QCD space-time description of high-density particle matter in high-energy collisions

... from clean e^+e^- to dirty nuclei ....

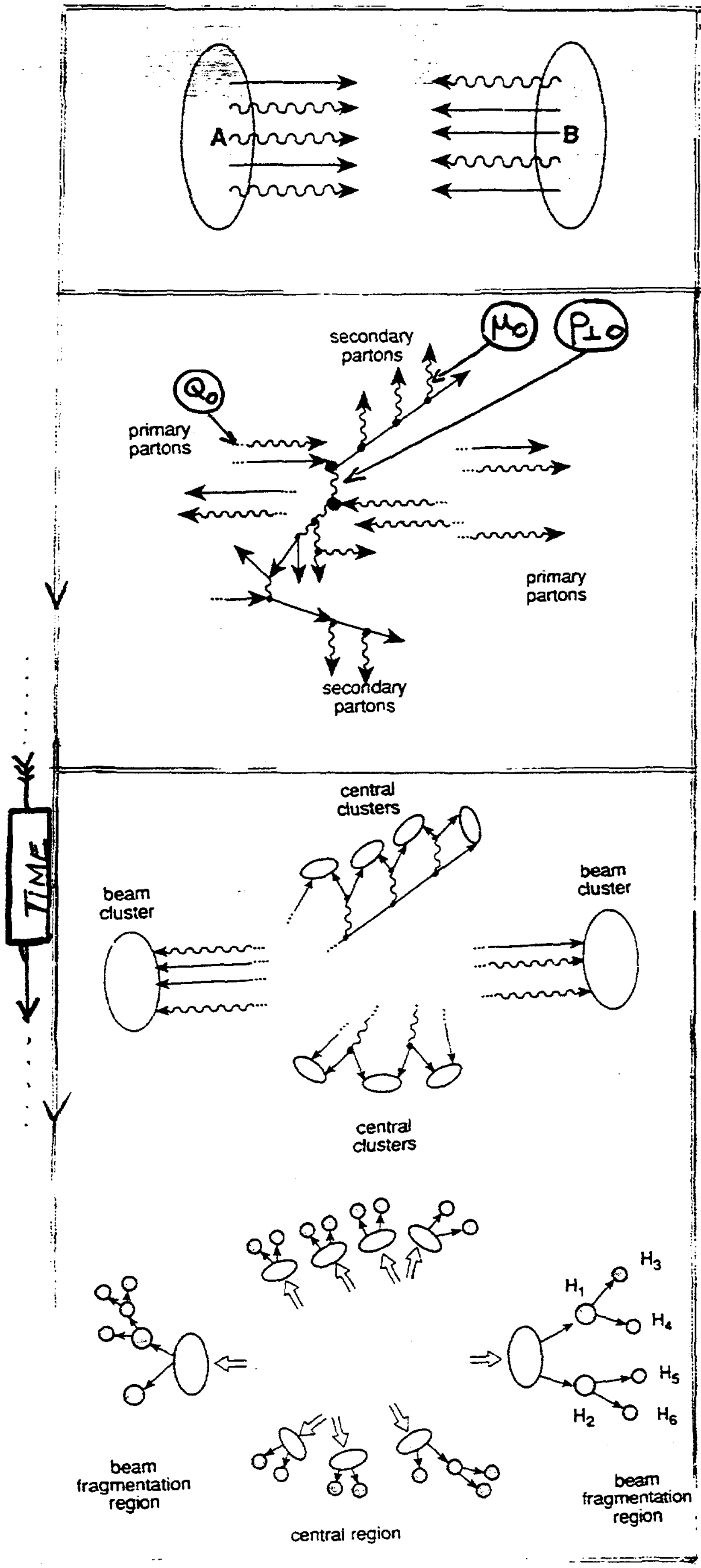
Klaus Kinder-Geiger

Kay Kay Gee

BNL

- Why at all ?
- How to ?
- Does it make sense ?
- What then ?

