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# EXPERIMENTAL TEST OF BELL'S INEQUALITIES USING ANGULAR CORRELATION OF COMPTON-SCATTERED ANNIHILATION PHOTONS

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## Introduction

Bell's inequalities apply to any correlated measurement on an ensemble of two correlated systems, for which the measurement taken separately appear random. For instance, quantum mechanics predicts strong correlations of measurements of linear polarizations performed on two photons belonging to the same pair. In 1935 Einstein, Podolsky, and Rosen (EPR) proposed that quantum mechanics was incomplete, in an effort to rescue locality [1]. In order to account for the observed correlations, supplementary parameter (common property of both members of the pair) was proposed in addition to the state vector which is the same for all emitted pairs. This natural supplement, called hidden variable leads to the conclusion that quantum mechanics is not a complete description of physical reality.

Assuming existence of hidden variables Bell showed that the expected correlations coefficients for the joint measurements taken at four possible sets of orientations of measuring instruments cannot take any set of values, but they are restricted by the so-called Bell's inequalities which state that the combination of four correlation coefficients S is between -2 and +2 for any hidden variable theory. Bell proved that any local hidden variable (LHV) theory based on the concepts of locality and reality would be inconsistent with some predictions of quantum mechanics [2]. In the experiment of EPR type measuring correlation of polarization of pairs of photons there exist a set of orientations for which the quantity S predicted by quantum mechanics violates Bell's inequality reaching the value of $2\sqrt{2}$.

When physicists realized the wide generality of Bell's theorem, they performed many experiments in order to test the inequalities derived for LHV models. Already on the road to Bell's Theorem the Wu-Shaknov [3] experiment was performed and identified by Bohm and Aharonov [4], as the first experimental refutation of the Schrodinger-Furry hypothesis for the EPR paradox. The first experiment by Wu-Shaknov was based on pairs of $\gamma$ -rays produced in the annihilation of positronium. The first measurements gave contradictory results, but in 1975 clear agreement with quantum mechanics was established [5].

Thanks to the progress in lasers, efficient source of pairs of visible photons was build. The pairs of EPR photons produced in radiative cascade in $^{40}$Ca were employed in the experiments of Clauser et al [6]. All early, above mentioned experiments, used only one channel polarizers, so the comparison of the experimental results with Bell's inequalities was indirect and based on supplementary assumptions. However all these tests had given convincing indications in favour of quantum mechanics and they opened the way to second generation experiments of Alain Aspect et al. [7]. In order to perform assumption of locality in the optical measurements of correlation coefficients of polarization of photon pairs, the result of a measurement by a polarizer cannot be directly influenced by the orientation of the other, remotely located. In order to check Einstein's causality one should consider experiments in which the settings of the polarizers are changed randomly in time which is short compared to the time of flight of photon at a distance between polarizers. This condition was fulfilled by Aspect et al. in the set-up in which the settings were changed during the flight of the photons, by a switch which was able to redirect the photon towards one of two polarizers in two different orientations.

## The experiment and results

Though ideal polarization analysers like polaroids or birefringent crystals do not exist for high energy $\gamma$ rays, Compton scattering offers to measure the relative polarization of $\gamma$-rays from two-quantum annihilation of positrons. The scattering of one photon is performing a partial analysis of the polarization of the other photon. One might expect that finding Compton scattered photon at certain azimuthal angle corresponds to finding the linear polarization at the perpendicular angle. Thus, in the case of two annihilation photons, one might guess that the Compton scattered quanta would tend to scatter in perpendicular directions.

The assumption of locality in the optical measurements of polarization of photon pairs, cannot be easily realized because the settings of the polaryzers cannot be changed randomly in short time. The Compton scattering, vice-versa gives unique opportunity to set a scheme in which several two-channel polarymeters can be adjusted to measure independently polarizations of two-photon state (EPR state). The relative linear polarization of $\gamma$-rays from two-quantum annihilation of positrons was measured by Compton scattering. The experimental arrangement used to measure the relative linear polarization of $\gamma$ -rays in order to investigate the Bell's inequalities is shown schematically in Fig.1. Positrons were emitted by $^{22}$Na radioactive source of activity A = 5 mCi (170 MBq). The source was placed in a cylindrical container made of copper which slid into rectangular hole, to fix it in the lead collimator.

