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Ideal MHD stability of plasmas with toroidal, helical and vertical field coils

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Abstract: A simple configuration consisting in a set of toroidal, helical and vertical field coils is used to calculate free boundary equilibria with nonzero plasma current and approximately helical plasma boundary. The amount of helical boundary deformation is controlled by the ratio of the current in the helical field coils to the current in the toroidal field coils. When this ratio is increased the $(m, n) = (2, 1)$ external kink is stabilized at $\beta \simeq 1\%$ for inverse rotational transform profiles in the region $q < 2$.

1. Introduction

In a previous work [1], we investigated the global ideal magnetohydrodynamic (MHD) stability of plasmas with a prescribed (fixed) helical boundary deformation and non vanishing toroidal current with respect to the (m, n) external kink modes $n = 1, 2, 3$ and $m = n + 1$. $L = 2$, 3 single helicity and mixtures (of both) configurations were studied by systematically varying parameters such as the type and amount of helical boundary deformation, the aspect ratio, the number of equilibrium field periods, the toroidal current density and the pressure profiles. Once these parameters were fixed, sequences of equilibria differing in the amount of helical boundary deformation and such that $1 \leq q \leq 2$, were calculated with the fixed boundary version of the VMEC [2] code. The stability analysis was performed with the TERPSICHORE [3] code. It was shown that increasing the helical boundary deformation leads to the stabilization of (m, n) external modes with $n = 1, 2, 3$, $m = n + 1$ at values of $\beta \simeq 1 - 2\%$. These modes are unstable in the circular tokamak at the same value of β. If δ is a measure of the plasma boundary deformation, then windows of stability $[\delta_{min} \;\; \delta_{max}]$ may exist and depend strongly on the equilibrium parameters.
We reconsider here the study of the $(2, 1)$ mode with equilibria calculated with a free boundary code. The aim is twofold:
1) to test how difficult it is to obtain free boundary equilibria with single helicity bounadry deformation at nonzero plasma current and positive β.
2) to check the results of the fixed boundary calculations in the sense that we search for stability windows when the amount of current in the helical coils is monotonously increased.

2. Equilibrium calculations

The calculations of the free boundary equilibria was performed in several steps. First, a system of coils producing a toroidal (TF), a vertical (VF) and a helical field (HF) is designed; the helical conductors are wound on a torus according to the following winding law

$$v = \frac{1}{N_{per}}(\tilde{u} + \alpha sin(\tilde{u})) - \frac{l}{L}\frac{2\pi}{N_{per}} \qquad\qquad \tilde{u} = u + 2\pi\frac{l-1}{L} \tag{1}$$

with u and v the geometrical poloidal and toroidal angles of a particular coil segment, N_{per} the number of field periods, $l = 1, ..L$ an index specifying a particular coil and α the pitch modulation coefficient of the helical coils. The magnetic field $\vec{B}$ is determined from the Biot-Savart law and a field line tracing code is used to find the coil geometrical parameters and currents such as to obtain closed helical flux surfaces in vacuum. The field produced by these external currents is given then as input to the free-boundary version of the equilibrium code VMEC [4]. At finite β and nonzero

plasma current, the plasma cross section is distorted; the currents in the coils are adjusted until the plasma cross section recovers an approximate helical shape.
Several types of coils systems (stellarator-, heliotron- and torsatron-like) were considered. We illustrate the results with the example of a L=2 stellarator-like configuration ($N_{per} = 4$) with 16 TF coils two piars of 2 HF coils and one pair of VF coils. Sequences of equilibria were calculated with the following parameters

$$R_0 = 5.0\,[m]\;\; r_t = 1.8\,[m]\;\; r_h = 1.4\,[m]\;\; r_v = 7.0\,[m]\;\; z_v = \pm\, 2.1\,[m]\;\; \alpha = -0.150$$
$$I_t = -1.6 \times 10^5[A] \quad I_v = 1. \times 10^4[A] \tag{2}$$
$$\beta = 1\%\ ,\ \text{parabolic pressure profile}\ ,\ J'(s) \sim (1 - s^{20})^8$$

The subscripts $_t$, $_h$ and $_v$ refer to the TF, HF and VF coils respectively, R_0 is the major radius, r and z are the coils radii and verical positions, I refers to the coils currents and J' is the toroidal plasma current density. The equilibria belonging to one particular sequence differ in the amount of HF coil current I_h. The coil system is illustrated in Fig.1 and plasma cross sections for two values of I_h are shown in Fig.2. In this case the plasma current was $J = 1.27 \cdot 10^5 [A]$. If the conventional (tokamak) definition of normalized beta $\beta_N = \beta / I_N$ with $I_N = J\,[MA]/(a\,[m]\; B_0\,[T])$ where a and B_0 are the averaged minor radius and the magnetic field intensity on the axis respectively, is used we obtain $\beta_N \sim 4 - 6$.