THE PARTON CASCADE MODEL



- INITIAL STATE

SCALE: Q_0^2

- PARTON CASCADE DEVELOPMENT

SCALE: $[Q_0^2, \mu_0^2]$

- PARTON RECOMBINATION (preconfinement)

SCALE: $[\mu_0^2, \Lambda^2]$

- CLUSTER FRAGMENTATION (confinement)

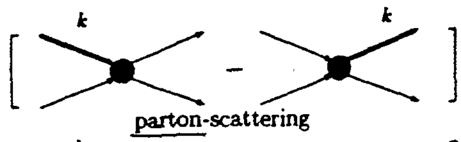
SCALE: Λ^2

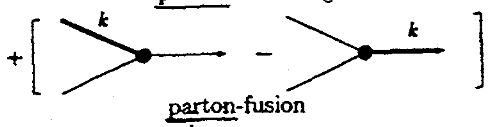
4/6

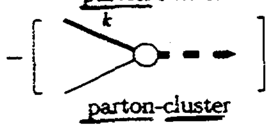
Closed set of "balance" equations

- **partons:**

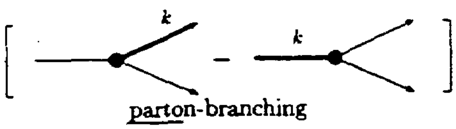
$$k \cdot \frac{\partial}{\partial r} F_p =$$





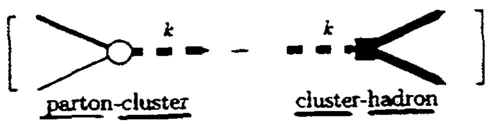


$$k^2 \frac{\partial}{\partial k^2} F_p =$$



- **pre-hadronic clusters**

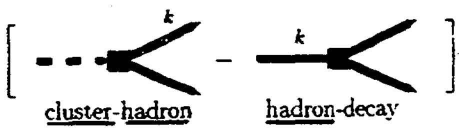
$$k \cdot \frac{\partial}{\partial r} F_c =$$



$$k^2 \frac{\partial}{\partial k^2} F_c \approx 0$$

- **hadrons**

$$k \cdot \frac{\partial}{\partial r} F_h =$$



$$k^2 \frac{\partial}{\partial k^2} F_h \approx 0$$

Microscopic dynamics ⟶ macroscopic observables

Lorentz-invariant 1-particle distribution ("Wigner function"):

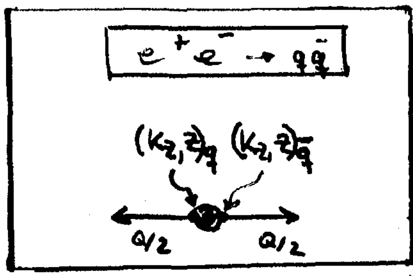
$$F_a(r,k)\, d^4r\, d^4k = \begin{cases} \# \text{ of particles } a = g, q, \text{ cluster}, \pi, K, p \ldots \\ \text{at time } t \text{ within } dt, \text{ position in } d\vec{r} \text{ around } \vec{r}, \\ \text{energy (momentum) in } dE\, (d\vec{k}) \text{ around } E\ (\vec{k}). \end{cases}$$

$$r^\mu = (t, \vec{r}) \; ; \quad k^\mu = (E, \vec{k}) \; ; \quad k^2 \begin{cases} = m^2 & \text{on shell} \\ < m^2 & \text{space-like} \\ > m^2 & \text{time-like} \end{cases}$$

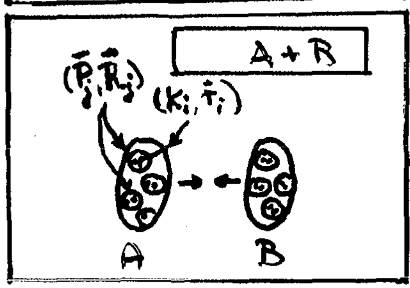
Initial value problem: specify initial state at time $t = t_0 \equiv 0$:

$$F_a^{(0)}(r,k) \equiv F_a(t=0, \vec{r} \,|\, E, \vec{k}) = \frac{d N_a(t=0)}{d^3r\, d^3k\, dE}$$

$$= \mathcal{P}_a(E,\vec{k}) \otimes \mathcal{R}_a(\vec{r})$$



e.g. $$\sum_{i=q,\bar{q}} \left[\delta(E_i - \tfrac{Q}{2})\, \delta(k_{z_i} \pm \tfrac{Q}{2})\, \delta^2(\vec{k}_\perp) \right] \left[e^{-z_i^2 Q^2}\, \delta^2(\vec{r}_\perp) \right]$$



