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สดชื่น วิบูลยเสข

ภาควิชาฟิสิกส์ คณะวิทยาศาสตร์ มหาวิทยาลัยเชียงใหม่

โทรศัพท์(053)943367 โทรสาร(053) 892270

บทคัดย่อ

นิวเคลียสหนักซึ่งมีนิวตรอนส่วนเกินสูงบางชนิดมีผิวนิวตรอนหนาบริเวณพื้นผิวนิวเคลียส งานวิจัยนี้จะได้พิจารณาถึง collective density excitations ในนิวเคลียสดังกล่าวโดยละเอียด โดยใช้แบบจำลองพลวัตของไหล (hydrodynamic model) และจะแสดงว่ามีความเป็นไปได้ที่จะเกิด isoscalar excitations พลังงานต่ำในนิวเคลียสแบบนี้ สถานะกระตุ้นนี้จะให้ข้อมูลเกี่ยวกับความหนาของผิวนิวตรอน สภาพอัด และแรงตึงผิวของสสารนิวตรอนที่ความหนาแน่นต่ำ

Theoretical Studies of Excited States of Heavy Nuclei With Large Neutron Excess

S. Wiboolsake

Department of Physics, Faculty of Science

Chiang Mai University, Chiang Mai 50200, THAILAND

Tel. (053)943367, Fax.(053)892270

ABSTRACT

Certain neutron excess heavy nuclei have a thick neutron skin on the nuclear surface. In this paper detailed collective densities excitations in heavy nuclei with a large neutron excess and a thick neutron skin will be considered within the framework of the "hydrodynamic" model. It will be shown that low energy isoscalar excitations are possible. The knowledge obtained will give information on the neutron thickness, the compressibility and the surface tension of neutron excess at low density.

1. Introduction

Recently the properties of neutron excess light nuclei have been studied extensively both theoretically and experimentally. The studies reveal the insight into the properties of the collective states in the consequence of excitations of a thick neutron skin on the nuclear surface such as low energy giant dipole resonance[4,5,6]. However, the results obtained cannot be extended to the case of neutron excess heavy nuclei as there exists the difference in the density of nuclear matter and the density of nucleons in the volume of the light nuclei and the heavy nuclei.

The main objective of the this paper is to try to give a possible description of the collective density excitations in large neutron excess heavy nuclei in a theoretical aspect. This type of survey, therefore, is quite sensitive to the model employed as will be shown tn the following calculation.

2. Equations of motion and the spectrum of isoscalar excitations

The model used is the hydrodynamic model with the first sound approximation which employs a two step radial density distribution as in Fig. 1. The main character of this model is that nucleons in the stable core nucleus and neutrons in the thick skin are both considered as nuclear matter with certain density. Thus the hydrodynamic wave equation will be used to describe the time and space evolution of these density excitations.

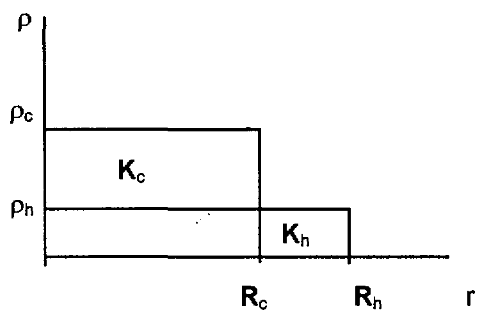

Fig. 1

This is justified due to the fact that nucleons in a nucleus acts strongly with a neutron halo as well as neutrons in the halo themselves. This gives rise to an abrupt change in the density both at the core radius, R_c, and at the radius of the neutron halo, R_h.

The equations of motion employed for both the neutron density excitations in the halo and the density excitations inside the core under this first sound approximation of the hydrodynamic model can be obtained easily obtained from the hydrodynamic wave equations:

$$\frac{\partial^2}{\partial t^2}\rho_h(r,t) - \frac{K_h}{9M}\Delta\rho_h(r,t) = 0 \qquad (1)$$

$$\frac{\partial^2}{\partial t^2}\rho_c(r,t) - \frac{K_c}{9M}\Delta\rho_c(r,t) = 0 \tag{2}$$

where ρ_h and ρ_c are the density excitations in the neutron halo and inside the corerespectively. K_h and K_c are the compressibilities of nuclear matter in the halo and inisde the core. M is the necleon mass. Obviously, the solutions of equations (1) and (2) must satisfy the boundary conditions at the surface of the core and the halo and at the outer surface of the neutron halo as follows:

The second term on the left hand side of both equations involve the pressure in the radial direction associated with the variation of neutron density.

i.e
$$P_h(r,t) = \frac{K_h}{9M}\rho_h(r,t) \tag{3}$$

and

$$P_c(r,t) = \frac{K_c}{9M}\rho_c(r,t) \tag{4}$$

where P_h and P_c denote the radial pressure in the halo and in the core respectively. It is easy to see that

$$P_c(r,t)\Big|_{R_c} = P_h(r,t)\Big|_{R_c} + P_s(R_c,t) \tag{5}$$

where $P_s(R_c,t)$ comes from the surface tension forces due to the change in the shape of the core surface.

The other two boundary conditions at the core surface are

$$V_c(r,t)\Big|_{R_c} = V_{sc}(R_c,t) \tag{6}$$

$$V_c(r,t)\Big|_{R_c} = V_h(r,t)\Big|_{R_c} \tag{7}$$

where $V_c(r,t)$ is the radial velocity of the nucleons in the core, $V_{sc}(R_c,t)$ is the velocity of the core surface and $V_h(r,t)$ is the radial velocity of the neutrons in the halo.

At the outer surface of the neutron halo, the boundary conditions are

$$V_h(r,t)\Big|_{R_h} = V_{sh}(R_h,t) \tag{8}$$

$$P_h(r,t)\Big|_{R_h} = P_{sh}(R_h,t) \tag{9}$$

The solutions of (1) and (2) are easily found.[1] to be of the form

$$\rho_h(r,t) = \rho_h\left[\alpha_h j_L(k_{nLh}r) + \beta_h y_L(k_{nLh}r)\right] Y_{LM}(\theta,\phi)T(t) \tag{10}$$

and

$$\rho_c(r,t) \quad = \quad \alpha_c \rho_c \left[j_L(k_{nLc} r) \right] Y_{LM}(\theta,\phi) T(t) \qquad (11)$$

α_c, α_h and β_h are the amplitudes ; $j_L(k_{nLh,c} r)$ the spherical Bessel function of the first kind; $y_L(k_{nLh} r)$ the spherical Bessel function of the second kind; $Y_{LM}(\theta.\phi)$ the spherical harmonics and $T(t)$ the harmonics time dependence. As $k_{nLh,c}$ are wave vectors of the n^{th} excitation of multipolarity L, thus the frequency, ω_{nL}, is

$$\omega_{nL} \quad = \quad \left(\frac{K_{c,h}}{9M} \right)^{1/2} k_{nLc,h} \qquad (12)$$

The spectrum of isoscalar collective excitations of multipolarity L can then be determined from

$$E_{nL} \quad = \quad \hbar \omega_{nL} \qquad (13)$$

3. Calculation of ω_{nL}

To find $\omega_{nL}$from (13) one must know $k_{nLc,h}$ which in turn involves the knowledge of $V_c, V_h, V_{sc}, V_{sh}, R_c,$and R_h. As we are now employing the hydrodynamic model ,the equation of continuity for nucleon and neutron flow in the core and in the halo applies here and yields,

$$V_c(r,t) \quad = \quad \frac{\alpha_c \nabla \left[j_L(k_{nLc} r) Y_{LM}(\theta,\phi) \right] \dot{T}(t)}{k_{nLc}^2} \qquad (14)$$

$$V_h(r,t) \quad = \quad \frac{\nabla \left\{ \left[\alpha_h j_L(k_{nLh} r) + \beta_h y_L(k_{nLh} r) \right] y_{LM}(\theta,\phi) \right\} \dot{T}(t)}{k_{nLh}^2} \qquad (15)$$

The second principle employed is that the multipole density oscillations in the volume of the nucleus give rise to multipole oscillations on the surface of the core and the neutron shell. Therefore the radius R_h, R_c depend on time and oscillate as

$$R_h(t) \; = \; R_h \left[1 + \alpha_{sh} Y_{LM}(\theta,\phi) T(t) \right] \qquad (16)$$

and

$$R_c(t) \; = \; R_c \left[1 + \alpha_{sc} Y_{LM}(\theta,\phi) T(t) \right] \qquad (17)$$

We then have

$$V_{sh}(R_h, t) \; = \; \dot{R}_h(t) \; = \; R_h \alpha_{sh} Y_{LM}(\theta,\phi) \dot{T}(t) \qquad (18)$$

and

$$V_{sc}(R_c,t) = \dot{R}_c(t) = R_c\alpha_{sc}Y_{LM}(\theta,\phi)\dot{T}(t) \tag{19}$$