3. Stability calculations

Let $(m_e, N_{per}n_e)$ and (m_l, n_l) represent the Fourier components in Boozer coordinates (TERPSICHORE) of the equilibrium and perturbation quantities respectively. As the equilibrim configurations has several field periods, a partial decoupling of the perturbation components occurs, depending on the values of the toroidal mode numbers. The coupling between two perturabtion components (m_{l1}, n_{l1}) and (m_{l2}, n_{l2}) is nonzero if the following relations hold between the mode numbers: [1]$m_e = m_{l1} \pm m_{l2}$ and $N_{per}n_e = n_{l1} \pm n_{l2}$. This means that the perturbation toroidal mode numbers are distributed in families of non-interacting modes.
If the mode studied is $(m, n) = (2, 1)$ and if $N_{per} = 4$, the contribution of the coupling $(m_e, N_{per}n_e) \times (2, 1) \times (m_l, n_l)$ to the potential energy δW_p is nonzero only if n_l = 3, 5, 7, 9, etc. When the numerical study is carried in the parameter region corresponding to $1 \leq q(s) \leq 2$, then a particular attention should be given to those (m_l, n_l) perturbation components which are resonat i.e. $m_l > n_l > n = 1$. Depending on the q profile, these components can be destabilized and could lead a priori to important couplings with $(2, 1)$. The contribution of a particular $(m_{l1}, n_{l1}) \times (m_{l2}, n_{l2})$ coupling to δW_p is determined by the amplitude $A^{eq}_{me,ne}$ of the $(m_e, N_{per}n_e)$ equilibrium coupling term - c.f. [1]. Typical equilibrium quantities appearing in these couplings are $\sqrt{g}_{m_e,n_e}$ (the Jacobian), $|B^2|_{me,ne}$ and in general combinations between the coefficients of the metric tensor [5]. If $N_{per} = 4$ the coupling between $(2, 1)$ and (m_l, n_l) resonant components with $n_l = 3, 5$ requires $(m_e, N_{per}n_e)$ components with $n_e = 1$ and $m_e > 4$; the coupling with resonant (m_l, n_l) having $n_l = 7, 9$ requires $n_e = 2$ and $m_e > 8$.
The systematic study performed in [1] for fixed helical boundary shapes, showed that the $A^{eq}_{me,ne}$ amplitudes of the $(m_e, N_{per}n_e)$ components involved in couplings between $(2, 1)$ and resonant (m_l, n_l) are negligible (compared to the dominant equilibrium components) for any equilibrium quantity $(\sqrt{g}_{m_e,n_e}$, $|B^2|_{me,ne}$, etc) and for any boundary deformation δ. If the free boundary equilibrium has an approximate helical boundary shape, the above mentioned property remains valid - see Fig.3. Thus, the study of a particular mode would not require the inclusion in the calculations of the resonant perturbation components (n and n_l $\in$ the same family). This hypothesis was verified in the sense that the ratio of any of the $(m, n) \times (m_l, n_l)$ coupling contributions ((m_l, n_l) resonant) to δW_p to the dominant contributions to δW_p is very weak i.e. $\leq 10^{-4}$.
At the beginning of the equilibrium sequence presented in Fig.4 i.e for $I_h = 0.6 \times 10^5\ [A]$ the q profile

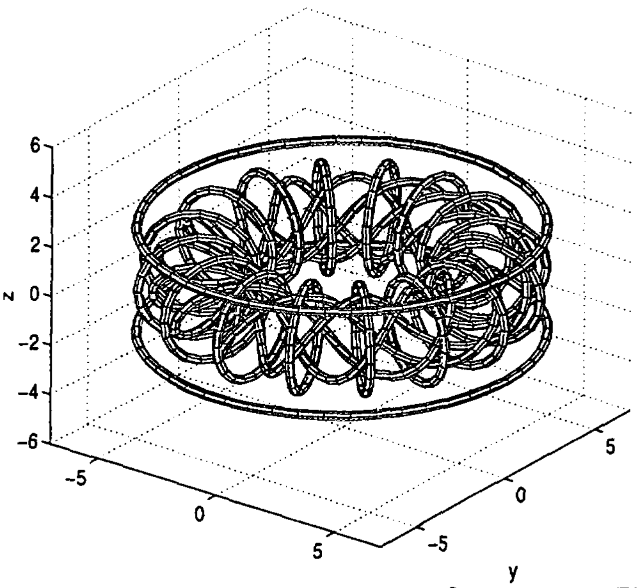

Figure 1 : *stellarator-like configuration with 16 TF coils, two pairs of 2 HF coils and one pair of VF coils*

