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OSCILLATIONS OF ATMOSPHERIC NEUTRINOS

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United Nations Educational Scientific and Cultural Organization  
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International Atomic Energy Agency  
THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

**NEUTRINO MASS SPECTRUM WITH  $\nu_\mu \rightarrow \nu_s$  OSCILLATIONS  
OF ATMOSPHERIC NEUTRINOS**

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**Abstract**

We consider the “standard” spectrum of the active neutrinos (characterized by strong mass hierarchy and small mixing) with additional sterile neutrino,  $\nu_s$ . The sterile neutrino mixes strongly with the muon neutrino, so that  $\nu_\mu \leftrightarrow \nu_s$  oscillations solve the atmospheric neutrino problem. We show that the parametric enhancement of the  $\nu_\mu \leftrightarrow \nu_s$  oscillations occurs for the high energy atmospheric neutrinos which cross the core of the Earth. This can be relevant for the anomaly observed by the MACRO experiment. Solar neutrinos are converted both to  $\nu_\mu$  and  $\nu_s$ . The heaviest neutrino ( $\approx \nu_\tau$ ) may compose the hot dark matter of the Universe. Phenomenology of this scenario is elaborated and crucial experimental signatures are identified. We also discuss properties of the underlying neutrino mass matrix.

MIRAMARE – TRIESTE

February 1998

**1. Introduction**

Reconstruction of the neutrino mass spectrum and lepton mixing is one of the fundamental problems of particle physics. This problem may be solved with new generation of the neutrino experiments.

It is believed that the most natural scenario of the neutrino mass and mixing is the one with strong mass hierarchy

$$m_3 = (1 - 5) \text{ eV}, \quad m_2 = (2 - 3) \cdot 10^{-3} \text{ eV}, \quad m_1 \ll m_2 \quad (1)$$

and small lepton mixing (comparable with mixing in the quark sector), so that  $\nu_e \approx \nu_1$ ,  $\nu_\mu \approx \nu_2$ ,  $\nu_\tau \approx \nu_3$ . In this scenario the heaviest neutrino,  $\nu_3$ , composes the hot dark matter (HDM) of the Universe, while the  $\nu_e \rightarrow \nu_\mu$  resonance conversion solves the solar neutrino ( $\nu_\odot$ ) - problem [1]. This scenario (which can be called the “standard scenario”) follows naturally from the see-saw mechanism [2,3] with the neutrino Dirac mass matrix similar to the quark mass matrix:  $m_\nu^D \sim m_{up}$  and the Majorana masses of the right-handed neutrinos at the intermediate mass scale:  $10^{10} - 10^{13}$  GeV [4]. Short length  $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$  and  $\nu_e \leftrightarrow \nu_\mu$  oscillations proceed via mixing of  $\nu_e$  and  $\nu_\mu$  in the heaviest state  $\nu_3$  [5]. The transition probability for the LSND experiment [6] is small,  $P \lesssim 10^{-3}$ ; it can be reconciled with data at  $3\sigma$  level only [7].

In the standard scenario, there is no oscillation solution of the atmospheric neutrino ( $\nu_{atm}$ ) - problem [8]. Recently, the Super-Kamiokande [9] and Soudan [10] experiments have confirmed the existence of the problem. Moreover, observations of the zenith angle as well as  $L/E$  (distance/energy) dependencies of the muon neutrino deficit strongly indicate an oscillation solution of the problem.

One can accommodate the oscillation solution of the  $\nu_{atm}$ - problem modifying the standard scenario. A straightforward possibility is to diminish  $m_3$  down to  $\sim (0.03 - 0.3)$  eV implied by the  $\nu_{atm}$ - data. For  $m_3 \sim (0.5 - 0.6)$  eV one can also try to explain the LSND result [11] ignoring the zenith angle dependence in the atmospheric neutrino deficit. Further modification is needed to get a sufficient amount of the hot dark matter in the Universe. In this connection the degenerate mass spectrum with  $m_1 \approx m_2 \approx m_3 \sim 1 - 2$  eV has been suggested [12]. Simultaneous explanation of the LSND result and the zenith angle dependence in the atmospheric neutrinos requires depart from the three neutrino scheme.

In this paper we will consider a possibility to rescue the “standard scenario”. The idea is to keep the standard structure with strong mass hierarchy (1) and small mixing for active neutrinos and explain the atmospheric neutrino problem by oscillations of  $\nu_\mu$  to new neutrino state  $\nu_s$ :  $\nu_\mu \leftrightarrow \nu_s$  [13]. This new neutrino should be a sterile neutrino (singlet of  $SU_2 \times U_1$ ) in order to satisfy the bound on the number of neutrino species from the  $Z^0$ -decay width. Large  $\nu_s - \nu_\mu$  mixing can be related to a singlet character of  $\nu_s$ .

The  $\nu_\mu \leftrightarrow \nu_s$  oscillations as a solution of the atmospheric neutrino problem (see *e.g.* [13]) have been considered previously in the context of the model with mirror symmetry

[14]. In contrast with [14] we introduce only one singlet state, and the phenomenology of our scenario substantially differs from that in [14].

The paper is organized in the following way. In sect. 2 we describe the scenario. Then we consider predictions for the atmospheric (sect. 3) and solar (sect. 4) neutrinos, for the LSND experiment (sect. 5) and neutrinos from supernovas (sect. 6). Implications to the primordial nucleosynthesis are discussed in sect. 7. We comment on properties of the underlying mass matrix in sect. 8.

## 2. Scenario

The level and mixing scheme is shown in (fig. 1). The mass states  $\nu_1, \nu_2, \nu_3$  have the hierarchy of the standard scenario (1). The mass of the additional state  $\nu_4$  equals  $m_4 \sim (0.5 - 7) \cdot 10^{-1}$  eV, so that  $\Delta m_{24}^2 \sim m_4^2$  is in the range of solution of the  $\nu_{atm}$ -problem. The singlet neutrino,  $\nu_s$ , and the muon neutrino,  $\nu_\mu$ , mix strongly in the mass states  $\nu_2$  and  $\nu_4$ . We define the elements of the mixing matrix  $U_{\alpha i}$  which relate the flavor  $\nu_\alpha \equiv (\nu_e, \nu_\mu, \nu_\tau, \nu_s)$  and the mass  $\nu_i \equiv (\nu_1, \nu_2, \nu_3, \nu_4)$  states as:

$$\nu_\alpha = U_{\alpha i} \nu_i, \quad \alpha = e, \mu, \tau, s; \quad i = 1, 2, 3, 4.$$

All mixings except for  $\nu_\mu - \nu_s$  are small:  $U_{\mu 2} \sim U_{\mu 4} \sim U_{s 2} \sim U_{s 4} = 0.5 - 0.8$ , and  $U_{\alpha, i} \ll 1$  for others.

## 3. Atmospheric Neutrinos

A solution of the atmospheric neutrino problem is based, mainly, on  $\nu_\mu \leftrightarrow \nu_s$  oscillations. The muon neutrino state can be written as:

$$\nu_\mu = \sqrt{1 - U_{\mu 3}^2 - U_{\mu 4}^2} \cdot \nu' + U_{\mu 3} \cdot \nu_3 + U_{\mu 4} \cdot \nu_4, \quad (2)$$

where  $\nu'$  is the combination of the light states  $\nu_1$  and  $\nu_2$ . According to our scenario the admixtures of the  $\nu_e$  in the heavy states  $\nu_3$  and  $\nu_4$  are small:  $U_{e 3}, U_{e 4} \ll 1$ . In fact, these matrix elements are strongly restricted by the recent CHOOZ result [15]. For small  $U_{e 3}$  and  $U_{e 4}$  the oscillation effects related to splitting between the states  $\nu_1$  and  $\nu_2$  are negligibly small. Note that splitting between these states in matter can be large enough, so that the oscillation length is smaller than the diameter of the earth. However, the effective mixing angle is strongly suppressed by matter. (The mixing may not be suppressed if  $U_{e 3}$  and  $U_{e 4}$  are large. In fact,  $U_{e 4}$  can be large, if  $m_3 < 3 \cdot 10^{-2}$  eV. This possibility will be described elsewhere [16].) Thus, the task is reduced to oscillations in the  $(\nu', \nu_3, \nu_4)$  system (see (2)) and the mixing elements  $U_{\mu 3}, U_{\mu 4}$  are relevant only. Being in the eV - range, the state  $\nu_3$  produces just the averaged oscillation effect, so that the  $\nu_\mu$  survival probability can be written as

$$P(\nu_\mu \rightarrow \nu_\mu) = (1 - |U_{\mu 3}|^2)^2 P_2 + |U_{\mu 3}|^4.$$

Here  $P_2$  is the survival probability for two neutrino oscillations in matter with parameters  $\Delta m_{42}^2$  and

$$\sin^2 2\theta_{atm} = \frac{4|U_{\mu 4}|^2(1 - |U_{\mu 3}|^2 - |U_{\mu 4}|^2)}{(1 - |U_{\mu 3}|^2)^2}. \quad (3)$$