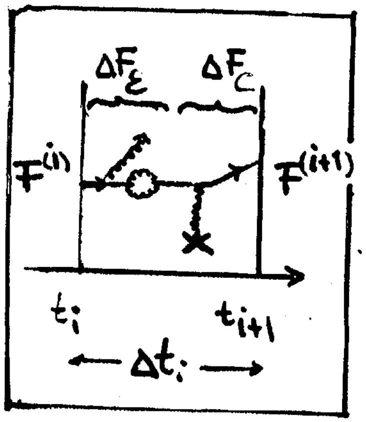
e.g. $$\sum_{j=1}^{A+B} \sum_{i=1}^{n(j)} \left[f_a^j\!\left(\frac{k_{z_i}}{P_j}, Q_0^2\right) g(\vec{k}_{\perp_i}) \right] \left[e^{-\nu^2 (\vec{r}_i - \vec{R}_j)^2}\; h^{A(B)}(\vec{R}_j, \vec{P}_j) \right]$$

For $t > t_0 = 0$, solve coupled evolutional & collisional cascade in small timesteps :

$$\frac{\partial}{\partial \ln k^2} F_a(r,k) = \mathcal{E}_a(F) \otimes F_a$$

$$k \cdot \frac{\partial}{\partial r} F_a(r,k) = C_a(F) \otimes F_a$$

Iterative 2-step procedure : trace particle history $t_i = t_{i-1} + \Delta t_i$ from t_0 with $F_a^{(0)}(t_0)$:



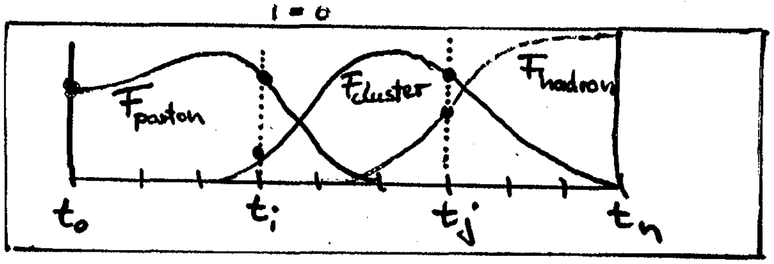
$$F^{(i+1)} = F^{(i)} + \Delta F_{\mathcal{E}}^{(i)} + \Delta F_C^{(i)}$$

$$\Delta F_{\mathcal{E}}^{(i)} = \Delta \ln k^2 \, \mathcal{E}(F^{(i)}) \otimes F^{(i)} \cdot \Theta(\Delta t_i - \Delta \ln k^2)$$

$$\Delta F_C^{(i)} = \frac{\Delta t}{E} \, C(F^{(i)}) \otimes F^{(i)} \cdot \Theta(\Delta t_i - \Delta |\vec{r}|)$$

At any finite $t = t_n > t_0$ microscopic particle history is contained in

$$F_a(t, \vec{r} | E, \vec{k}) = \sum_{i=0}^{n} F_a^{(i)}(t_i, r | E, \vec{k})$$



From statistical physics: extract from F macroscopic, spacetime-dep. quantities:

particle current: $n_a^{\mu}(t,\vec{r}) = d_a \int \frac{d^4k}{(2\pi)^4}\, k^{\mu}\, F_a(t,\vec{r}\,|\,E,\vec{k})$

energy mom. tensor: $T_a^{\mu\nu}(t,\vec{r}) = d_a \int \frac{d^4k}{(2\pi)^4}\, k^{\mu} k^{\nu}\, F_a(t,\vec{r}\,|\,E,\vec{k})$

entropy current: $S_a^{\mu}(t,\vec{r}) = d_a \int \frac{d^4k}{(2\pi)^4}\, k^{\mu} \left\{ F_a \ln(F_a) - \theta_a (1+\theta_a F_a) \ln(1+\theta_a F_a) \right\}$

Define 'local flow velocity

$$u^{\mu}(t,\vec{r}) = \frac{n^{\mu}}{\sqrt{n_{\nu}\, n^{\nu}}}$$

Get Lorentz-invariant, scalar density profiles

number density: $\rho_a(t,\vec{r}) = n_{\mu_a} n_a^{\mu}$ $\quad\rightsquigarrow\quad \frac{dN}{d^2k_\perp}$