The third principle is that the surface tension comes from the gradient terms in the density functional. Therefore, this tension forces act on the surface of the halo and on the surface of the core as well. Assuming that the nucleus is an incompressible fluid, the two forces then have the same form.

$$P_{sh}(R_h,t) = \frac{\sigma_h(L-1)(L+2)\,\alpha_{sh}Y_{LM}(\theta,\phi)\,T(t)}{R_h} \tag{20}$$

and

$$P_{sc}(R_c,t) = \frac{\sigma_c(L-1)(L+2)\,\alpha_{sc}Y_{LM}(\theta,\phi)\,T(t)}{R_c} \tag{21}$$

where σ_h is the surface tension of the outer surface of the neutron halo and σ_c is the surface tension of the core surface.

Substituting (10) - (12) and (14) - (21) into (5) - (9), the following five equations are obtained;

$$\frac{\alpha_c j_L'(k_{cnL}R_c)}{k_{cnL}R_c} = \alpha_{sc} \tag{22}$$

$$\frac{K_c}{9M}\rho_c\alpha_c j_L(k_{cnL}R_c) = \frac{\sigma_c}{R_c}(L-1)(L+2)\sigma_{sc} + \frac{K_h}{9M}\rho_h\left[\alpha_h j_L(k_{hnL}R_c)+\beta_h y_L(k_{hnL}R_c)\right] \tag{23}$$

$$\frac{\left[\alpha_h j_L'(k_{hnL}R_c)+\beta_h y_L'(k_{hnL}R_c)\right]}{k_{hnL}R_c} = \alpha_{sc} \tag{24}$$

$$\frac{K_h}{9M}\rho_h\left[\alpha_h j_L(k_{hnL}R_h)+\beta_h y_L(k_{hnL}R_h)\right] = \frac{\sigma_h}{R_h}(L-1)(L+2)\alpha_{sh} \tag{25}$$

$$\frac{\left[\alpha_h j_L'(k_{hnL}R_h)+\beta_h y_L'(k_{hnL}R_h)\right]}{k_{hnL}R_h} = \alpha_{sh} \tag{26}$$

These five equations may then be solved for the five unknowns $\alpha_h, \alpha_c, \beta_h, \alpha_{sc}, \alpha_{sh}$ as a nonzero oscillation frequency ω_{nL} occurs only when the determinant of these equation vanishes. These unknowns enable us to find the frequency of the density excitations of multipolarity L in a heavy nucleus with thick neutron skin which in turn yields the excitation energy E_{nL}.

4.Results

To test the validity of the model proposed, we calculated the excitation energies for tin, Sn whose proton number corresponds to a magic nucleus. The number of neutrons in the halo is $N_h = 40$ and the number of nucleons in the core is $A_1 = 120$.The results optained are shown below:

Table 1 shows the excitation energy as calculated from the above procedure. χ_n is the ratio between the core energies E_{nLc} and the halo energies E_{nLh}. These χ_n determine the nature of the states. The energy of monopole resonance is about 18 MeV whereas the dipole isoscalar resonance is about 25 MeV. The results shows taht there are many new levels which do not occur when a thick neutron skin is not considerd and that these states have much lower energies than these of giant resonance in nuclei taht do not have a halo.

It is impossible to study resonance at multipolarity ≥ 2 in the first sound approximation. Therefore, this work should be extended to cover those collective states.

5. References

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Table 1

n	L	E_{nL}	χ_n
1	0	8.1	0.112×10^{-1}
2	0	18.4	4.16
3	0	21.0	0.321
4	0	30.4	0.752×10^{-1}
5	0	34.6	14.3
6	0	43.1	0.311×10^{-1}
7	0	51.2	10.7
8	0	54.6	0.821×10^{-1}
9	0	67.2	0.205
10	0	68.7	7.62
1	1	6.1	0.291×10^{-1}
2	1	19.2	0.401×10^{-1}
3	1	26.1	18.4
4	1	31.5	0.436×10^{-1}
5	1	43.2	2.22
6	1	44.1	0.648
7	1	51.1	0.91×10^{-1}
8	1	59.4	17.6
9	1	65.8	0.421×10^{-1}
10	1	77.2	3.46

Experimental diffuseness of proton density(2) 0.48 F

Core compressibility(3) 220 MeV

Halo compressibility(3) 50 MeV