The collimator of 30 cm length has cylindrical holes of 2 cm diameter. The positrons annihilate in the Cu stopper placed 1.5 mm from the source. The annihilation photons are emitted in opposite directions along the symmetry axis which is selected by lead collimator. Both annihilation photons are scattered by cylindrical plastic scintillator of dimensions $\Phi$ = 3 cm x 5 cm, placed symmetrically at the distance of 18.5 cm from the source (see Fig.2). The dimensions of the scatterer were limited in order to minimize the chance of multiple Compton scattering.

The total energy resolution of the scatterer was measured to be 89 keV for registration of electrons of energy 236 keV, corresponding to annihilation photons scattered for 82°. The scattered photons are registered by eight $BaF_2$ scintillators set on both sides at polar angle 82°, at the distance from the scattering axis of 35.8 cm forming set of four polarizers at selected orientations $\vec{a},\vec{a}'$, and $\vec{b},\vec{b}'$. Each pair of $\gamma$-detectors registering Compton scattered $\gamma$-rays at polar angle 82° and relative azimuthal scattering angles of 90°, establish one two-channel polarymeter. For each polarymeter assembly consisting of scatterer S and two analysing detectors $D_{\parallel}$, $D_{\perp}$ there are only two possible outcomes: coincidence count in S - $D_{\parallel}$ and coincidence count S - $D_{\perp}$.

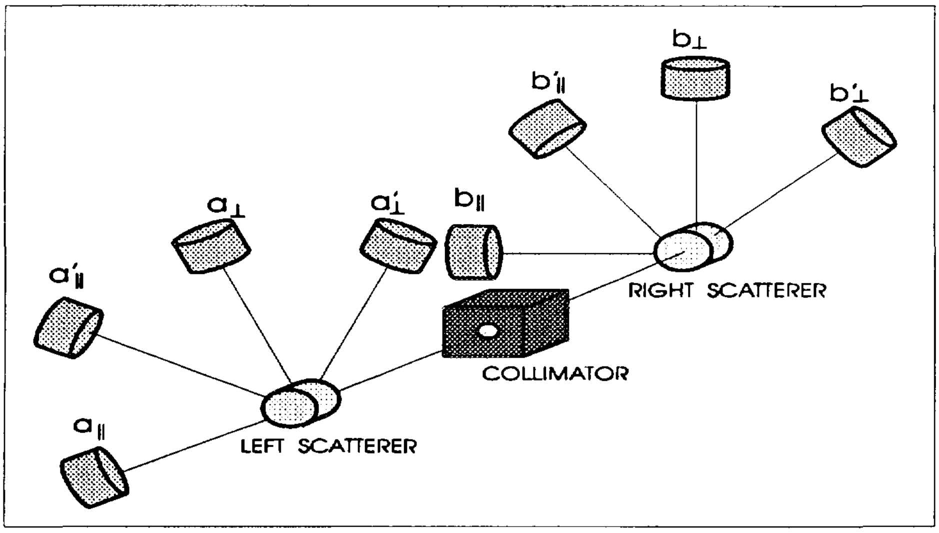

Fig. 1 Schematic diagram of scattering geometry for the experiment of the test of Bell's inequality violation.