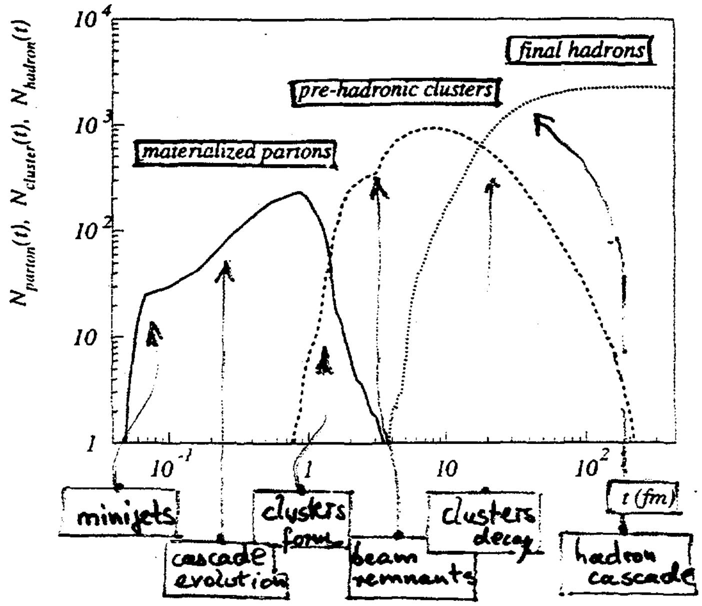
energy density: $\mathcal{E}_a(t,\vec{r}) = -\frac{1}{3} T_a^{\mu\nu} \left(g_{\mu\nu} - u_{\mu_a} u_{\nu_a}\right)$ $\quad\rightsquigarrow\quad \frac{dE_\perp}{dy}$

pressure: $P_a(t,\vec{r}) = T_a^{\mu\nu}\, u_{\mu_a} u_{\nu_a}$ $\quad\rightsquigarrow\quad \frac{d\langle k_\perp\rangle}{dz}$

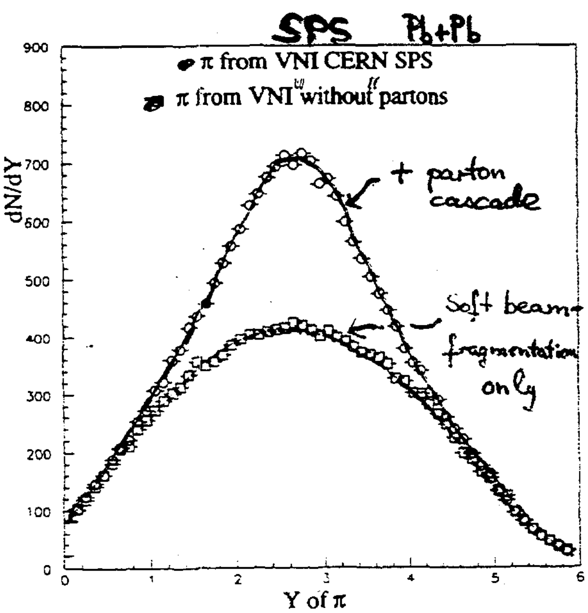
entropy density: $S_a(t,\vec{r}) = S_a^{\mu}\, u_{\mu a}$

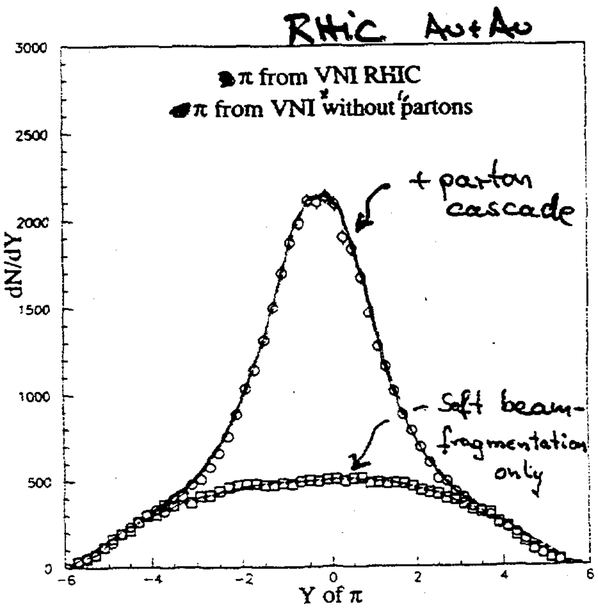
eff. temperature: $T_a(t,\vec{r}) = \frac{\mathcal{E}_a + P_a}{S_a}$ $\quad\rightsquigarrow\quad \frac{dT}{d^2r_\perp}$

