ON THE ANGULAR DISTRIBUTIONS IN THE PROCESSES WITH POLARIZED PARTICLES

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The "geometric part" of the differential cross sections for the processes with polarized particles is expressed in terms of tensorial products of spherical harmonics $Y_{lm}(\mathbf{n})$. The simplest example of such products are so called bipolar harmonics (BH), i. e. the following tensorial products:

$$Y_{LM}^{ll'}(\mathbf{n},\mathbf{n}') = \left\{Y_l(\mathbf{n}) \otimes Y_{l'}(\mathbf{n}')\right\}_{LM} = \sum_{m,m'} C_{lml'm'}^{LM} Y_{lm}(\mathbf{n}) Y_{l'm'}(\mathbf{n}'), \qquad (1)$$

where $\mathbf{n}$, $\mathbf{n}'$ may be, for example, the direction of the atomic polarization and the momentum direction of the photoelectron.

In some problems the final expressions involve BH with small L, but l, l' may be large. So, in the study of the polarization and angular dependencies of atomic photoionization processes (including the autoionizing resonances), at the present time, the explicit expressions for the components of $Y_{LM}^{ll'}$ are obtained, proceeding from (1), as a functions of angles, defining $\mathbf{n}$, $\mathbf{n}'$ in a some coordinate system[1,2]. In this case the above-mentioned procedure should be repeated anew for each pair l, l', and, with increasing the latter ones, the expressions for $Y_{LM}^{ll'}$ become more and more complicated.

At the present work we derive for $Y_L^{ll'}$ the following expansion

$$Y_L^{ll'}(\mathbf{n},\mathbf{n}') = \sum_{\lambda=\lambda_p}^{L} a_\lambda Y_L^{\lambda, L+\lambda_p-\lambda}(\mathbf{n},\mathbf{n}'), \qquad (2)$$

$$\lambda_p = \frac{1}{2}\left(1-(-1)^{l+l'-L}\right) = \begin{cases} 0, & \text{for even } (l+l'-L), \\ 1, & \text{for odd } (l+l'-L), \end{cases}$$

which allows to represent the arbitrary BH in terms of BH's with minimal values of the internal tensor ranks. The coefficients $a_\lambda = a_\lambda(l,l',L,\theta)$ (here $\cos\theta \equiv \mathbf{n}\mathbf{n}'$) may be expressed in the general case in terms of the associated Legendre polynomials $P_m^n(\cos\theta)$. For $L=1$ we have, for example,

$$Y_1^{l,l}(\mathbf{n},\mathbf{n}') = i\frac{(-1)^l}{4\pi}\sqrt{\frac{3(2l+1)}{l(l+1)}}\frac{P_l^1(\cos\theta)}{\sin\theta}[\mathbf{n}\times\mathbf{n}'],$$

$$Y_1^{l,l-1}(\mathbf{n},\mathbf{n}') = \frac{(-1)^l}{4\pi}\sqrt{\frac{3}{l}}\frac{1}{\sin\theta}\left(P_l^1(\cos\theta)\mathbf{n} - P_{l-1}^1(\cos\theta)\mathbf{n}'\right).$$

The expressions (2) allow to write down the differential cross sections in a compact invariant form, for the arbitrary magnitudes of the angular moments l and l'. As an example, the cross sections of the bremsstrahlung and angular distributions of photoelectrons ejected from polarized atoms with arbitrary initial angular moment j_0 are presented.

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2. S. Baier, A. N. Grum-Grzhimailo, N. M. Kabachnik // J.Phys.B. 1994, v.27, p.3363.