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MESOSCOPIC FLUCTUATIONS, TWO-PARAMETER SCALING AND CONCOMMITANT UNUSUAL LEVEL SPACING DISTRIBUTIONS IN FINITE 1D DISORDERED SYSTEMS

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Abstract

We study level spacing distributions of finite-sized one-dimensional disordered systems. As the system evolves from a quasi-ballistic to a strongly localized regime, the system crosses over from a strongly non-Wigner-Dyson type level spacing distribution to a universal Poisson distribution in the thermodynamic ($L \to \infty$) limit. In between it goes through regimes where the distribution seems to be a mixture of Wigner-Dyson type and Poisson type distributions, thus indicating existence of pre-localized states before the thermodynamic limit sets in.

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In a disordered chain with random but real-valued site-potentials, almost all the states are exponentially localized and hence an incident wave ($\sim e^{ikx}$) propagating in the positive x-direction is completely backscattered due to the well-known localization effects [1]. Because the hamiltonian is random, its eigenvalues (energy levels) are also random. There have been a lot of works [2] on the level spacing distribution (LSD) as well in such random disordered systems. The motivation for such studies come from the original works of Wigner [3], Dyson [4], and of Mehta [5] on the nuclear level spectra of heavy elements, where the number of levels are so large that they warrant a statistical description. But, for disordered systems, whose hamiltonian may be represented by a random matrix, randomness in the spectra and consequently the randomness in the level spacings come in a natural way. It was known for a long time that in the disordered metallic regime, the LSD becomes one of the three Wigner-Dysonian (depending upon the three possible symmetry classes), whereas in the insulating regime the LSD assumes a Poissonian form. These are the only four universal behaviors in the large length (thermodynamic) limit. More recent works [6] indicate that very near the critical disorder strength for the Anderson's metal-insulator (M-I) transition (which occurs only above 2D if there is no magnetic field), the LSD is neither Wigner-Dysonian on the metallic side, nor Poissonian in the insulating (localized) side, but it takes one of the universal forms as one renormalizes to large length limit. Further, the behaviour at the transition with a critical disorder, is non-universal (i.e., none of the above four) even in the thermodynamic limit.

One of the characteristic features of disordered mesoscopic systems is the statistics of its anomalously large conductance fluctuations and its universality in the diffusive regime. A lot of works [7] have been done in this area starting with the numerical work of Stone and Lee and the analytical work of Altshuler and others [8], to demonstrate that in the diffusive (disordered but metal-like) regime the sample-to-sample fluctuations of the two-probe conductance or the transmittance becomes universal. This is called the universal conductance fluctuations (UCF). Our recent works on mesoscopic conductance fluctuations indicate that the universality exists even in one dimension [9, 10]. The universality relates to the fact that the fluctuation (say the standard deviation) is independent of the size of the system, the strength of disorder, the Fermi energy of the charge carriers, or the type of hamiltonian (say, Schrodinger with Kronig-Penney model, or the nearest neighbour tight binding one). Hence the UCF is expected to be independent of the specific material parameters, but does depend on the dimensionality of the system. Until recently, it was believed that strictly speaking, UCF cannot occur in 1D. But, in a series of recent works (starting with the ref.[9]), we have shown that an almost diffusive regime does occur even in 1D (where $\xi = 4l_e$) and hence UCF is achieved in 1D as well starting from a length of about $2l_e$ and persisting upto a length slightly larger than ξ. To put things in perspective, we just quote the UCF values (in the unit of e^2/h) for electrons of one spin variety: they are 0.544, 0.431, 0.365 [8] and 0.30 [9] in three, two, quasi-one, and one dimensions respectively.

More recently, we have demonstrated [10, 11] the inadequacy of the one parameter scaling [12] in 1D from the quasi-ballistic upto some mildly localized regime (i.e., in finite-sized samples upto about 2 times the localization length). In our previous work [11] (referred to as I from now on), we had reported that there are at least two relevant parameters in this regime in the sense that the mean and the variance of the variable $u = \ln(1 + \mathrm{R}_4)$ increase as a function of the length L with two different and independent power-law exponents. In this regime $< u > \sim L$ for any Fermi-energy (E_F), but $var(u) \sim L^{\kappa}$, where κ depends on the E_F and is always greater than 1. Beyond this length scale, the behaviour slowly crosses over towards the one parameter scaling behaviour in the thermodynamic limit in the sense that both $< u >$ and $var(u)$ diverge with the same exponent 1, irrespective of the E_F. In I, we had presented a typical case of $W/V = 1.0$ and $E_F/V = 1.6$ where V is the hopping term and found that $\kappa = 1.57$. We give here the values of κ for some other Fermi energies (but the same W): (i) for $E_F/V = 0.1$, $\kappa = 1.54$, (ii) for $E_F/V = 0.5$, $\kappa = 1.68$, and (iii) for $E_F/V = 1.9$, $\kappa = 1.39$. Thus, it appears that from the quasi-ballistic to the mildly localized regime, the $var(u)$ increases in an independent and faster power law fashion than $< u >$ does and that other than the disorder strength (or, equivalently, the Thouless energy) there is another relevant energy scale, namely the Fermi energy itself. We reiterate that our main focus is on the finite-size effects on the scaling and the level spacing distributions.