Timevolution of Pb+Pb @ CERN SPS



"Partonic contribution" in final hadron yield

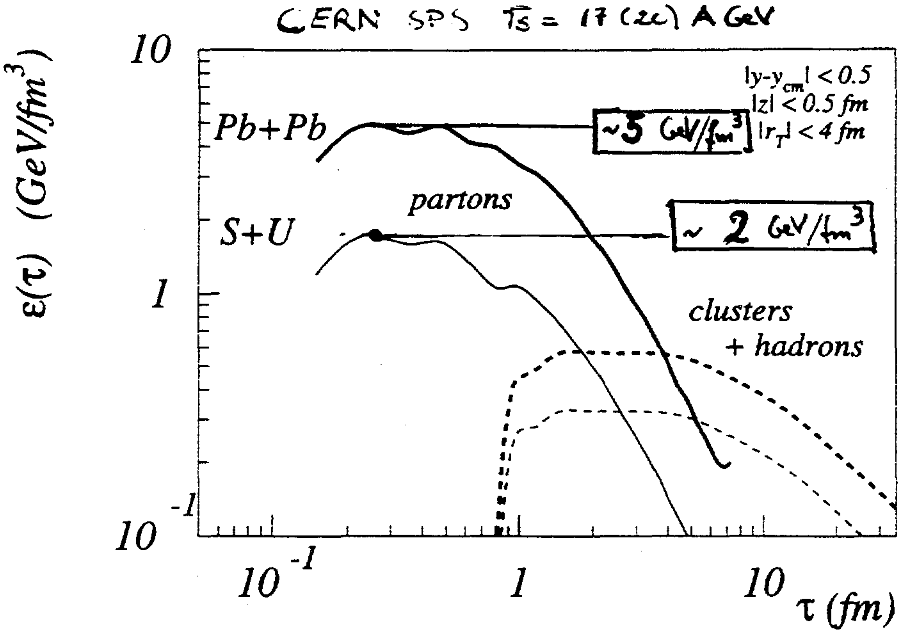




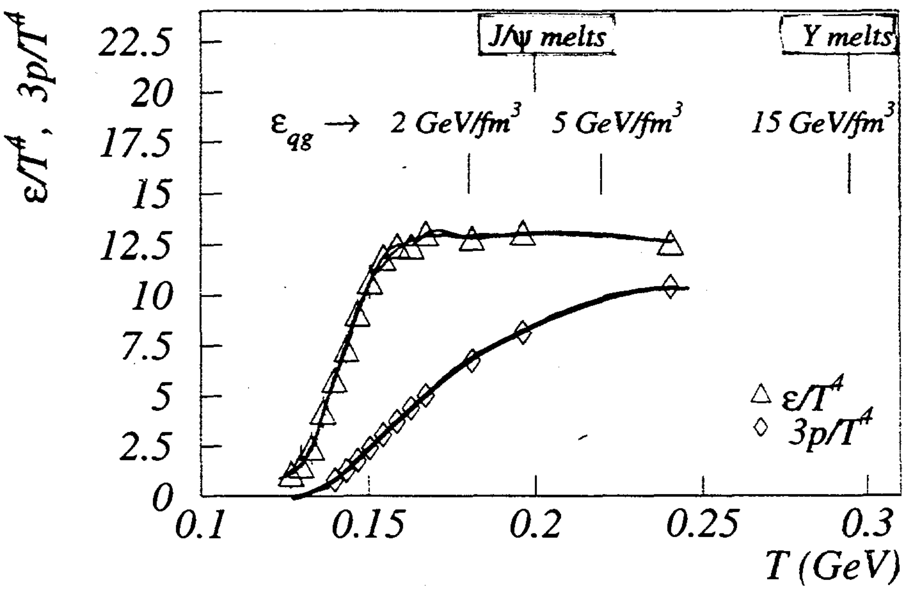
7/5