These oscillations are described by the effective Hamiltonian

$$H = \begin{pmatrix} 0 & \frac{\Delta m_{42}^2}{4E} \sin 2\theta_{atm} \\ \frac{\Delta m_{42}^2}{4E} \sin 2\theta_{atm} & \frac{\Delta m_{42}^2}{2E} \cos 2\theta_{atm} + V \end{pmatrix},$$

with the matter potential

$$V = \pm \sqrt{2} G_F \left[ \frac{1}{2} N_n + (1 - k)(N_e - \frac{1}{2} N_n) \right],$$

where  $k \approx |U_{s 4}|^2 / (|U_{s 4}|^2 + |U_{e 4}|^2) \leq 1$ . (The minus sign of  $V$  is for anti-neutrino.)  $N_n(t)$  and  $N_e(t)$  are the neutron and the electron number densities correspondingly. In the pure  $\nu_\mu \leftrightarrow \nu_s$  case one has  $k = 1$ , and  $V = \sqrt{2} G_F N_n / 2$ , so that the neutron density contributes only. With decrease of  $k$  the potential increases.

If  $\Delta m_{42}^2 \leq 10^{-2}$  eV<sup>2</sup>, the oscillations are not averaged and  $P_2$  leads to the zenith angle dependence of the  $\nu_\mu$  - deficit. For large  $U_{\mu 3}$  the total effect is the combination of the averaged  $\nu_\mu \leftrightarrow \nu_\tau$  oscillations and non-averaged  $\nu_\mu \leftrightarrow \nu_s$  oscillations.

The double ratio equals to survival probability:

$$R \equiv \frac{(\mu/e)_{osc}}{(\mu/e)_{MC}} = P(\nu_\mu \leftrightarrow \nu_\mu), \quad (4)$$

as in the case of  $\nu_\mu \leftrightarrow \nu_\tau$  oscillation. In (4)  $(\mu/e)_{osc}$  and  $(\mu/e)_{MC}$  are the ratios of numbers of the  $\mu$ -like to  $e$ -like events in the presence of oscillations and without oscillations correspondingly.

The difference between the  $\nu_\mu \leftrightarrow \nu_\tau$  and  $\nu_\mu \leftrightarrow \nu_s$  channels can follow from matter effect which influences the latter channel only. For maximal mixing the oscillation length  $l_m$  and the effective mixing  $\theta_m$  which determines the depth of oscillations can be written as

$$l_m = \frac{2\pi}{V} [1 + \xi]^{-1/2}, \quad \sin^2 2\theta_m = \frac{\xi}{1 + \xi}, \quad \xi \equiv \left[ \frac{\Delta m_{42}^2}{2EV} \right]^2. \quad (5)$$

For  $\Delta m_{42}^2 > 4 \cdot 10^{-3}$  eV<sup>2</sup> both for the sub-GeV and for multi-GeV ( $E \sim 3 - 5$  GeV) events we get  $\xi > 10$  and the matter effect is negligibly small ( $< 10\%$  in probability). Thus, in significant region of parameters (masses and mixing) at low energies the  $\nu_\mu \leftrightarrow \nu_s$  oscillations reproduce results of the  $\nu_\mu \leftrightarrow \nu_\tau$  oscillations, as far as the charged current (CC) interactions are concerned. (The number of the tau-lepton events is small.)

The matter effect can be important, for multi-GeV events, if  $\Delta m_{42}^2 < 3 \cdot 10^{-3}$  eV<sup>2</sup>. In particular, the zenith angle dependence of ratios can be modified. Since matter suppresses the  $\nu_\mu \leftrightarrow \nu_s$  oscillations, one expects a weaker oscillation effect for the upward-going events, and therefore, flattening of the zenith angle,  $\Theta$ , dependence. In particular,  $R(\cos \Theta = -1) > 1/2$  in the  $\nu_\mu \leftrightarrow \nu_s$  case, whereas for the  $\nu_\mu \leftrightarrow \nu_\tau$  case one gets  $R(\cos \Theta = -1) = 1/2$  provided the mixing is maximal.

The matter effect becomes important for through-going and stopping muons [13] produced by high energy neutrinos even for large  $\Delta m_{42}^2$ . With increase of energy (decrease of

$\Delta m^2$ )  $\xi \rightarrow 0$  and the oscillation length increases approaching the asymptotic value determined by the potential only:  $l_m \approx 2\pi/V$ . At the same time, the effective mixing angle decreases, so that the oscillation effects become weaker.

The zenith angle dependence of the survival probability for fixed energies is shown in fig. 2. According to fig. 2 the dependence has two dips: the wide dip with minimum at  $\cos \Theta \sim (-0.5 - 0.4)$  and the narrow one with minimum at  $\cos \Theta \sim -0.9$ . With increase of energy the depths of dips (peaks in the transition probability) decreases, whereas the shape of dips, and in particular, positions of minima change weakly. This behavior can be understood in the following way. The positions of minima and maxima are determined by the phase of oscillations:

$$\Phi = 2\pi \int \frac{dL}{l_m} \approx \int dLV. \quad (6)$$

For  $E > 20$  GeV and  $\Delta m^2 < 5 \cdot 10^{-3}$  eV<sup>2</sup> we have  $\xi < 0.2$ , so that the length of oscillations and therefore the phase are determined by the potential  $V$  and only weakly depend on energy. As a consequence, the phase of oscillations is fixed by the zenith angle. According to the model of the Earth [17], the trajectories touch the core at  $\cos \Theta \approx -0.8$ . For  $\cos \Theta > -0.8$  neutrinos cross the mantle only, and the wide dip at  $\cos \Theta > -0.8$  is due to the oscillations in the mantle. At  $\cos \Theta \approx -0.4$  the phase  $\Phi$  equals  $\pi$  which corresponds to the minimum of  $P$ . It turns out that at  $\cos \Theta \approx -0.8$  the phase is  $\Phi = 2\pi$  and the oscillation effect is zero.

At  $\cos \Theta < -0.8$  neutrinos cross both the mantle and the core. That is, the narrow dip is due to oscillations of neutrinos whose trajectories cross the core. In this dip the oscillation effect is even stronger than in the wide dip, in spite of larger density of the core. This dip (or peak in the transition probability) is the parametric resonance peak [18,19]: the parametric enhancement of the  $\nu_\mu \leftrightarrow \nu_s$  oscillations occurs for neutrinos which cross the core of the Earth. Indeed, these neutrinos cross three layers with approximately constant densities: the mantle, the core, and again the mantle, divided by sharp change of density on the borders. Let us denote the phases acquired in these layers by  $\Phi_{m1}$ ,  $\Phi_c$ ,  $\Phi_{m2}$  (obviously,  $\Phi_{m1} = \Phi_{m2}$ ). It turns out that for  $\cos \Theta \sim -0.88$  the following equality is realized:

$$\Phi_{m1} \approx \Phi_c \approx \Phi_{m2} \approx \pi. \quad (7)$$

That is, for each layer the size of the layer coincides with half of the averaged oscillation length:  $l_{mi}/2 \approx l_i$ . This is nothing but the condition of the parametric resonance [18,19]. The amplitude of oscillations is enhanced. Maximal transition probability which can be achieved in the case of three layers equals:

$$P_{tr} = \sin^2(4\theta_m^{mant} - 2\theta_m^{core}), \quad (8)$$

where  $\theta_m^{mant}$  and  $\theta_m^{core}$  are the averaged mixing angles in the mantle and in the core correspondingly. Let us stress that for sufficiently large energies the equality (7) does not depend on neutrino masses. The equality is determined basically by the density distribution in the Earth and by the potential which in turn is fixed by the channel of oscillations and the Standard Model interactions. The equality is fulfilled for oscillations into sterile neutrinos

only. For the  $\nu_\mu \leftrightarrow \nu_e$  channel the potential is two times larger and (7) fails. Note that for  $\cos \Theta = -1$  the oscillation effect is small.

Qualitatively the zenith angle dependence shown in fig. 2 is similar to that observed in the MACRO experiment [20]. There are two dips with minima at  $\cos \Theta = (-0.4 - 0.6)$  and  $\cos \Theta = (-1.0 - 0.8)$ . In our interpretation the latter dip could be due to the parametric enhancement of the  $\nu_\mu \leftrightarrow \nu_s$  oscillations of neutrinos which cross both the mantle and the core of the Earth. Similar features, although not so profound, have been observed in the Baksan experiment [21] which has energies of the detected neutrinos similar to those in the MACRO experiment. In the Super-Kamiokande experiment the effect is expected to be weaker, since the energy threshold of muon detection and corresponding energies of neutrinos are higher. Detailed comparison of the effect with data will be given elsewhere [22].