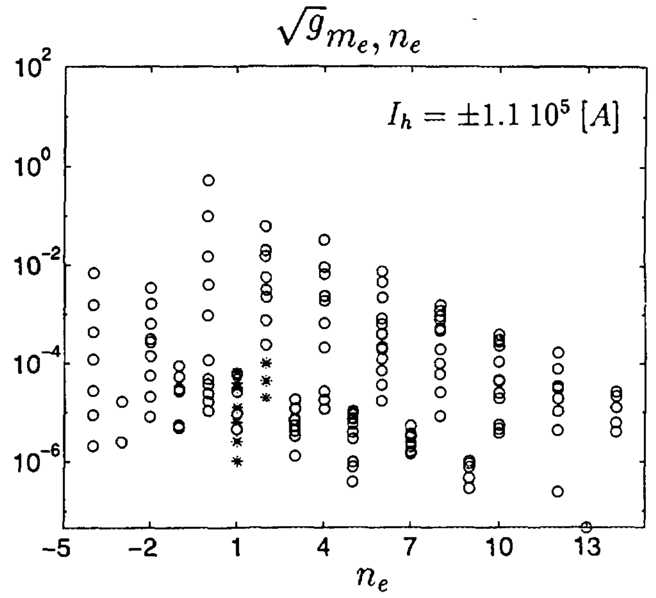

Figure 3 : $\sqrt{g}_{m_e,n_e}$ *amplitudes for the free boundary equilibrium described by Eq.(2) The x-axis corresponds to the n_e equilibrium mode number. The points marked with '*' represent the equilibrium components responsible for couplings between the (2,1) mode and the (m_l, n_l) perturbation components with $m_l > n_l > 1$ (only the $n_e \leq 2$ i.e. $n_l \leq 9$ are shown). All other equilibrium components are marked with 'o'.*

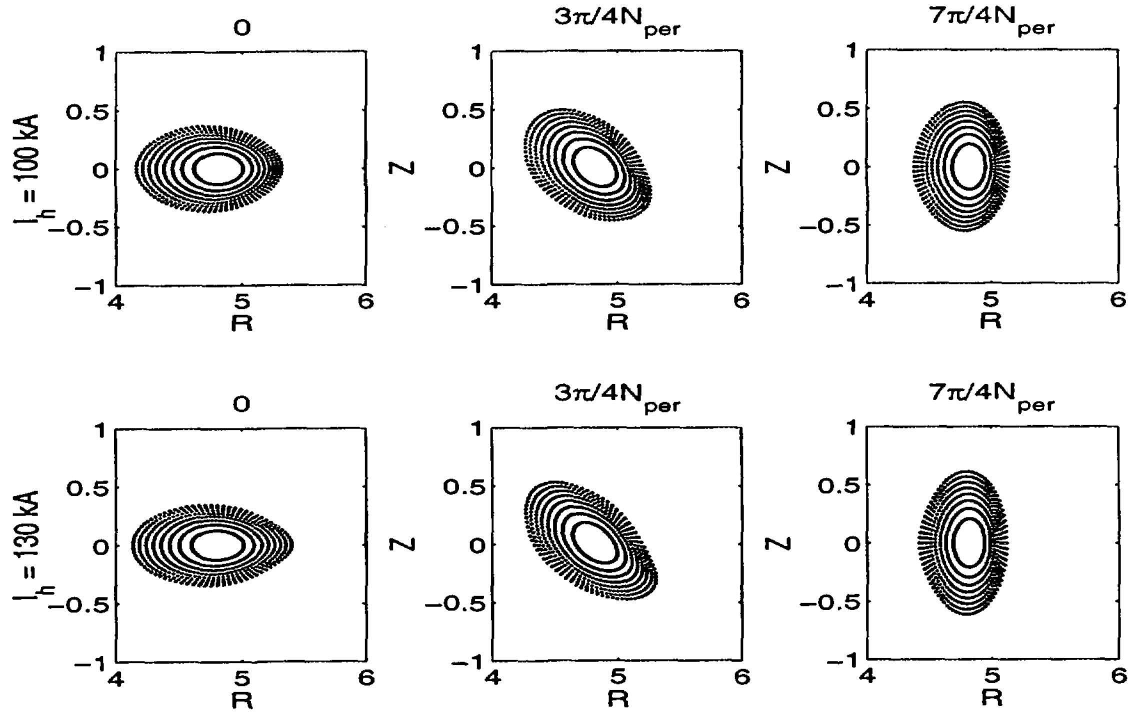

Figure 2 : *Free boundary equilibrium flux surfaces produced with VMEC. Each column represents the cross sections at one toroidal angle and each of the two rows are associated with one value of I_h. The coil system is represented in* Figure 1. *The equilibrium parameters are those of Eq.(2); the two equilibria correspond to the points just before and just after the stability window in Fig.4*