Next, one observes that conductance as a probe samples local LSD in the vicinity of the Fermi-energy. This is explicitly seen in the Kubo formula for conductivity which connects the properties of an open quantum system (conductance or reflectance) to those of a closed quantum system (energy level spectra). For transport to take place in a Fermi system, a particle from a state below the E_F, must be excited to a state above it, thereby creating a particle-hole pair. The steady-state dynamics of such pairs is responsible for the conduction in the sample. Since conductance in finite-sized systems is non-universal (beyond one-parameter scaling theory), we had conjectured in I that the LSD for finite-size systems should also be non-universal upto about that length scale (i.e., about 2ξ) for the state with the largest ξ (usually the band centre $E = 0$ of a pure system obeying a tight binding hamiltonian). Thus, we focus on presenting the LSD of finite and *closed quantum* chains. The results presented below should amply support our conjecture in I. The other reason for focussing on finite chains is that many of the current experiments on mesoscopic fluctuations and quantum chaos are indeed done on low-dimensional and small-sized systems (even zero-dimensional, for a quantum dot).

We consider a quantum chain of N lattice points (lattice constant unity), represented by the standard single band, tight binding equation:

$$(E - \epsilon_n)c_n = V(c_{n-1} + c_{n+1}). \tag{1}$$

Here E is the fermionic energy, V is the constant nearest neighbour hopping term, ϵ_n is the random site-energy, and c_n is the site amplitude at the n-th site. Without any loss of generality, we choose $V = 1$ to set the energy scale. Further, we choose ϵ_n randomly from an uniform distribution with $P(\epsilon_n) = 1/W$ only inside the real, symmetric interval $[-W/2, W/2]$. The $N \times N$ tridiagonal matrix (random in the diagonal entry) represented by the above equation is diagonalized using the standard procedures to obtain the N energy levels for various configurations with the same W. Since we are dealing with disorder, we have not used any periodic boundary condition and kept it free. The set of levels $(E_n; n = 1, 2, ..., N)$ for each configuration is then sorted in an increasing order. Thus we obtain $N - 1$ level spacings $s = E_{n+1} - E_n$ for each configuration. Next we obtain the normalized histogram $P(x)$ representing the LSD with the scaled level spacing $x = s/ < s >$, where $< s >$ is the average level spacing. The more the number of configurations we choose, the smoother is the LSD. We would like to present our results in two different ways. In the former, keeping the length $L = N - 1$ fixed, we will keep on changing the disorder W from very small to quite large values. In the latter presentation we will do just the opposite, namely, that we will hold the disorder W fixed and change L from very small to fairly large values. As we find out, both the ways of presentation has some complementary aspects to display.