Energy density: Parton Cascade vs. Lattice QCD

B. Mueller & KKG '97



VNi-3.2
KKG, 97

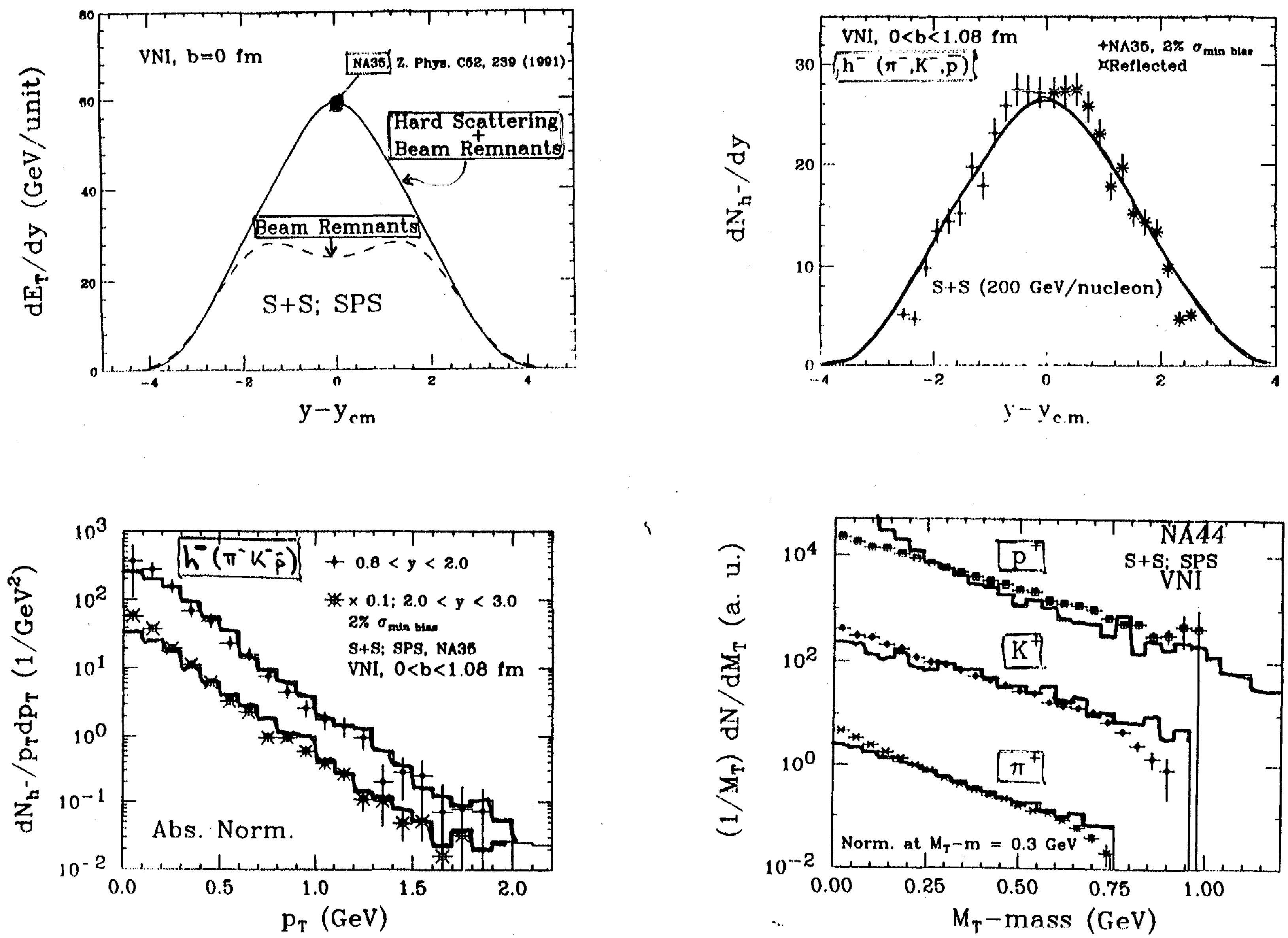


Karsch & Satz '96

MILC Coll., Blum et al. '96