Another consequence of the  $\nu_\mu \leftrightarrow \nu_s$  oscillations is that the region excluded by the IMB analysis of the stopping and through-going muons is shifted by factor  $\sim 3$  to larger values of  $\Delta m^2$  [13]. This allows one to reconcile the excluded IMB region with the preferable Super-Kamiokande domain.

A crucial check of the  $\nu_\mu \leftrightarrow \nu_s$  oscillations can be obtained from studies of events produced by the neutral current ( $NC$ -) interactions of the atmospheric neutrinos. The number of the  $NC$ -events is suppressed by the  $\nu_\mu \leftrightarrow \nu_s$  oscillations and unchanged by the  $\nu_\mu \leftrightarrow \nu_\tau$  oscillations. It is difficult to detect the elastic neutral current interactions. In this connection, it was proposed to study  $\pi^0$ 's produced, mainly, by the neutral currents [23,24]:

$$\nu N \rightarrow \nu \pi^0 N$$

(in [23] the long base line experiment K2K was discussed). Subsequent decay  $\pi^0 \rightarrow \gamma\gamma$  is identified as the two showering event with certain invariant mass. Practically, one can measure the number of  $\pi^0$ -events,  $N_{\pi^0}$ , and find the ratios  $N_{\pi^0}/N_e$  and  $N_{\pi^0}/N_\mu$ , where  $N_e$  and  $N_\mu$  are the numbers of  $e$ -like and  $\mu$ -like events induced, mainly, by the charged currents. This eliminates uncertainties related to absolute values of the atmospheric neutrino fluxes. Another proposal [24] is to measure the ratio of  $N_{\pi^0}$  and the number of events with 1  $\pi$ -production by the charged currents. The latter dominate in the two-prong (ring) events sample,  $N_{2ring}$ , and experimentally it is possible to determine the ratio

$$R_{\pi^0/2ring} \equiv \frac{N_{\pi^0}}{N_{2ring}}. \quad (9)$$

This allows one to diminish uncertainties related to cross-sections. Also one can study the ratio  $N_{\pi^0}/N_{multi-ring}$ . Since  $\nu_\mu \leftrightarrow \nu_s$  oscillations suppress equally the  $CC$ - and  $NC$ -interactions of the muon neutrinos, while the  $\nu_\mu \leftrightarrow \nu_\tau$  oscillations suppress the  $CC$ - events only, one expects

$$R_{\pi^0/multi-ring}(\nu_\mu \rightarrow \nu_s) < R_{\pi^0/multi-ring}(\nu_\mu \rightarrow \nu_\tau).$$

Similar inequality is expected for  $N_{\pi^0}/N_e$ . For the survival probability  $P \approx 0.6$  the difference can be as big as (30–50)% [24]. Thus with about 300  $\pi^0$ -events which will be detected by the

Super-Kamiokande experiment within 2 - 3 years one will be able to disentangle solutions. An additional check will be done by the long baseline experiment K2K (KEK to Super-Kamiokande) [23]. In particular, one will be able to compare the number of  $\pi^0$ 's produced in the front (close to neutrino production target) detector and in the Super-Kamiokande detector. However, the number of  $\pi^0$  expected in the Super-Kamiokande is expected to be small ( $\sim 30$ ).

#### 4. Solar Neutrinos

Due to strong mass hierarchy, a dynamics of conversion of the solar neutrinos is reduced to one  $\Delta m^2$  task. Indeed, the electron neutrino state can be written as

$$\nu_e = \cos \phi \cdot \nu'_e + \sin \phi \cdot \nu_H, \quad (10)$$

where

$$\sin \phi \equiv \sqrt{U_{e3}^2 + U_{e4}^2}, \quad (11)$$

and  $\nu'_e$  is the combination of the light states:

$$\nu'_e = \cos \omega \cdot \nu_1 + \sin \omega \cdot \nu_2 \quad (12)$$

with

$$\cos \omega = \frac{U_{e1}}{\sqrt{U_{e1}^2 + U_{e2}^2}}. \quad (13)$$

The  $\nu_H$  is a combination of the heavy states:

$$\nu_H = \frac{U_{e3} \cdot \nu_3 + U_{e4} \cdot \nu_4}{\sqrt{U_{e3}^2 + U_{e4}^2}}. \quad (14)$$

For evolution of the solar neutrinos produced as  $\nu_e$  we get the following picture. The states  $\nu_3$  and  $\nu_4$  decouple from the system, thus leading to the averaged oscillations result. The state  $\nu'_e$  converts resonantly to its orthogonal state:

$$\nu_x = \cos \omega \cdot \nu_2 - \sin \omega \cdot \nu_1. \quad (15)$$

Using this picture we find that the survival probability for the solar  $\nu_e$  can be written as

$$P(\nu_e \rightarrow \nu_e) = \cos^4 \phi P_{2MSW} + |U_{e4}|^4 + |U_{e3}|^4. \quad (16)$$

(Here we also take into account that oscillation effects in  $\nu_3 - \nu_4$  system are averaged out.)  $P_{2MSW}$  is the  $\nu'_e$  survival probability of  $\nu'_e \rightarrow \nu_x$  resonance conversion. The amplitude of the probability can be obtained by solving the evolution equation

$$i \frac{d}{dt} \nu = H_2 \nu \quad (17)$$

for the two neutrino system  $\nu \equiv (\nu'_e, \nu_x)^T$ . The effective Hamiltonian

$$H_2 = \begin{pmatrix} 0 & \frac{\Delta m_{21}^2}{4E} \sin 2\omega - \frac{1}{4} U_{e4} \sin 2\psi \cdot \sqrt{2} G_F N_n \\ \frac{\Delta m_{21}^2}{4E} \sin 2\omega - \frac{1}{4} U_{e4} \sin 2\psi \cdot \sqrt{2} G_F N_n & \frac{\Delta m_{21}^2}{2E} \cos 2\omega - \sqrt{2} G_F N_n^{eff} \end{pmatrix} \quad (18)$$

can be found from the whole 4-neutrino Hamiltonian. In (18)  $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$ , and  $\psi$  is determined by

$$\sin^2 \psi = \frac{|U_{\mu 4}|^2}{|U_{\mu 4}|^2 + |U_{s 4}|^2}. \quad (19)$$

Note that for small  $U_{\mu 3}$  the angle  $\psi$  coincides practically with  $\theta_{atm}$  (3) responsible for oscillations of the atmospheric neutrinos. In (18) the effective density equals

$$N_n^{eff} = N_e \cos^2 \phi - \frac{1}{2} N_n (\sin^2 \psi - |U_{e4}|^2 \cos^2 \psi). \quad (20)$$

If the mixing of  $\nu_e$  in heavy states is negligibly small, the whole survival probability is reduced to  $P_{2MSW}$  according to (16).

In our scenario new features of the matter effect appear in comparison with pure active ( $\nu_e \rightarrow \nu_\mu$ ) or pure sterile ( $\nu_e \rightarrow \nu_s$ ) neutrino conversions. In particular, for non-zero admixture of the  $\nu_e$  - flavor in the state  $\nu_4$  ( $U_{e4} \neq 0$ ) the off-diagonal elements of the Hamiltonian (18) depend on matter density, so that the mixing can be induced, at least partly, by the matter effect. Let us consider the ratio of the vacuum and the matter contributions to mixing:

$$R_{m/\nu} \equiv \frac{U_{e4} \sin 2\psi \cdot \sqrt{2} G_F N_n^{res}}{\frac{\Delta m_{21}^2}{E} \sin 2\omega}. \quad (21)$$

Here the neutron density is taken in the resonance  $N_n = N_n^{res}$  which is justified for small  $\omega$  (narrow resonance). Determining  $N_n^{res}$  from the resonance condition ( $H_{22} = 0$ , where  $H_{22}$  is the 22-element of the Hamiltonian (18)), we find:

$$R_{m/\nu} = \frac{U_{e4}}{4 \tan 2\omega} \left[ r_R \cos^2 \phi - \frac{1}{2} (\sin^2 \psi - |U_{e4}|^2 \cos^2 \psi) \right]^{-1} \approx \frac{U_{e4}}{4 \tan 2\omega} \left[ r_R - \frac{1}{4} \right]^{-1}, \quad (22)$$

where the last equality holds for small  $\phi$  and  $\psi \approx \pi/4$ . Here  $r_R \equiv N_e/N_n$  is the ratio of the electron and neutron densities in resonance. The ratio increases from 2 in the center of the Sun to about 6 at the surface [25]. Using the upper bound  $U_{e4} < 0.2$  from the CHOOZ experiment [15] and the lower bound,  $\tan 2\omega > 0.06$ , implied by a small mixing solution of the  $\nu_\odot$ -problem we get from (22)  $R_{m/\nu} \leq 0.4$ . That is, the matter effect is smaller than the vacuum effect, although the difference is not large. The matter effect can both enhance and suppress the vacuum effect, thus enlarging a range of possible vacuum mixing angles:  $\sin^2 2\omega = (0.5 - 3) \sin^2 2\omega_0$ ; here  $\omega_0$  is the angle without off-diagonal matter effect.