In the Figs.1(a)-1(j), we present the LSD for a system of size $N = 51$ and for various W starting from an almost pure sample with $W = 10^{-5}$ to a strongly disordered system with $W = 10.0$. In the case of the Fig.1(a), the system is almost periodic and the $P(x)$ has a highly peaked structure with large gaps for small x. The gaps tend to disappear only near large x. All the peaks have the same height equal to 100, except a smaller single one in the middle reminding us that there is a minute disorder in the system. In the Fig.1(b) where $W = 10^{-3}$, the gaps are almost as before, but the maximum peak height is smaller (about 80), and hence the peaks have become broader (indeed at large x some of them have even merged together). Further, peak heights have become more random in response to the stronger disorder than in Fig.1(a). In the next Fig.1(c) for $W = 10^{-2}$, the peak heights become even smaller but wider, with the largest peak very close to the upper bound of x, where $P(x)$ drops down to zero with a very sharp band-edge like discontinuity. It may be noted that the fragmented structure of $P(x)$ still remains. The gaps are the signatures of an almost ballistic regime, which implies that the localization lengths of almost all the states are larger than the system size chosen ($L = 50$). In Fig.1(d) for $W = 0.05$, the gaps between the peaks have completely disappeared, and the distribution at low x has a very interesting oscillatory form, but for a strong repulsion (gap) upto $x \le x_0 \simeq 0.06$. This seems to be the typical behaviour of $P(x)$ in the quasi-ballistic regime. In this regime, the band-edge like behaviour (at large x) beyond the largest peak has transformed into a less sharp decay. In Fig.1(e) for $W = 0.1$, the oscillatory pattern of $P(x)$ has completely vanished, indicating that the localization effects have started becoming prominent. But, the behaviour is still metal-like in the sense that $P(x = 0) = 0$ (level-repulsion). Yet the LSD is far from a Wigner-Dyson (WD) type first because $P(x) = 0$ upto $x_0 = 0.03$ (even though it is smaller than the same for $W = 0.05$) and next because a small hump appears near $x = 0.2$ followed by a flat region and a much larger second peak (at $x \simeq 1.5$) which finally decays very sharply quite close to zero near $x \simeq 1.7$. Thus this case still indicates a much stronger level repulsion than in the case of a Wigner-Dyson distribution, particularly because $P(x)$ has an interesting *double-peaked structure*. This double peaked structure is more prominent in Fig.1(f) for $W = 0.5$. But, the peaks have come closer together with the smaller one at $x \simeq 0.5$ and the larger one at $x \simeq 1.2$. Further the decay beyond the larger peak is slower than before ($P(x > 2.4) \simeq 0$), and while $P(x = 0) = 0$, $P(x > 0) > 0$. Clearly, this LSD is approaching a WD distribution, even though it is qualitatively very different from a WD form. For a mild disorder $W = 1.0$ as in Fig.1(g), the double-peaked structure has disappeared (the two peaks merge), and the LSD is qualitatively quite similar to a WD type. In the case of both the Fig.1(h) and Fig.1(i) for $W = 2.5$ and 5.0 respectively, again we have something very different from a WD in the sense that $P(x = 0) > 0$ (level repulsion has disappeared), but on the other hand there is a single, broad peak at a finite $x > 0$, indicating the presence of some remnants of metal-like correlations. Thus these two $P(x)$'s seem to be a combination of a WD distribution and a Poisson distribution. Physically, this seems to indicate that some of the states are localized and others are extended for this finite sized sample. It is reminiscent of the existence of pre-localized states [13]. Finally, for a large disorder ($W = 10.0$) as shown in Fig.1(j), even the largest localization length at $E = 0$ is much smaller than the system size ($L = 50$). Hence all the states are strongly localized, and we get back the universal (Poisson) distribution in 1D in the thermodynamic ($N \to \infty$) limit.

Next, we would like to show pictorially in the Fig.2, the evolution of the LSD as a function of the system size N, for a fixed, mild disorder strength $W/V = 1.0$ ($V = 1.0$ as stated before). Note that for this disorder, $\xi \simeq 100$. In contrast with Fig.1, all the LSD's are continuous here because we are probing the quasi-ballistic regime not with an extremely small disorder, but with a mild disorder and very small length of, say $N = 3$, as in Fig.2(a). The LSD shows no fragmentation or any oscillatory behaviour and is rather narrow with a single global peak at $x \simeq 1.0$ and a fast decay practically to zero beyond $x = x_0 \simeq 1.4$. Thus there is no sign of quasi-ballisticity and the effect of disorder is felt even at this small length. But the $P(x)$ shows a much stronger level repulsion than in the WD case in the sense that $P(x) = 0$ upto a $x_0 \simeq 0.7$ [compare with the cases of Fig.1(d) and Fig.1(e)]. Almost similar is the case of Fig.2(b) for $N = 11$, where the very strong level repulsion is somewhat reduced in the sense that the initial gap in $P(x)$ extends only upto an $x_0 \simeq 0.075$ (this is not so clear from the figure, but becomes evident on a log-log plot, which gives a power-law behaviour for small x: $P(x) \sim (x - 0.075)^{\beta}$ with $\beta \simeq 2.2$). The cases of Figs.2(c-f) for $N = 21$, 31, 41 and 51 respectively, looks qualitatively like WD, but the exponent β keeps decreasing from about 2 to 0.58 for these cases. While a $\beta < 1$ indicates a weaker level repulsion than in WD and hence seems surprising, one may find its origin in the Figs.1(e-g), where the weaker peak at small x slowly comes closer to the stronger peak and gives the LSD a convex shape upto the single global peak (implying a $\beta < 1$). For still larger sizes of $N = 101$, 201, and 501 as shown in the Figs.2(g-i) respectively, the LSD seems to be a mixture of WD and Poisson type distributions and hence seems to indicate the existence of pre-localized states [13] as in the cases of Figs.1(h) and 1(i). Finally, in the case of Fig.2(j) for $N = 1001$, $L \simeq 10\xi(E = 0)$, but the LSD does not yet seem to have reached the universal Poisson form (consistent with I). In contrast, for the case of Fig.1(j), $L \simeq 20\xi(E = 0)$ and hence in conformity with our work in I, shows a Poisson distribution.

