical pipe, it is easy to find the portion of wall current picked up by the strips. Consider again the 2 delta function line charge system. Suppose its centroid is on the centre of the pipe, the distance from one delta function to the centre is r. Then the peak signal on the wall , normalized by the $\frac{I}{2\pi a}$, is $2 + 4\sum_{k=1}^{\infty}(\frac{r}{a})^{2k}$. For a beam of two line charges, if distance between them is 10% of the pipe ($r/a = 0.1$), for a 10 cm pipe, this will be equivalent to a beam of 2 cm x 1cm, the quadrupole amplitude will be 2% of the total beam signal. This is not a problem since the signal is still well within the accuracy of our measurement for a mV signal.

3 Conclusion

The above discussion shows that from the wall current distribution, when beam electrical centroid position shift is smaller than beam size, the transverse high order components in the wall current can be obtained to good accuracy by subtracting the distribution caused by a delta function line with the total charges at the position of the centroid. The quadrupole and high order moments can be deduced from the FFT. This should be very useful for measuring the beam emittance or energy spread[2] non-destructively and continuously.

4 Acknowledgements

The author gives her thanks to R.Miller, D.Witkover for their suggestions on the subject, and also thanks to S.Koscielniak for a good discussions.

References

[1] R.Miller et al:*Nonintercepting Emittance Monitor*,XII High Energy Accelerator Conference, 1983, p.602

[2] J.C.Sheppard: *Implementation of Nonintercepting Energy Spread Monitors*, Proceedings of the Particle Accelerator Conference, Washington , 1987.p.757

[3] G.Nassibian:*The Measurement of the Multipole Coefficients of a Cylindrical Charge Distribution*,CERN, SI/note EL/70-13

[4] G.Nassibian:*The Basic Limits to the Measurement of the Charge Distributions in a Cylindrical Beam by Means of Electrostatic Induction Electrodes*, CERN, SI/note EL/71-2

[5] R.W.Hamming: *Digital Filters* Prentice Hall, Englewood Cliffs, New Jersey

[6] Robert C.Webber: *Measurement techniques Including design of a Wall current Monitor*, AIP Conference Proceedings 212, Upton, NY (1989)