CERN SPS S+S @ 200 A GeV

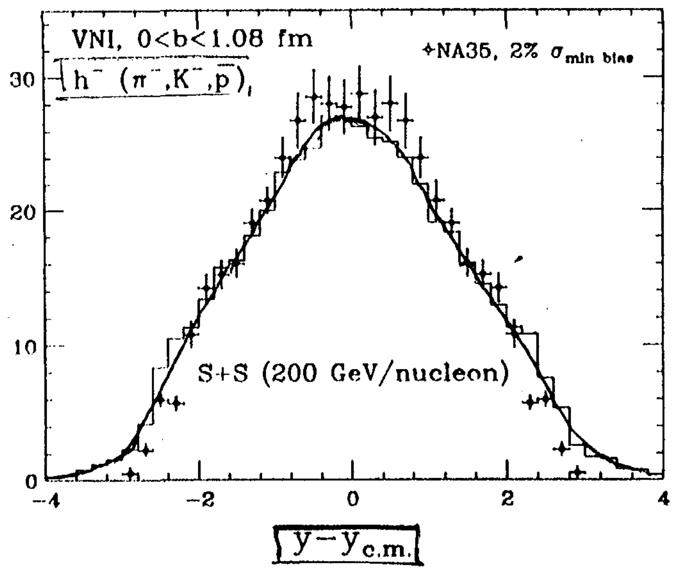
Srivastava & KKG 97

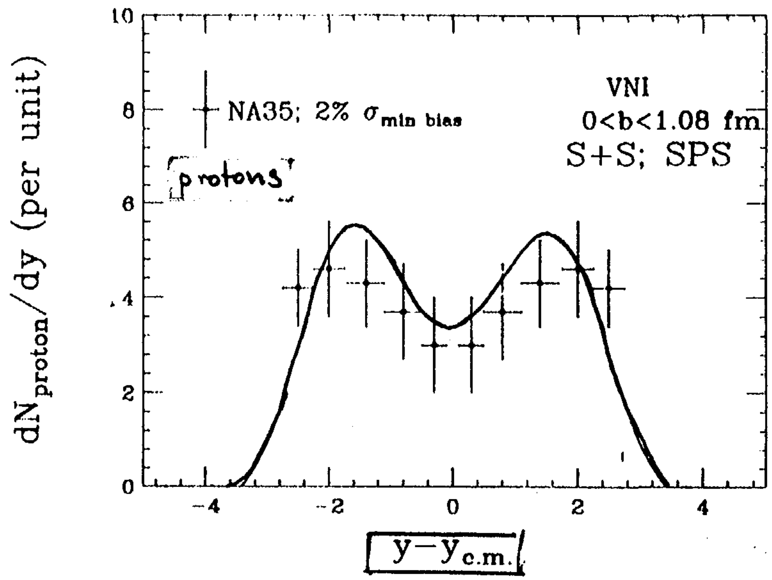


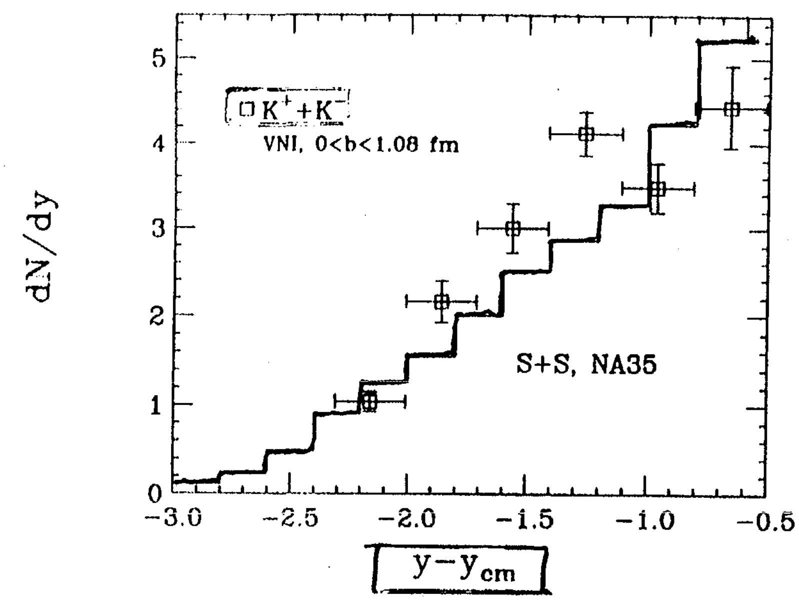
7/3

