Orthogonality and Boundary Conditions in Quantum Mechanics

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One dimensional particle states are constructed according to orthogonality conditions, without requiring boundary conditions. Free particle states are constructed using Dirac's delta function orthogonality conditions. The states (doublets) depend on two quantum numbers: energy and parity ("+" or "-"). With the aid of projection operators the particles are confined to a constrained region -a¡x¡a, in a way similar to the action of an infinite well potential. From the resulting overcomplete basis only the mutually orthogonal states are selected. Four solutions are found, corresponding to different non-commuting Hamiltonians. Their energy eigenstates are labeled with the main quantum number n and parity "+" or "-". The energy eigenvalues are functions of n only. The four cases correspond to different boundary conditions: (I) the wave function vanishes on the boundary (energy levels: $1^+, 2^-, 3^+, 4^-,$) , (II) the derivative of the wavefunction vanishes on the boundary (energy levels $0^+, 1^-, 2^+, 3^-,$) , (III) periodic, symmetric boundary conditions (energy levels: $0^+, 2^+, 2^-, 4^+, 4^- 6^+, 6^-,$) , (IV) periodic, antisymmetric boundary conditions (energy levels: $1^+, 1^-, 3^+, 3^-, 5^+, 5^-,$). Orthogonality seems to be a more basic requirement than boundary conditions. By using projection operators, confinement of the particle to a definite region can be achieved in a simple and unambiguous way, and physical operators can be written so that they act only in the confined region.