If the original boron neutrino flux is smaller than the SSM one, then smaller total mixing is needed and  $R_{m/\nu}$  can be of the order one. In this case the matter effect can give dominant

contribution or even substitute the vacuum mixing in (18).

For  $U_{e4} \ll 0.2$  the matter term in the off-diagonal element of  $H_2$  is much smaller than the vacuum term, and  $P_{2MSW}$  reduces to usual  $2\nu$  survival probability characterized by  $\omega$ ,  $\Delta m_{21}^2$  and  $N^{eff} \approx \cos^2 \phi N_e - \frac{1}{2} \sin^2 \psi N_n$  (see (20)). Using the reactor bound on the admixture of  $\nu_e$  in the state  $\nu_3$ :  $\cos^2 \phi > 0.95$ , we get  $N^{sterile} < N^{eff} < N_e$ , where  $N^{sterile} = N_e - N_n/2$  is the effective density for pure  $\nu_e \rightarrow \nu_s$  conversion. As a consequence, in our case the survival probability at the adiabatic edge of the MSW suppression pit (survival probability as a function of  $E$ ) is in between the probabilities for the  $2\nu$  sterile and  $2\nu$  active cases; the non-adiabatic edges (determined by the derivative  $d[\ln(N^{eff})]/dx$ ) practically coincide for all three cases.

The main feature of solution of the solar neutrino problem in our scenario which can be used for its identification is that solar neutrino flux should contain both  $\nu_\mu$  and  $\nu_s$  neutrinos. Moreover, due to large  $\nu_\mu - \nu_s$  mixing in the  $\nu_2$ -state implied by the atmospheric neutrino data the transition probabilities  $P(\nu_e \rightarrow \nu_s)$  and  $P(\nu_e \rightarrow \nu_\mu)$  should be comparable. (The  $\nu_e \rightarrow \nu_\tau$  probability is small, since  $\nu_\tau$  is weakly mixed with other neutrinos.) We find:

$$P(\nu_e \rightarrow \nu_\mu) \approx \cos^2 \phi \left[ \sin^2 \psi \sin^2 \phi P_{2MSW} + \cos^2 \psi (1 - P_{2MSW}) + \sin \phi \sin 2\psi \tan 2\omega \left( \frac{1}{2} - P_{2MSW} \right) + \sin^2 \phi \sin^2 \psi \right], \quad (23)$$

$$P(\nu_e \rightarrow \nu_s) \approx \cos^2 \phi \left[ \cos^2 \psi \sin^2 \phi P_{2MSW} + \sin^2 \psi (1 - P_{2MSW}) - \sin \phi \sin 2\psi \tan 2\omega \left( \frac{1}{2} - P_{2MSW} \right) + \sin^2 \phi \cos^2 \psi \right]. \quad (24)$$

In the limit of small  $\omega$  and small  $\phi$  we get from (23, 24):

$$\begin{aligned} P(\nu_e \rightarrow \nu_\mu) &\approx \cos^2 \psi (1 - P_{2MSW}) \\ P(\nu_e \rightarrow \nu_s) &\approx \sin^2 \psi (1 - P_{2MSW}). \end{aligned} \quad (25)$$

That is, the relative contributions of these channels are determined by the angle  $\psi$ . In particular, for  $\sin^2 \psi \approx 0.5$  we have  $P(\nu_e \rightarrow \nu_\mu) \approx P(\nu_e \rightarrow \nu_s)$ . Taking  $\sin^2 2\psi \approx 0.70 - 1.0$  as it follows from the atmospheric neutrino data we find

$$\frac{P(\nu_e \rightarrow \nu_s)}{P(\nu_e \rightarrow \nu_\mu)} = \tan^2 \psi = \frac{1}{4} - 4. \quad (26)$$

This ratio does not depend on the neutrino energy (fig. 3).

In what follows we will refer to this solution as to the mixed  $\nu_e \rightarrow \nu_\mu, \nu_s$  solution. As we will see, its properties are intermediate between properties of the pure flavor  $\nu_e \rightarrow \nu_\mu$  and pure sterile  $\nu_e \rightarrow \nu_s$  solutions.

Some general tests of presence of the sterile neutrinos in the solar neutrino flux have been elaborated in [27]. Transition to sterile neutrinos suppresses the number of neutral

current events. Let us consider a modification of the charged-to-neutral current events ratio,  $(CC/NC)$ , which will be measured in the SNO experiment. Let us introduce the double ratio:

$$r_d \equiv \left( \frac{CC}{NC} \right)_{osc} : \left( \frac{CC}{NC} \right)_0,$$

where  $(CC/NC)_0$  and  $(CC/NC)_{osc}$  are the ratios in absence of oscillations and with oscillations. For three possible solutions of the  $\nu_\odot$ -problem we get:

$$r_d = \begin{cases} \langle P_{ee} \rangle & \nu_e \rightarrow \nu_\mu \\ \frac{\langle P_{ee} \rangle}{1 - \langle P_{es} \rangle} & \nu_e \rightarrow \nu_\mu, \nu_s \\ \sim 1 & \nu_e \rightarrow \nu_s \end{cases} \quad (27)$$

where  $\langle \dots \rangle$  means averaging over energy spectrum of the boron neutrinos. According to (27):

$$r_d(\nu_e \rightarrow \nu_\mu) < r_d(\nu_e \rightarrow \nu_\mu, \nu_s) < r_d(\nu_e \rightarrow \nu_s). \quad (28)$$

For small mixing solution and the SSM boron neutrino flux [25], we find intervals

$$r_d = \begin{cases} 0.3 - 0.4 & \nu_e \rightarrow \nu_\mu \\ 0.5 - 0.8 & \nu_e \rightarrow \nu_\mu, \nu_s \\ 1.07 - 1.09 & \nu_e \rightarrow \nu_s \end{cases} \quad (29)$$

which are well separated from each other. (In (29) difference of energy dependences of the  $CC$ - and  $NC$ - current cross-sections is taken into account.) Thus, if the original flux of the boron neutrinos,  $F_B$ , is known, it will be easy to disentangle solutions by measuring  $r_d$ . However, uncertainties in  $F_B$  make the task to be ambiguous. If, *e.g.*,  $F_B$  is smaller than the one in the SSM [25]:  $F_B \sim 0.7 F_B^{SSM}$ , then for the  $(\nu_e \rightarrow \nu_\mu)$  solution we get  $r_d \approx 0.4 - 0.6$  which overlaps with the interval expected for  $(\nu_e \rightarrow \nu_\mu, \nu_s)$ .

Due to the presence of both  $\nu_s$  and  $\nu_\mu$  in the final state, the effect of the  $NC$ - interactions in the  $\nu e$ - scattering (the Super-Kamiokande experiment) is intermediate between the effects in the pure active and pure sterile neutrino cases. Correspondingly, the  $\sin^2 2\omega - \Delta m_{21}^2$  region of solutions is intermediate between regions for pure active and sterile [26] neutrino conversions. This gives  $\sin^2 2\omega = (4.5 - 11) \cdot 10^{-3}$  and  $\Delta m_{21}^2 = (3 - 9) \cdot 10^{-6} \text{ eV}^2$  in the SSM framework.

Let us consider the distortion of the energy spectrum of the recoil electrons. In the SNO detector ( $\nu_e d \rightarrow epp$ ) the distortion is determined by the probability  $P(\nu_e \rightarrow \nu_e) \approx \cos^4 \phi \cdot P_{2MSW}$ . The factor  $\cos^4 \phi$  leads to smoothing of energy dependence of the probability, and therefore the distortion of the recoil electron spectrum can be weaker than in the pure  $\nu_e \rightarrow \nu_\mu$  or  $\nu_e \rightarrow \nu_s$  case. However for  $\sin^2 2\phi < 0.2$  this effect is small ( $\cos^4 \phi \sim 0.95$ ).

The Super-Kamiokande experiment ( $\nu_e \rightarrow \nu_e$ ) is sensitive to both  $CC$ - and  $NC$ - interactions. The  $NC$ -scattering of  $\nu_\mu$  leads to smoothing of the spectrum distortion. The transition to sterile neutrinos removes this effect. Therefore in the case of mixed conversion,  $\nu_e \rightarrow \nu_\mu, \nu_s$ , the spectrum is distorted stronger than in the  $\nu_e \rightarrow \nu_\mu$  case but weaker than due to  $\nu_e \rightarrow \nu_s$  conversion [28] (fig. 4), although the difference is small.