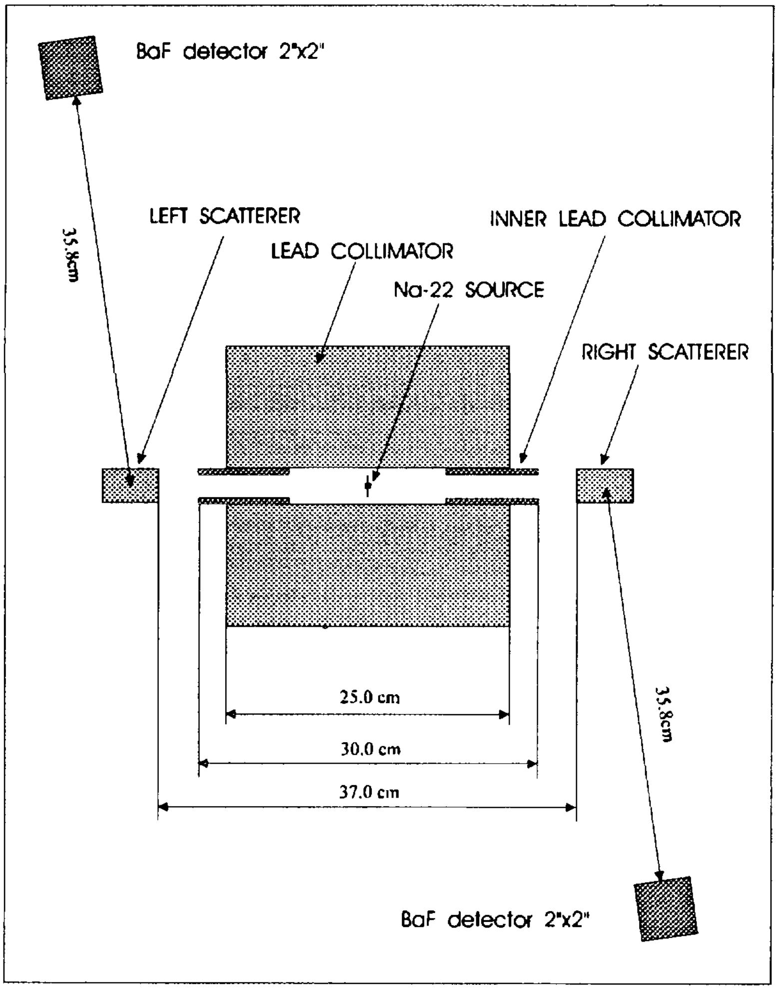

Fig. 2 Schematic view, to scale, of the experimental arrangement. The length and diameter of the collimator is 30 cm and 2cm respectively. The distance between the source and face of the scatterer is 18.5 cm. The scattered photons are registered by $BaF_2$ scintillators set at polar angle $82^{o}$, at the distance from the scattering axis of 35.8 cm.

The polarization measurements then become similar to Stern-Gerlach measurements for spin 1/2 particles except for analysing power of our polarymeters A < 1. On the other hand, in our set-up the polarization analysers are kept at two fixed orientations $\vec{a}, \vec{a}'$ on one side, and $\vec{b}, \vec{b}'$ on the other side, what allows to determine two different orientations of polarization. Four identical polarymeters adjusted at orientations $\vec{a}, \vec{a}'$, and $\vec{b}, \vec{b}'$ measure polarization of both annihilation photons emitted in opposite direction. The polarymeter consists of scatterer and two analysing detectors placed at relative azimuthal angles of $90^{o}$. In our set-up action-at-a-distance in the relativistic sense is precluded, since both photon impinging scatterers have choice to demonstrate various polarizations until they impinge scatterer (first photon have choice to demonstrate polarization $\vec{a}$ or $\vec{a}'$, but the second photon can demonstrate polarization $\vec{b}$ or $\vec{b}'$). It is easy to imagine that instrumental mechanism in producing highly correlated coincidence events is excluded. To test locality condition, we do not require to change the orientations $\vec{a}$ and $\vec{a}'$ of the polarymeters while the correlated photons are in flight.

In order to test the Bell's inequalities we consider to measure the combination S of four possible correlation coefficients, in one setting of four polarymeters. The polarymeters at adjusted orientations (a,b), (a,b'), (a',b) and (a',b') will measure the corresponding correlation coefficients $E(\vec{a},\vec{b})$, $E(\vec{a},\vec{b}')$, $E(\vec{a}',\vec{b})$ and $E(\vec{a}',\vec{b}')$. Each correlation coefficient can be determined as:

$$E(\vec{a},\vec{b}) = \frac{R(\vec{a}_{\parallel},\vec{b}_{\parallel})+R(\vec{a}_{\perp},\vec{b}_{\perp})-R(\vec{a}_{\parallel},\vec{b}_{\perp})-R(\vec{a}_{\perp},\vec{b}_{\parallel})}{R(\vec{a}_{\parallel},\vec{b}_{\parallel})+R(\vec{a}_{\perp},\vec{b}_{\perp})+R(\vec{a}_{\parallel},\vec{b}_{\perp})+R(\vec{a}_{\perp},\vec{b}_{\parallel})}$$

where $R(\vec{a}_{\parallel},\vec{b}_{\parallel})$, $R(\vec{a}_{\perp},\vec{b}_{\perp})$, $R(\vec{a}_{\parallel},\vec{b}_{\perp})$ and $R(\vec{a}_{\perp},\vec{b}_{\parallel})$ denotes respective coincidence counting rate between two polarymeters set at opposite sides of the source with orientations (a,b). We required a 4-fold time coincidence among the two scatterers and two analysing detectors responding on opposite sides of the symmetry axis selected by the collimator. Additionally we required relevant energy deposit in each detector, 236 keV in scatterer and 275 keV in analysing detector. This condition is equivalent to requirement of the total energy loss equal to the energy of the annihilation photon (511 keV).