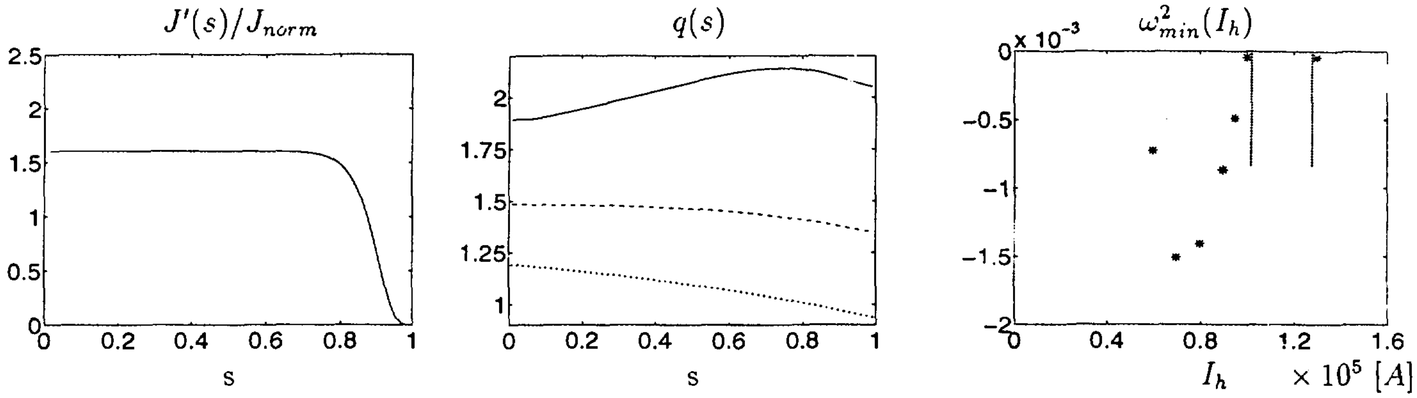

Figure 4 : *Study of the (2,1) mode:* (a) *plasma current density profile,* (b) $q(s)$ *profile,* (c) *sequence of most unstable eigenvalues* $\omega^2_{min}(I_h)$ *when the coil geometry and plasma parameters are given by Eq.(2) and the lines that follow. The aspect ratio is* $1/\epsilon \simeq 10$ *and the toroidal plasma current is* $J = 1.27 \cdot 10^5[A]$. *The inverse rotational transform profile is represented for* $I_h = 0.60 \times 10^5 A$ *(-),* $1.0 \times 10^5[A]$ *(- -) and* $I_h = 1.30 \times 10^5[A]$ *(·). The stability window is delimited by the two vertical lines and is associated with values of* I_h *between* $1 \times 10^5[A]$ *and* $1.3 \times 10^5[A]$

is such that the $(2,1)$ component is already destabilized. The effect of increasing I_h is to lower the inverse rotational transform. The $(2,1)$ component is strongly destabilized and becomes the dominant perturtbation component - we speak then about the $(2,1)$ mode. The most unstable eigenvalue $\omega^2_{min}(I_h)$ decreases until a minimum is attained, after which it starts increasing again; this is the stabilizing effect associated with an increasing (near)helical boundary deformation. Depending on the equilibrium parameters, a stability window $I_{stb} = [I_h^{min}, I_h^{max}]$ may appear in the sense that all $(m,1)$ components with $m > 1$ are stable and the $(1,1)$ component is not yet destabilized. Fig.4 (c) illustrates a stability window bounded by $I_h^{min} \simeq 1.0 \times 10^5[A]$ and $I_h^{max} \simeq 1.3 \times 10^5[A]$.
When the resonant perturbation components i.e. $(m_l, 3)$ with $m_l > 3$, $(m_l, 5)$ with $m_l > 5$, etc are taken into account and the stability calculations are performed again for those equilibria in the stable window, then several unstable eigenvalues may appear for each $I_h \in I_{stb}$. Each of these eigenvalues is associated to one of the resonant (m_l, n_l) modes (component whith largest amplitude). We systematically checked each of the $(2,1) \times (m_l, n_l)$ and $(1,1) \times (m_l, n_l)$ coupling contribution to δW_P and found that they are indeed negligible i.e. $< 10^{-4}$ compared to the dominant contributions.

4. Aknowledgements

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