Let us discuss the day-night effect [29]. The  $\nu_e \rightarrow \nu_s, \nu_\mu$  gives weaker asymmetry than  $\nu_e \rightarrow \nu_\mu$ , but stronger asymmetry than  $\nu_e \rightarrow \nu_s$ . The last two cases were studied in [26] and [30].

An important signature of our scenario is the day-night effect for events induced by the neutral currents,  $N^{NC}$ . (This in pure  $\nu_e \rightarrow \nu_s$  conversion case has been studied in [26].) Let us introduce the day-night asymmetry as

$$A_{D/N}^{NC} = \frac{N_N^{NC} - N_D^{NC}}{N_N^{NC} + N_D^{NC}}.$$

(Similarly, the asymmetry for  $CC$ -events,  $A_{D/N}^{CC}$ , can be defined.) The asymmetry can be used to distinguish our scenario from pure  $2\nu$ -cases. Indeed,  $A_{D/N}^{NC}$  is zero for the  $\nu_e \rightarrow \nu_\mu$  conversion and non-zero in the presence of sterile neutrinos. The intermediate case ( $\nu_e \rightarrow \nu_\mu, \nu_s$ ) can be distinguished from the pure sterile case by comparison of asymmetries in the charged currents,  $A_{D/N}^{CC}$ , and in the neutral currents. For the pure sterile case:  $A_{D/N}^{CC} - A_{D/N}^{NC} \approx 0$  [26]. In the ( $\nu_e \rightarrow \nu_\mu, \nu_s$ ) case we predict  $A_{D/N}^{CC} - A_{D/N}^{NC} > 0$ . Studies of these asymmetries will be possible in the SNO experiment.

### 5. Short range oscillation experiments. LSND.

In our scenario, two neutrino states with large masses,  $\nu_3$  and  $\nu_4$ , can be relevant for short range experiments. The probability of  $\nu_\mu \leftrightarrow \nu_e$  ( $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$ ) oscillations can be written as

$$P(\nu_\mu \rightarrow \nu_e) \approx 4|U_{\mu 3}|^2|U_{e 3}|^2 \sin^2\left(\frac{m_3^2 t}{4E}\right) + 4|U_{\mu 4}|^2|U_{e 4}|^2 \sin^2\left(\frac{m_4^2 t}{4E}\right) + 4\text{Re}[U_{e 3}^* U_{\mu 3} U_{e 4} U_{\mu 4}^*] \cdot \left\{ \sin^2\left[\frac{(m_4^2 - m_3^2)t}{4E} + \frac{\delta}{2}\right] - \sin^2\left(\frac{m_4^2 t}{4E} + \frac{\delta}{2}\right) - \sin^2\left(\frac{m_3^2 t}{4E} - \frac{\delta}{2}\right) \right\}, \quad (30)$$

where  $\delta = \text{arg}(U_{e 3}^* U_{\mu 3} U_{e 4} U_{\mu 4}^*)$ . For real mixing matrix we get  $\delta = 0$ . The first two terms in (30) correspond to "indirect" oscillations [5] related to levels  $\nu_3$  and  $\nu_4$  correspondingly. The third term is the result of interference of oscillations due to  $\nu_3$  and  $\nu_4$ . For  $m_3 \sim O(\text{eV})$  we get the following results depending on value of the mass  $m_4$ :

1) The mass  $m_4 \leq 0.3 \text{ eV}$  is too small to contribute to the LSND effect and the problem is reduced to the one level problem with probability described by the first term in (30). For values of  $U_{\mu 3}$  and  $U_{e 3}$  similar to those in quark sector we get negligibly small probability for LSND:  $P \leq 10^{-6}$ . If  $U_{\mu 3}$  and  $U_{e 3}$  are at the upper experimental bounds, then  $P \leq 10^{-3}$  which could be reconciled with data at  $3\sigma$ -level [7].

2) For  $m_4 \geq 0.3 \text{ eV}$  both  $\nu_3$  and  $\nu_4$  can give sizable effect to the short range oscillations. Also interference between two different modes of oscillations becomes important. In certain ranges of parameters the interference is constructive, thus leading to substantial enhancement of the probability. For instance, if  $m_3^2 = 2 \text{ eV}^2$  and  $m_4^2 = 1 \text{ eV}^2$ , the probability  $P(\nu_\mu \rightarrow \nu_e)$  is enhanced by factor 3, as compared to the one level case. Now the probability  $P \sim 3 \cdot 10^{-3}$  turns out to be in a good agreement with LSND result. In this case one also predicts  $\nu_e \leftrightarrow \nu_\tau$  and  $\nu_\mu \leftrightarrow \nu_\tau$  oscillations at the level of present experimental bounds. However, no zenith angle dependence of atmospheric neutrino deficit is expected.

### 6. Supernova neutrinos

Since  $\nu_\mu$  and  $\nu_s$  are (almost) maximally mixed, their level crossing occurs at zero (small) density. Let us introduce the eigenstates of system ( $\nu_\mu, \nu_s$ ) in medium  $\nu'_{2m}, \nu'_{4m}$ , in absence of mixing with  $\nu_e$  and  $\nu_\tau$ . At zero density  $\nu'_{2m} \approx \nu_2$  and  $\nu'_{4m} \approx \nu_4$ . Using the eigenvalues of  $\nu'_{2m}$  and  $\nu'_{4m}$  it is easy to find that the whole system ( $\nu_e$  and  $\nu_\tau$  included) has three resonances (level crossings) in the neutrino channels and one resonance in the antineutrino channels (fig. 5): (i) ( $\nu_e - \nu_\tau$ ) crossing at density  $N_{e\tau}$ , (ii) ( $\nu_e - \nu'_{4m}$ ) crossing at  $N_{e4}$ , (iii) ( $\nu_e - \nu'_{2m}$ ) crossing at  $N_{e2}$  and (iv) ( $\bar{\nu}_\tau - \bar{\nu}'_{4m}$ ) crossing at  $N_{\tau 4} \geq N_{e\tau}$ . The mass hierarchy leads to hierarchy of the resonance densities (see fig. 5):

$$N_{\tau 4} \geq N_{e\tau} \gg N_{e4} \gg N_{e2}.$$

For  $m_3 \leq 5 \text{ eV}$  we get  $m_N N_{\tau 4} \leq 10^8 \text{ g/cm}^3$  (here  $m_N$  is the nucleon mass). That is, the  $\bar{\nu}_\tau \rightarrow \bar{\nu}_s$  resonance conversion occurs outside the core. Moreover, the effective  $\bar{\nu}_\tau - \bar{\nu}_s$  mixing in the core is very small:  $\sin^2 2\theta_{\tau 4}^m \approx 4|U_{s 3}|^2 \cdot N_{\tau 4}/N_{\text{core}}$ , and generation of the sterile neutrinos is strongly suppressed. Consequently, there is no influence of the sterile neutrinos on the gravitational collapse and cooling of the core.

The nucleosynthesis of heavy elements due to the  $r$ -processes in the inner parts of supernovas implies that  $\nu_\tau \rightarrow \nu_e$  conversion does not occur efficiently above certain densities [31]. This gives the restrictions:  $m_3 \leq 2 \text{ eV}$  for  $\sin^2 2\theta_{e\tau} > 10^{-2}$ , so that the conversion takes place above the nucleosynthesis region, or  $\sin^2 2\theta_{e\tau} < 10^{-5}$  for  $m_3 > 2 \text{ eV}$ , so that the adiabaticity is strongly broken in the resonance and the conversion is not effective.

Let us consider consequences of transitions for neutrino fluxes detectable at the earth. Suppose first that all resonances are effective (which implies that  $m_3 \leq 2 \text{ eV}$ ). In this case the following transitions occur:

$$\begin{aligned} \nu_e \rightarrow \nu_\tau, \quad \nu_\mu \rightarrow \nu_e \rightarrow \nu_2 (\approx \nu_s, \nu_\mu), \quad \nu_\tau \rightarrow \nu_e \rightarrow \nu_4 (\approx \nu_s, \nu_\mu), \\ \bar{\nu}_e \rightarrow \bar{\nu}_\tau, \quad \bar{\nu}_\mu \rightarrow \bar{\nu}_2 (\approx \bar{\nu}_s, \bar{\nu}_\mu), \quad \bar{\nu}_\tau \rightarrow \bar{\nu}_4 (\approx \bar{\nu}_s, \bar{\nu}_\mu). \end{aligned} \quad (31)$$

Thus one expects (i) disappearance (or strong suppression) of the  $\nu_e$ -flux which can be established in experiment. (ii)  $\nu_\tau$ 's will have soft spectrum (corresponding to the original  $\nu_e$ -spectrum), and  $\bar{\nu}_\tau$ 's disappear. (iii) Fluxes of  $\nu_s$ 's and  $\bar{\nu}_s$ 's are produced, and total energy

released in neutrinos is about 50% larger than that in observable neutrinos. In contrast, in the standard scenario one would expect  $\nu_\mu \rightarrow \nu_e$  transition which leads to  $\nu_e$  with hard energy spectrum (corresponding to the original  $\nu_\mu$ -spectrum).