To summarise and to draw some conclusions, we hope to have shown that finite-sized 1-D disordered systems do possess interesting properties in the mesoscopic regime (either for $L \to \infty$ at a temperature $T \to 0^+$, or $L < l_\phi$, where l_ϕ is the inelastic scattering length or the phase decoherence length due to scatterings with phonons at a $T > 0$. For example, we had already shown that a UCF of about $0.3e^2/h$ *does* occur in the 1-D system [9] around the quasi-diffusive to the weakly localized regime ($L \sim \xi/2$ to about ξ). Further, from the ballistic to the mildly localized regime ($\sim 2\xi$, independent of the Fermi energy E_F), we observe a two-parameter scaling [11], in the sense that the average $< u > \sim L$ almost right from the quasi-ballistic regime onwards. But, as we show in I as well as with additional work here, var(u) $\sim L^\kappa$ where $\kappa = \kappa(E_F) > 1$ is an independent exponent in the regime $L = 0$ to $L \simeq 2\xi$. Of course, in the asymptotic limit $L \to \infty$ (indeed for $L \geq 20\xi$), the behaviour crosses over to $\kappa \to 1^+$, which is consistent with the one-parameter scaling [12]. The reason for the above behaviour for finite size open quantum systems was found to be that the phase (coherent wave nature) ϕ of the electron's reflection co-efficient follows a distribution $P_L(\phi)$ which is most of the times far from uniform, and continues evolving as a function of L towards its stationary form $P_\infty(\phi)$ only for $L >> 2\xi$. In the case of a finite, closed quantum system, we did look at the finite number of random energy eigenstates and the corresponding finite number of random nearest level spacings (s). Kubo formula indicates that the $P_L(s)$ (random LSD) should also be unusual in the same regime in which the mesoscopic conductance fluctuation is so, i.e., in the two-parameter scaling regime. Indeed, one observes a very strong level-repulsion (much stronger than in the case of Wigner-Dyson) in the quasi-ballistic regime. Typically, the exponent β in the power-law prefactor of $P_L(x)$ where $x = s/ < s >$ is the scaled level spacing, as obtained from small x is significantly larger than unity. As the system crosses over through the diffusive regime, β seems to decrease to values less than unity. In the mildly localized regime ($L \sim 2\xi$), there seems to be a non-zero probability of having $x = 0$, and yet there is a Wigner-Dyson-like peak before the eventual decay of the $P_L(x)$ for large x. This is certainly quite unusual and seems to indicate that there are small clusters of pre-localized states [13] in some energy regime and some infinite clusters of extended states in another energy regime. Finally, in the large length/ disorder limit, the extended states vanish, and one gets back the universal Poissonian behaviour for $P_{L\to\infty}(x)$.

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References

[1] P.A. Lee and T.V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985). See also *Scattering and Localization of Waves in Random Media*, ed. P. Sheng (World Scientific, Singapore, 1990).

[2] C. W. J. Beenakker, Rev. of Mod. Phys. **69**, 731 (1997) for an excellent review and the detailed references therein.

[3] E.P. Wigner, Ann. Math. **53**, 36 (1951); **62**, 548 (1955); **65**, 203 (1957); **67**, 325 (1958).

[4] F.J. Dyson, J. Math. Phys. **3**, 140 (1962); **3**, 157 (1962); **3**, 1191 (1962); **3**, 1199 (1962).

[5] F.J. Dyson and M.L. Mehta, J. Math. Phys. (N.Y.) **4**, 701 (1963); M.L. Mehta, *Random Matrices* (Academic, New York, 1991).