Using fourfold coincidence technique, we have measured in the single run the 16 coincidence rates R, which determines four correlation coefficients yielding directly S values defined as:

$$S = E(\vec{a},\vec{b}) - E(\vec{a},\vec{b}') + E(\vec{a}',\vec{b}) + E(\vec{a}',\vec{b}')$$

The quantity S is subject to the Bell's inequalities:

$$-2 \le S \le 2$$

The coincidence rate was measured as a function of azimuthal angle (defined as it is shown on Fig.3) between polarymeters $\Phi$. In the ideal polarymeter the probability p of correct response is equal to 1. In the case of compton polarymeter, the probability p of the right response is smaller ($p < 1$), on the other hand the probability of the false response $(1-p) > 0$. For the compton polarymeter consisting of a point source, point scatterer and point analysing detectors, for annihilation photons scattered at 82° the probability $p=0.846$. In order to determine p value for our polarymeters, the correlation function $E(90^{o})$ was measured. The average value of the probability p was determined from the formula :

$$E = p^2 + (1-p^2) - 2p(1-p) = (2p - 1)^2 \Rightarrow p = \frac{\sqrt{E} + 1}{2}$$

The probability of the right response was determined to be $p = 0.816$. Using p value one can define analysing power A of the polarymeter.

$$A = \frac{p - (1-p)}{p - (1-p)} = 2p - 1$$

Thus it can be shown that for real polarymeter with analysing power A, the measured value of function S reduces to:

$$S_r = S \cdot A^2$$

The Bell's inequality tested using real polarymeters with the value of analyzing power of a polarymeter equal to A, takes the form :

$$|S_r| \le 2\ A^2$$

For our polarymeter (see Table 1) the Bell's inequality takes the form:

$$|S_r| \le 0.802$$

The review of the properties of the polarymeters discussed above is shown in Table 1.

The analysing power of the polarymeter $A \le 1$ reduces the value of function "S" calculated according to quantum mechanics for pairs of annihilation photons what is demonstrated on Fig.4.

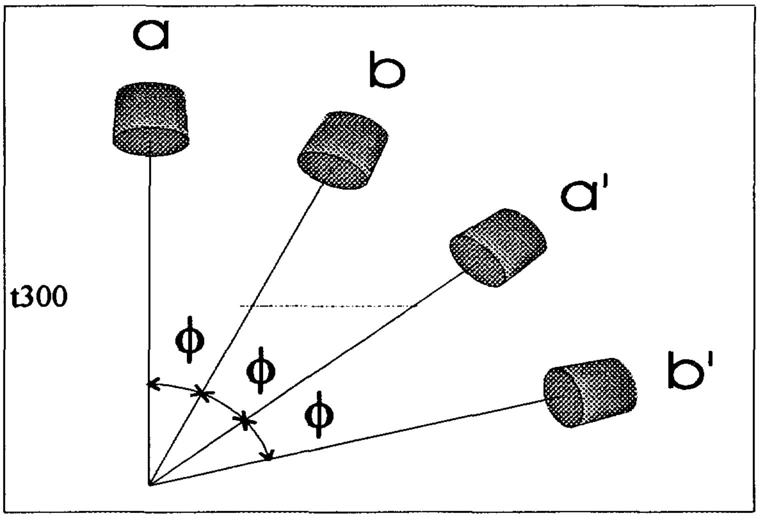

Fig.3. Polarymeters orientations used in the measurement; ϕ is azimuthal angle.