Suppose now that  $\nu_e - \nu_{2m}$  resonance is not efficient (which may occur in compact stars with small mass and large density gradient in the outer layers). Then transitions proceed as follows:

$$\nu_e \rightarrow \nu_\tau ; \quad \nu_\mu \rightarrow \nu_e ; \quad \nu_\tau \rightarrow \nu_e \rightarrow \nu_4 (\approx \nu_\mu, \nu_s) \quad (32)$$

(transitions in the antineutrino channels are as in (31)). Thus now  $\nu_e$ 's do not disappear, and moreover, the  $\nu_e$ -flux has hard spectrum (corresponding to the original  $\nu_\mu$ -spectrum).

Finally, suppose that  $\nu_e \rightarrow \nu_\tau$  resonance is not efficient because of smallness of  $U_{e3}$ . (In this case the mass  $m_3$  can be larger than 2 eV.) The following transitions take place:

$$\nu_e \rightarrow \nu_4 (\approx \nu_\mu, \nu_s) ; \quad \nu_\mu \rightarrow \nu_e \rightarrow \nu_2 (\approx \nu_\mu, \nu_s) ; \quad \nu_\tau \rightarrow \nu_\tau , \quad (33)$$

and transitions in the antineutrino channels are as in (31). Again the  $\nu_e$ -flux disappears. Half of the  $\nu_\mu$ -flux is converted to the  $\nu_s$ -flux, and it acquires a soft component due to the  $\nu_e \rightarrow \nu_4$  transition. New detectors (the Super-Kamiokande, SNO, ICARUS) will be able to identify these changes of fluxes.

## 7. Primordial Nucleosynthesis

The oscillations  $\nu_\mu \leftrightarrow \nu_s$ ,  $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_s$  with parameters implied by the atmospheric neutrino problem result in appearance of the equilibrium concentration of sterile neutrinos in the early Universe. Therefore, during the time of nucleosynthesis,  $t \geq 1$  s, the effective number of neutrino species is  $N_\nu = 4$ . This disagrees with bound  $N_\nu < 2.6$  based on low values of deuterium abundance [32]. Four neutrino species are, however, in agreement with recent conservative estimations  $N_\nu < 4.5$  [33,34] which use large abundance of  ${}^4\text{He}$ . In view of these uncertainties let us comment on the possibility to suppress generation of sterile neutrinos. It was shown that leptonic asymmetry in active neutrinos  $L_\nu \equiv (n_\nu - n_{\bar{\nu}})/n_\gamma > 7 \cdot 10^{-5}$  at the time  $t > 10^{-1}$  s [35], is enough to suppress  $\nu_\mu \leftrightarrow \nu_s$  oscillations. Such a  $L_\nu$  produces the potential due to  $\nu - \bar{\nu}$  scattering:  $V \sim G_F L_\nu n_\gamma$  which suppresses the effective mixing angle in matter:  $\sin^2 2\theta_m \sim \sin^2 2\theta \Delta m^2 / TV$  ( $T$  is the temperature). Although there are no strong restrictions on leptonic asymmetry one would expect that  $L_\nu$  is of the order of the baryon asymmetry:  $L_\nu \sim 10^{-10}$ . It is argued [36] (see also [37]) that large  $L_\nu$  can be produced in oscillations of the tau neutrinos to sterile neutrinos:  $\nu_\tau \rightarrow \nu_s$ ,  $\bar{\nu}_\tau \rightarrow \bar{\nu}_s$  in earlier epoch ( $t < 10^{-2}$  s). In the presence of small original leptonic asymmetry,  $L_\nu \sim 10^{-10}$ , these oscillations can lead to an exponentially fast increase of  $L_\nu$  (the task is non-linear). For  $m_3 \geq 1$  eV, the increase occurs at  $T \sim 16$  MeV. The lower value of  $m_3^2$  required by this mechanism is approximately proportional to  $\Delta m_{42}^2$ . Thus, for  $\Delta m_{42}^2 = 3 \cdot 10^{-3}$  eV<sup>2</sup> we get from [36]  $m_3^2 \geq 6$  eV<sup>2</sup> and  $\sin^2 2\theta_{\tau s} \leq 8 \cdot 10^{-6}$ . For  $\Delta m_{42}^2 = 10^{-3}$  eV<sup>2</sup> the corresponding bounds are  $m_3^2 \geq 2$  eV<sup>2</sup> and  $\sin^2 2\theta_{\tau s} \leq 4 \cdot 10^{-5}$ . For larger angles the  $\nu_\tau \rightarrow \nu_s$  oscillations themselves produce the equilibrium concentration of sterile neutrinos. These conditions can be satisfied in our scenario.

## 8. Properties of mass matrix

Let us find the elements of the neutrino mass matrix in the flavor basis,  $\|m_{\alpha\beta}\|$ , ( $\alpha, \beta = e, \mu, \tau, s$ ) which lead to the scenario under discussion (fig. 1). Suppose first that mixing of  $\nu_\tau$  is very small and it does not influence masses and mixing of lighter neutrinos. Then in the rest system:  $(\nu_s, \nu_e, \nu_\mu)$  the sub-system  $(\nu_s, \nu_\mu)$  with bigger masses and approximately maximal mixing dominates. This means that we can find masses and mixing of  $\nu_s$  and  $\nu_\mu$  considering the  $2 \times 2$  sub-matrix  $\|m_{\alpha\beta}^{(2)}\|$ , with  $\alpha, \beta = s, \mu$  only. The effect of mixing with  $\nu_e$  gives small corrections.

The main feature of our scenario is the maximal mixing of two states with at least one order of magnitude mass hierarchy:  $\epsilon \equiv m_2/m_4 = 0.03 - 0.10$ . Maximal mixing and strong hierarchy imply that sub-matrix  $\|m_{\alpha\beta}^{(2)}\|$  has small determinant and all its elements are of the same order. Indeed, we find the following relations:

$$\frac{m_{ss} - m_{\mu\mu}}{m_{\mu s}} = \frac{2}{\tan 2\theta_{atm}}$$

$$\frac{m_{\mu\mu}m_{ss} - m_{\mu s}^2}{(m_{\mu\mu} + m_{ss})^2} = \frac{\epsilon}{(1 + \epsilon)^2}$$

and  $m_4 \approx m_{\mu\mu} + m_{ss}$ . For maximal mixing the masses are determined by

$$m_{ss} = m_{\mu\mu} \approx m_{\mu s} \cdot \frac{1 + \epsilon}{1 - \epsilon}. \quad (34)$$

For  $0.03 - 0.1$ , the eq. (34) gives  $m_{ss} \approx (1.05 - 1.2) \cdot m_{\mu s}$  or  $m_{ss} \approx -(0.95 - 0.8) \cdot m_{\mu s}$ . In the case of non-maximal mixing larger spread of masses is possible. For  $\sin^2 2\theta_{atm} = 0.9$  we find:  $m_{ss} = m_{\mu\mu} + m_{\mu s}$ , and  $m_{\mu\mu} : m_{\mu s} : m_{ss} = 0.67 : 1 : 1.3$ . Thus, a spread of the elements can be within a factor of 2. Taking  $m_4 \approx 2m_{\mu s}$ , we get:  $m_{\mu s} \sim (1 - 3) \cdot 10^{-2}$  eV.

Diagonalizing the sub-matrix  $m^{(2)}$  one can find the mass matrix for the light neutrino system  $(\nu_e, \nu'_2)$ , where  $\nu'_2$  is the neutrino with mass  $m_2$ . From this we get the mixing of the light neutrinos:

$$\sin \theta_{\odot} \approx \frac{1}{m_2} (m_{e\mu} \cos \theta_{atm} - m_{e s} \sin \theta_{atm}) \approx (3 - 5) \cdot 10^{-2}. \quad (35)$$

and if there is no strong cancellation in (35):

$$m_{e\mu}, m_{e s} \leq \sqrt{2} \sin \theta_{\odot} \cdot m_2 \approx 2 \cdot 10^{-4} \text{ eV}.$$

The structure of the mass matrix can be substantially different if the mixing of the tau neutrino is not small. In particular, the desired mass  $m_{ss}$  can be generated by this mixing:  $m_{ss} \approx -(m_{s\tau})^2 / m_{\tau\tau}$ , if, e.g.,  $m_{s\tau} \sim 0.1$  eV and  $m_{\tau\tau} \approx (1 - 2)$  eV. This corresponds to a rather large  $\nu_\tau - \nu_s$  mixing parameter:  $\sin^2 2\theta_{\tau s} \sim (2 - 4) \cdot 10^{-2}$  for which the equilibrium concentration of  $\nu_s$  is produced in the early Universe already in  $\nu_\tau \leftrightarrow \nu_s$  oscillations. The mechanism of generation of the lepton asymmetry [35] does not work.