[6] B.I. Shklovskii, B. Shapiro, B.R. Sears, P. Lambrianides, and H.B. Shore, Phys. Rev. B **47**, 11487 (1993); V.E. Kravtsov, I.V. Lerner, B.L. Altshuler, and A.G. Aronov, Phys. Rev. Lett. **72**, 888 (1994); A.G. Aronov, V.E. Kravtsov, and I.V. Lerner, Phys. Rev. Lett. **74**, 1174 (1995).

[7] For a detailed review see *Mesoscopic Phenomena in Solids*, eds. B.L. Al'tshuler, P.A. Lee, and R.A. Webb (Elsevier, North Holland, Amsterdam, 1991).

[8] B.L. Al'tshuler, Pis'ma Zh. Eksp. Teor. Fiz. **41**, 530 (1985) [JETP Lett. **41**, 648 (1985); P.A. Lee and A.D. Stone, Phys. Rev. Lett. **55**, 1622 (1985); P.A. Mello, Phys. Rev. Lett. **60**, 1089 (1988).

[9] S. Gangopadhyay and A.K. Sen, Phys. Rev. B **46**, 4020 (1992).

[10] M.N. Ganguli and A.K. Sen, Phys. Rev. B **52**, 17342 (1995) for the appearence of two length scales in the weakly localized regime.

[11] A.K. Sen, Mod. Phys. Lett. B **11**, 555 (1997).

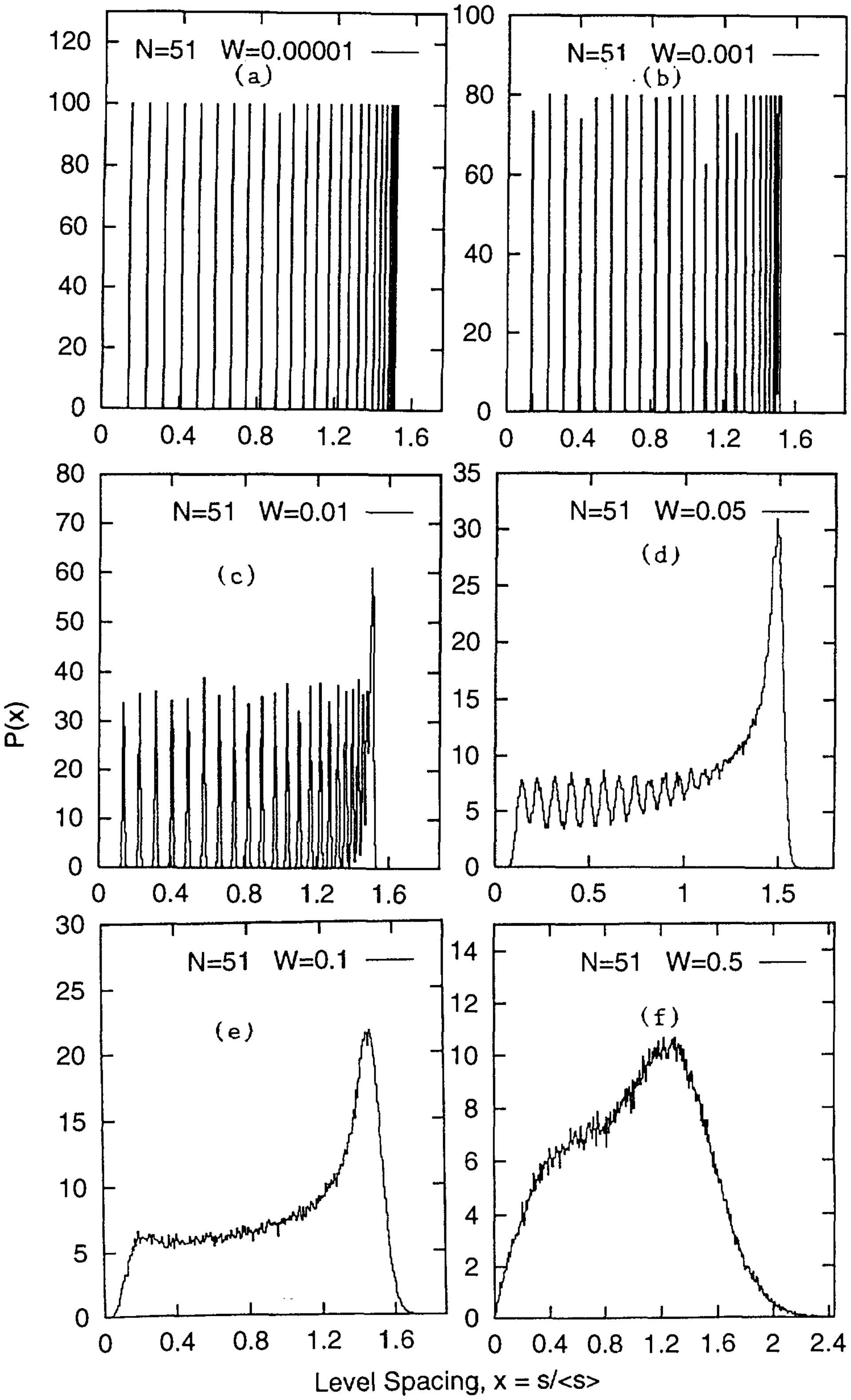
[12] E. Abrahams, P.W. Anderson, D.C. Licciardello, and T.V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979)