Table 1. Some properties of selected polarymeters.

| Type of polarymeter | p | A | E(90°) |
|---|---|---|---|
| Ideal polarymeter | 1 | 1 | 1 |
| The ideal compton polatrymeter | 0.846 | 0.691 | 0.478 |
| The real compton polatrymeter | 0.817 | 0.633 | 0.401 |

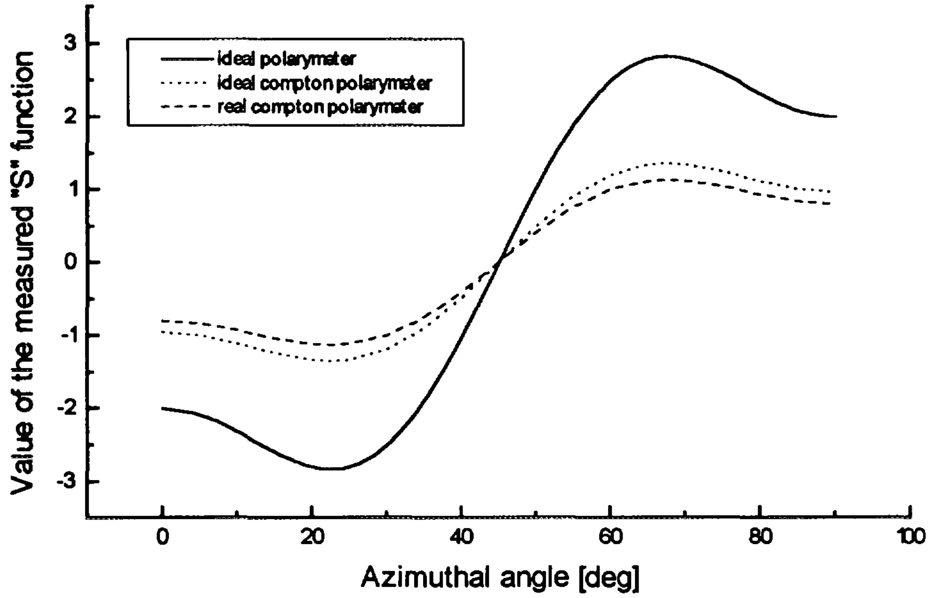

Fig.4. The influence of the analysing power of the polarymeter on the function "S" calculated according to quantum mechanics for pairs of annihilation photons assumed to be polarized at right angles.

The values of "S" function determined from the measurements of coincidence rates at different orientations of the polarymeters parameterized by the azimuthal angle $\phi$ are shown on Fig 5. As it is seen a good agreement between the prediction of quantum mechanics and experimental data is observed ($\chi^2$=1.34). Our data shows also violation of the Bell's inequality $|S_r| \leq 0.802$, what can lead to the rejection of the local hidden variable hypothesis.

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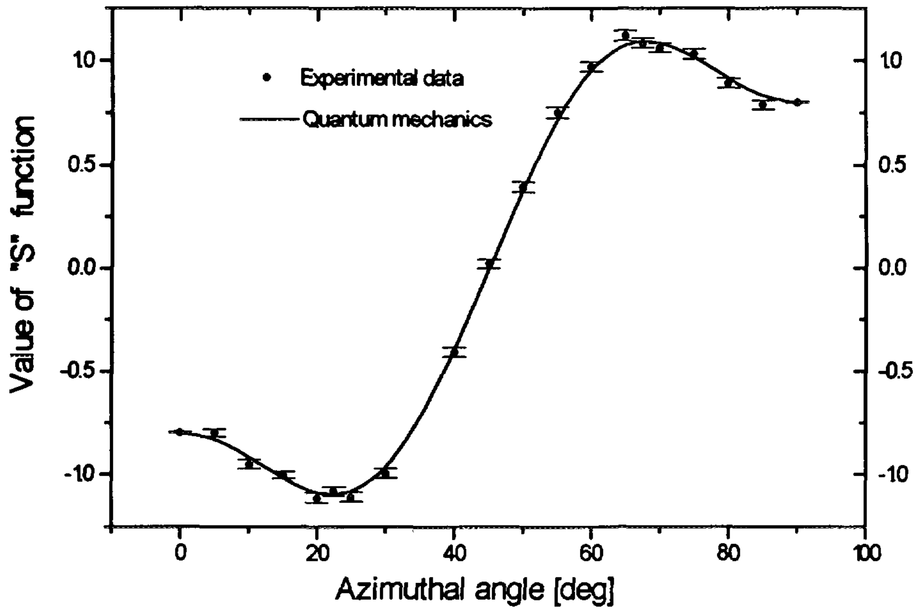

Fig. 5. Plot of experimental values "S" vs. relative azimuthal angle $\phi$. The solid line represents the function "S" calculated according to quantum mechanics with corrections for the analysing power of the polarymeter.