Error Reduction in Perturbative Approximations to Nonlinearly Coupled Harmonic Oscillators

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Error estimates for perturbative approximations to solutions of systems with small nonlinearities are limited in scope. For single oscillator systems, and for systems with more than one degree of freedom that depend on a single phase, the error between the approximation and the full solution is bounded by $O(\epsilon)$ for times of $O(1/\epsilon)$, where —ϵ—=AB1 is the strength of the perturbation. Otherwise, error estimates exist only on approximations to the *action* variables. Most of the error estimates have been derived for systems in which the unperturbed Hamiltonian is itself nonlinear. No information is available on errors in the *phases.* For the study of the topological structure of the solutions in phase space this may be sufficient. However, for good long-term approximations to the full solution of a dynamical system, this situation is not satisfactory.
Through a sequence of examples of growing complexity, of nonlinearly coupled harmonic oscillators, we have shown [1] that, by applying *minimal normal forms* (MNF) to the dynamical equations obeyed by the phases of all the degrees of fredom, the errors in the approximations to solutions are reduced siginficantly in comparison with the usual procedure for computing the perturbative approximations.
The normal form is given as a formal power series in the expansion parameter of the problem. Using the freedom inherent in any perturbation expansion, the normal form may be formally truncated into a finite sum, the MNF. In the present work this idea is applied to the phase part of the normal form equations.

[1] P. B. Kahn and Y. Zarmi, Physica D, in press (1998).