[13] V.I. Falko and K.B. Efetov, Phys. Rev. B **52**, 17413 (1995).

Figure Captions:

Fig.1 Normalized level spacing distributions $P(x)$ (as the function of a dimensionless level spacing $x = s/ < s >$ where $< s >$ is the average level spacing for a particular case) of a 1-D disordered system of a fixed size $N = 51$ and varying disorder strengths (W/V) equal to (a) 0.00001, (b) 0.001, (c) 0.01, (d) 0.05, (e) 0.1, (f) 0.5, (g) 1.0, (h) 2.5, (i) 5.0, and (j) 10.0.

Fig.2 Normalized level spacing distributions (as in Fig.1) of a 1-D disordered system for a fixed and small disorder $W/V = 1.0$ and varying sizes (N) equal to (a) 3, (b) 11, (c) 21, (d) 31, (e) 41, (f) 51, (g) 101, (h) 201, (i) 501, and (j) 1001.



N=51 W=0.00001
(a)
N=51 W=0.001
(b)
N=51 W=0.01
(c)
N=51 W=0.05
(d)
N=51 W=0.1
(e)
N=51 W=0.5
(f)
P(x)
Level Spacing, x = s/<s>

Fig.1

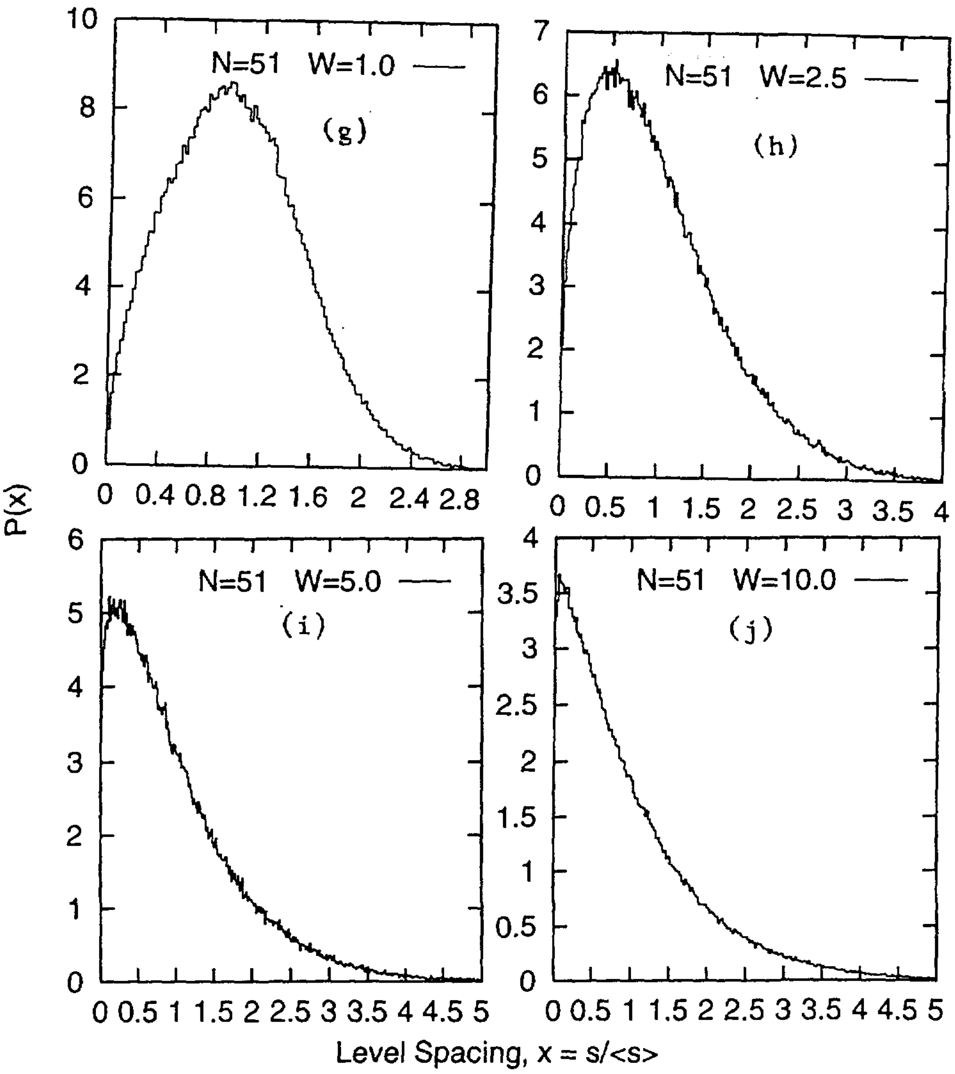
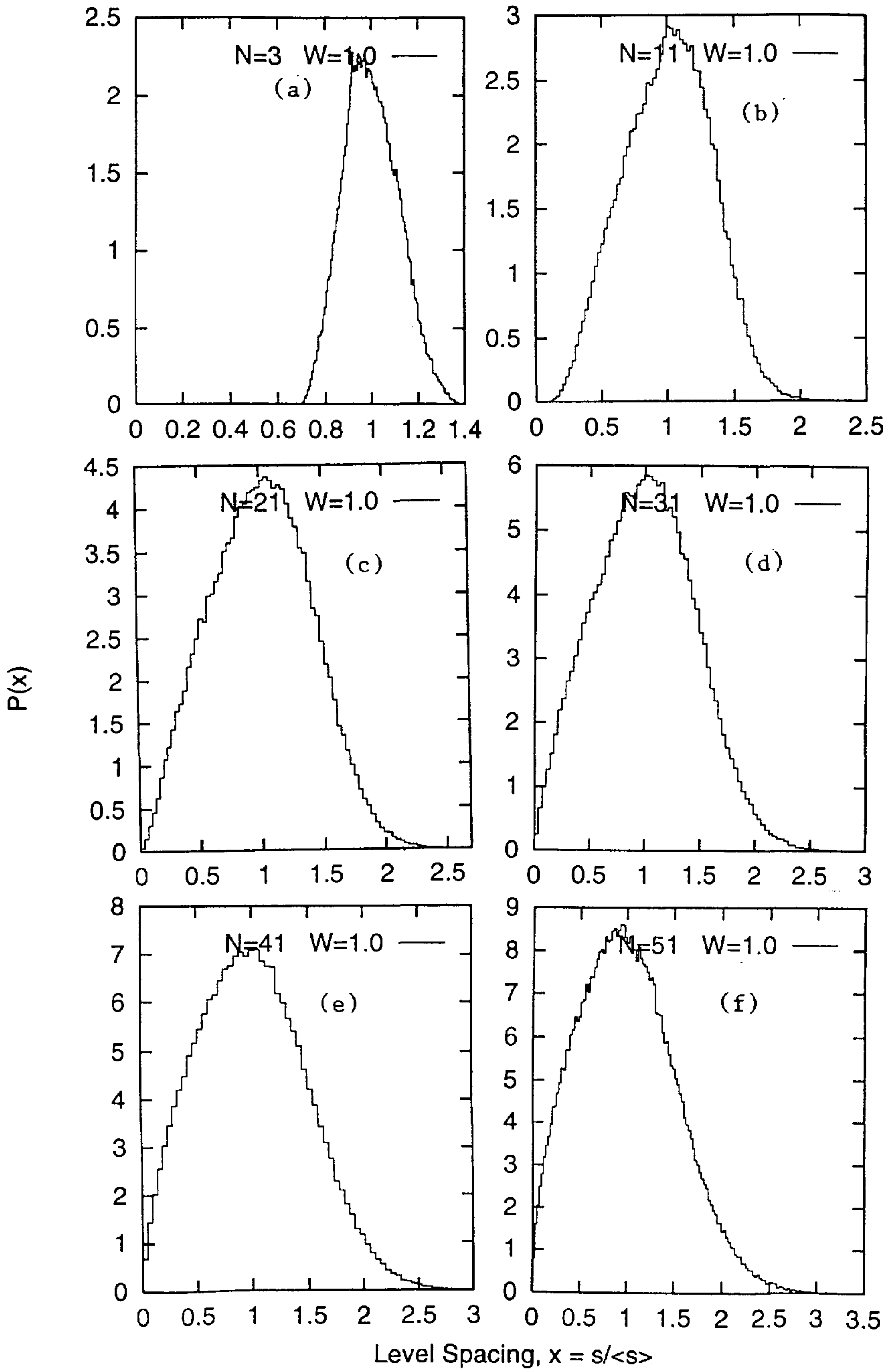


Fig.1 (cont.)