As follows from the above estimations of the mass terms, the mass matrix for the active neutrinos can have usual hierarchical structure with small mixing. It can easily be generated



by the see-saw mechanism with linear hierarchy of masses of the right-handed neutrinos. Also  $\nu_s$  can have the hierarchy of couplings with different generations:  $m_{e s} \ll m_{\mu s} \ll m_{\tau s}$ . Some elements can be zero: *e.g.*,  $m_{e s} = m_{\tau s} = 0$ .

## 9. Conclusion

We have considered a minimal modification of the “standard” scenario for the neutrino mass introducing one additional sterile neutrino. The sterile neutrino mixes strongly with muon neutrino, so that the  $\nu_\mu \leftrightarrow \nu_s$  oscillations solve the atmospheric neutrino problem.

We show that the parametric enhancement of the  $\nu_\mu \leftrightarrow \nu_s$  oscillations occurs when high energy atmospheric neutrinos cross the core of the Earth. This effect can be relevant for the explanation of the anomaly in the zenith angle distribution observed by the MACRO experiment. Suppression of the  $NC$ -events in the atmospheric neutrinos ( $\pi^0$  - like events) is another signature of the scenario.

The scenario can supply the hot dark matter of the Universe.

The solar neutrino problem is solved by  $\nu_e$  conversion to  $\nu_\mu$  and  $\nu_s$ . The solution has characteristics (spectra distortion, day-night effect,  $NC/CC$  ratio) being intermediate between characteristics of  $\nu_e \rightarrow \nu_\mu$  and  $\nu_e \rightarrow \nu_s$  conversions.

The probability of  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  oscillations in the LSND experiment can reach  $O(10^{-3})$ . It can be further enhanced if new neutrino state has the mass  $m_4 \geq 0.3$  eV. (In this case, however, the atmospheric neutrino deficit will have no zenith angle dependence.)

The disappearance of the  $\nu_e$ -flux from supernovas is another possible signature.

The scenario can be well identified or rejected by the neutrino experiments of new generation (Super-Kamiokande, SNO, Borexino, ICARUS and others).

## Acknowledgments.

The authors are grateful to E. Akhmedov and S. P. Mikheyev for useful discussions, and to R. Volkas and X. Shi for valuable comments. This work has been accomplished during the Extended Workshop on Highlights in Astroparticle Physics (ICTP, Trieste, October - December, 1997). The work of Q.Y.L. is supported in part by the EEC grant ERBFM-RXCT960090.

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## Figure Captions

Fig. 1. Qualitative pattern of the neutrino masses and mixing. Boxes correspond to different mass eigenstates. The sizes of different regions in the boxes determine flavors of mass eigenstates,  $|U_{oi}|^2$ . White regions correspond to the sterile flavor, light and dense shadowed regions fix the electron and muon flavors correspondingly; admixtures of the tau flavor are shown in black.

Fig. 2. The zenith angle dependence of the survival probability  $P(\nu_\mu - \nu_\mu)$  for atmospheric neutrinos for different values of the neutrino energy (figures at the curves in GeV) and  $\Delta m^2 = 5 \cdot 10^{-3} \text{ eV}^2$ . Solid lines correspond to  $\nu_\mu \leftrightarrow \nu_\mu$  oscillations, the dashed line is for  $\nu_\mu \leftrightarrow \nu_\tau$  oscillations.

Fig. 3. The conversion probabilities of solar neutrinos as functions of  $E/\Delta m^2$  for  $\sin^2 2\omega = 0.01$ ,  $\cos^2 \psi = 0.5$  and  $\sin \phi = -0.2$ ;  $P(\nu_e \rightarrow \nu_e)$  is shown by the solid line,  $P(\nu_e \rightarrow \nu_\mu)$  - dotted line,  $P(\nu_e \rightarrow \nu_s)$  - dashed line.

Fig. 4. The expected distortion of the recoil electron energy spectrum in the Super-Kamiokande experiment. The ratio of numbers of events with and without conversion  $R_e$  is shown by histograms: the bold solid line corresponds to  $\nu_e \rightarrow \nu_s, \nu_\mu$  conversion considered in this paper; the solid line is for  $\nu_e \rightarrow \nu_s$  conversion, and dotted line is for  $\nu_e \rightarrow \nu_\mu$  conversion. The following values of parameters were used:  $\Delta m^2 = 5 \cdot 10^{-6} \text{ eV}^2$ ,  $\sin^2 2\omega = 8.8 \cdot 10^{-3}$ , and (for our scenario)  $\cos^2 \psi = 0.5$  and  $\sin \phi = 0$ . Also the Super-Kamiokande experimental points from 306 days of observation are shown (statistical errors only).

Fig. 5. The level crossing scheme for supernova neutrinos. Solid lines show the eigenvalues of  $\nu_{3m}, \nu_{4m}, \nu_{2m}$  and  $\nu_{1m}$ . Black dots represent 4 resonances. Dashed lines correspond to energies of  $\nu_e, \nu_\tau$  (straight lines) and  $\nu'_{4m}, \nu'_{2m}$ .

## FIGURES

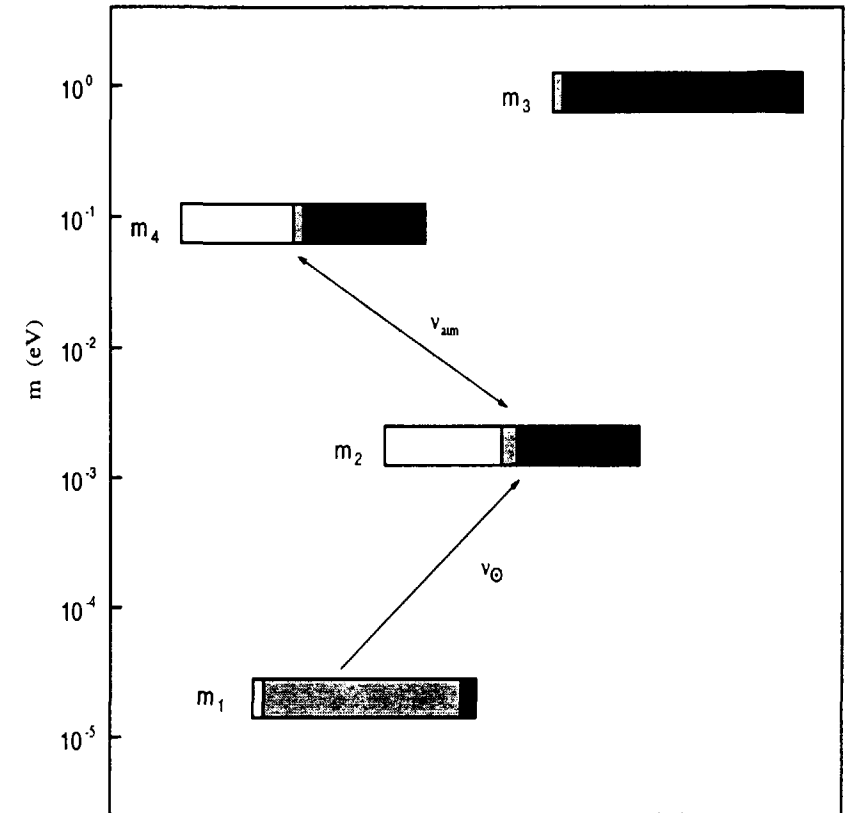


FIG. 1.