N=3 W=1.0
(a)
N=11 W=1.0
(b)
N=21 W=1.0
(c)
N=31 W=1.0
(d)
N=41 W=1.0
(e)
N=51 W=1.0
(f)
P(x)
Level Spacing, x = s/<s>

Fig.2

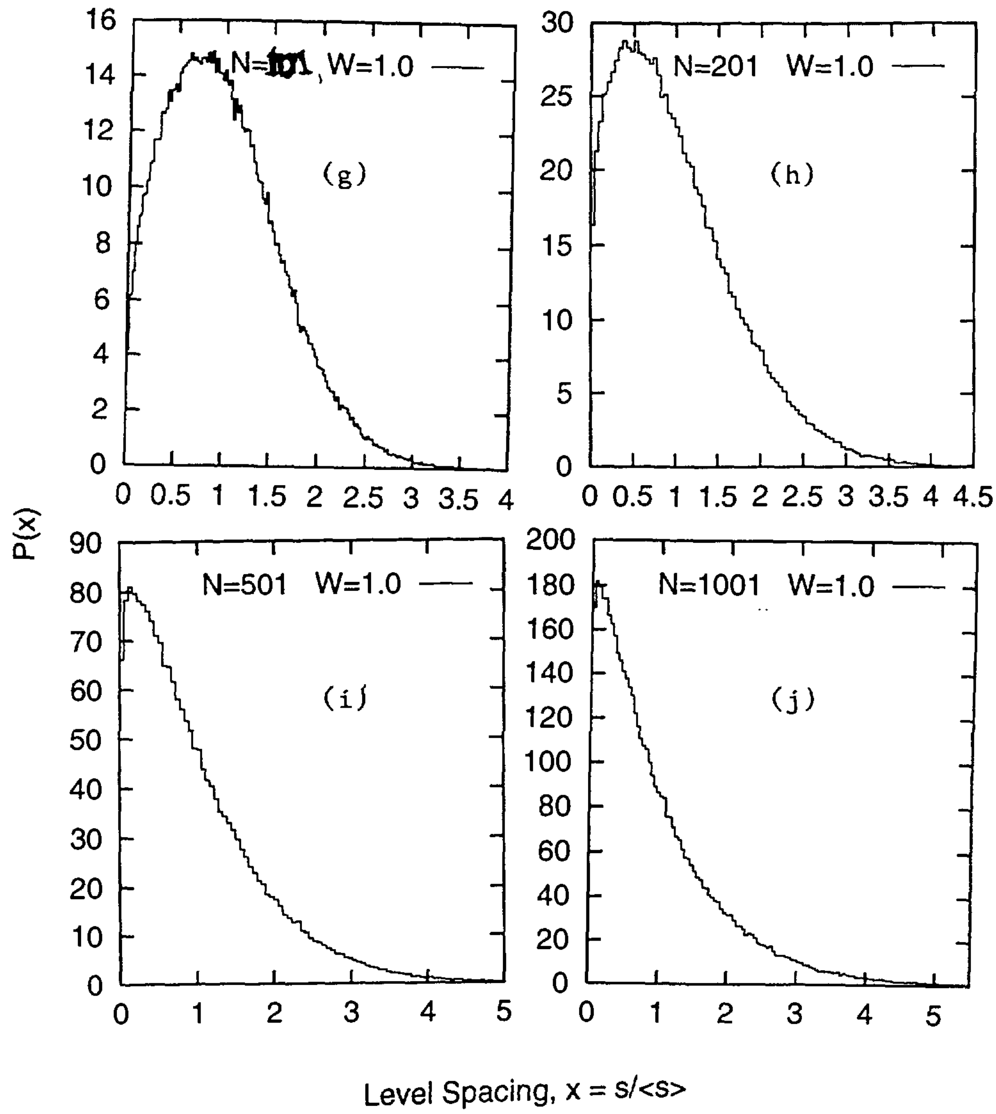


Fig.2 (cont.)