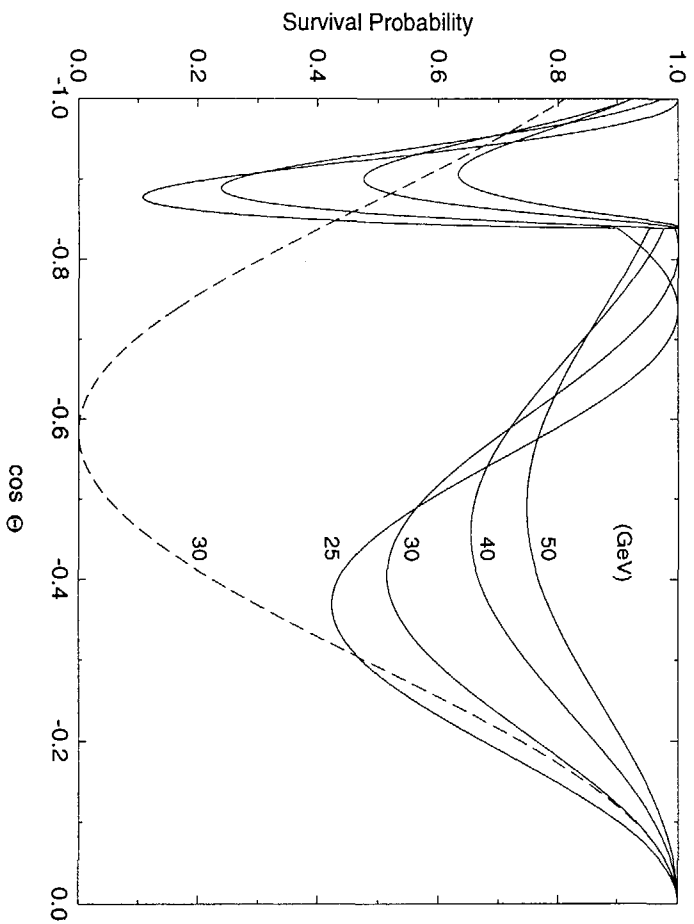


FIG. 2.

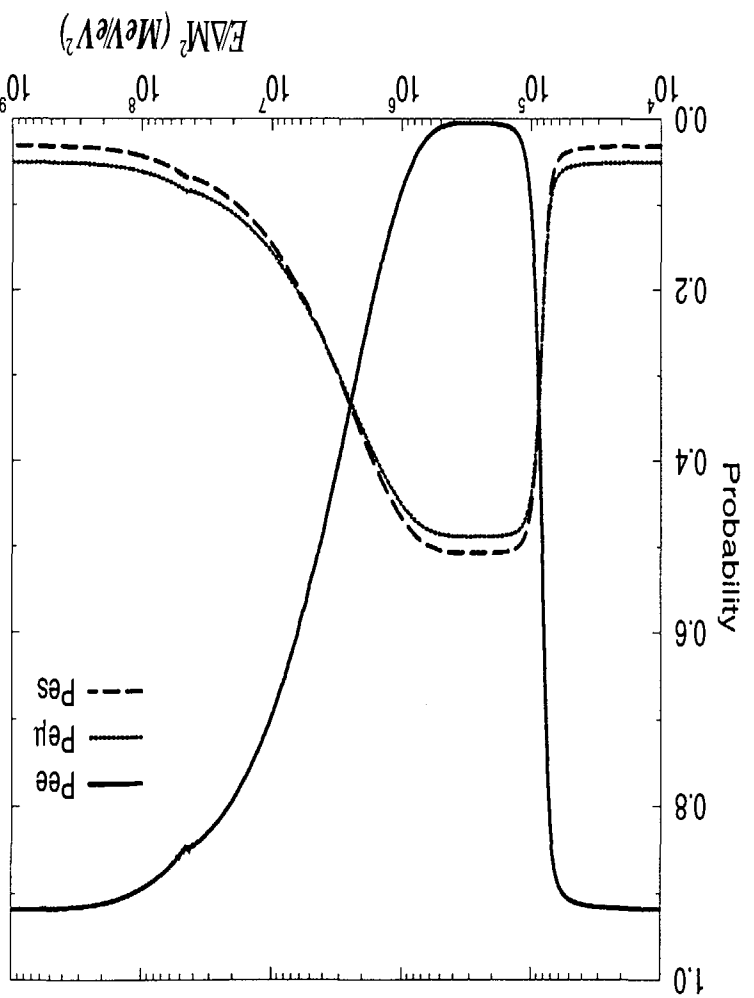


FIG. 3.

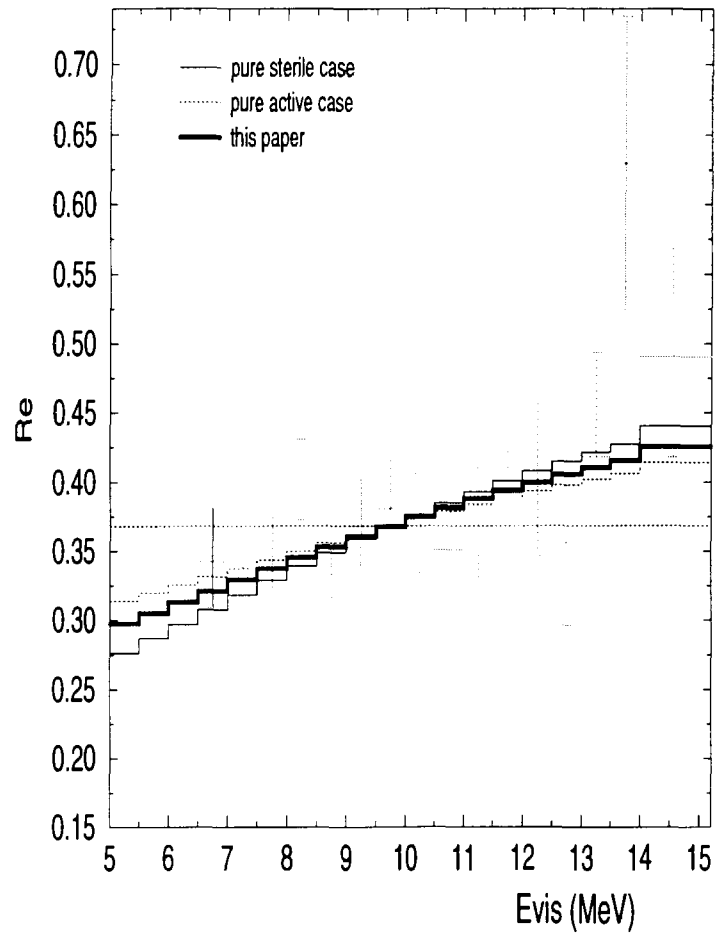


FIG. 4.

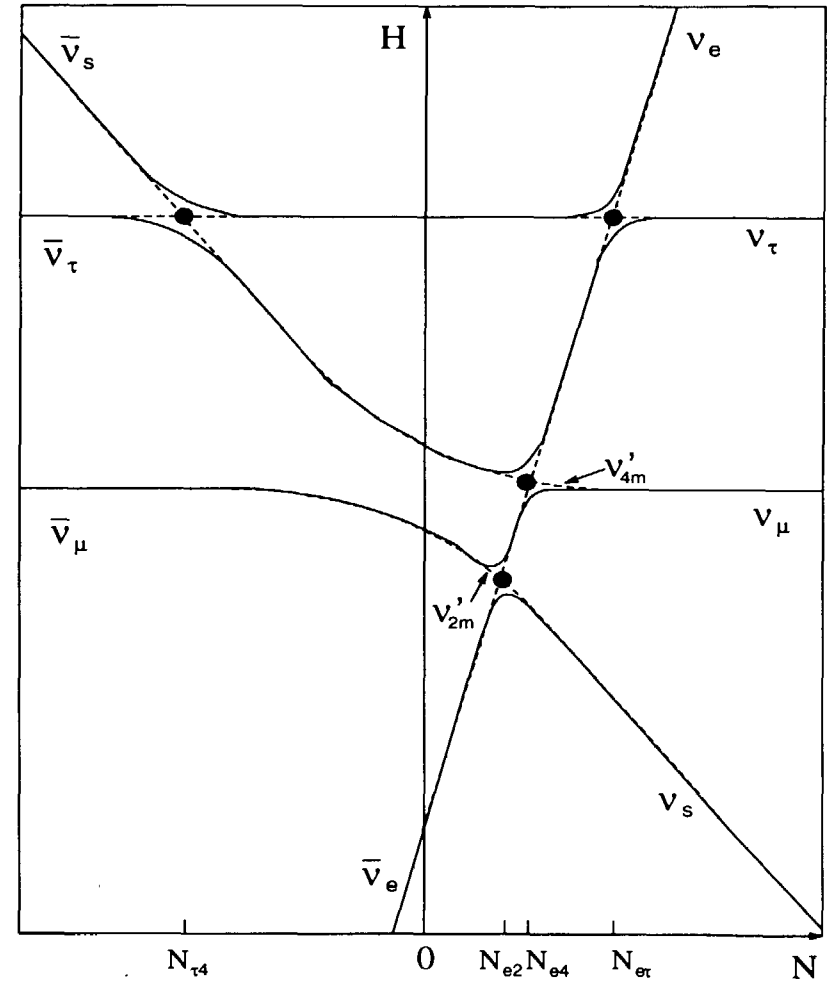


FIG. 5.