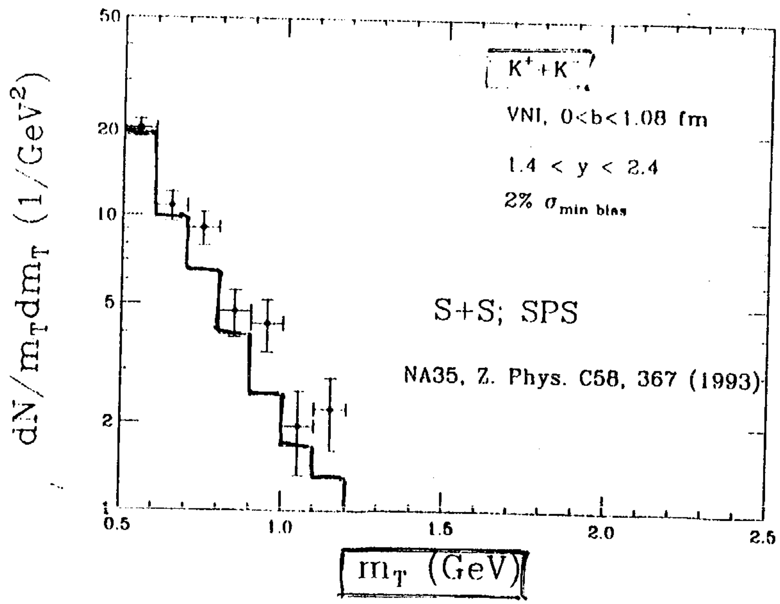
CERN SPS S+S @ 200 AGeV

Srivastava '97
& KKG





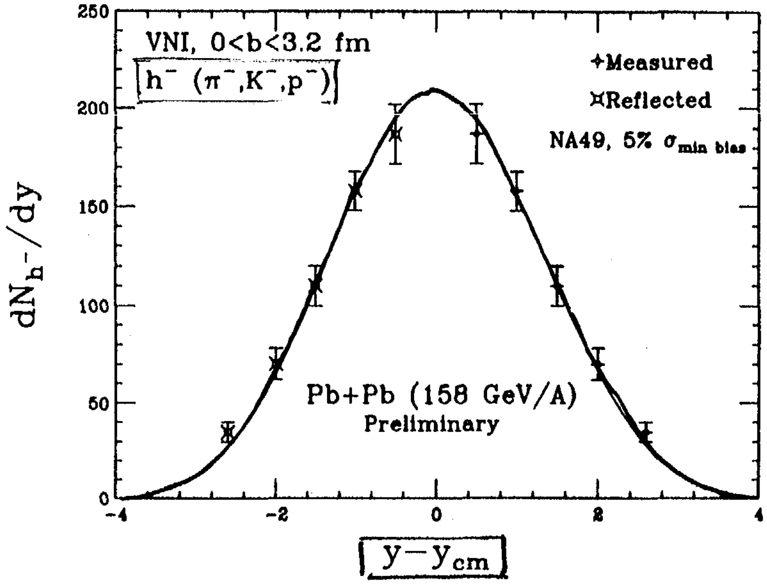


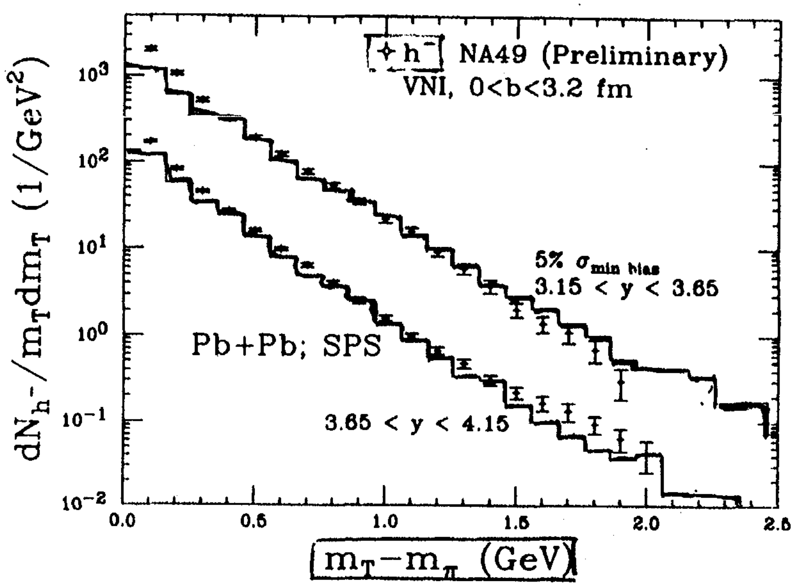


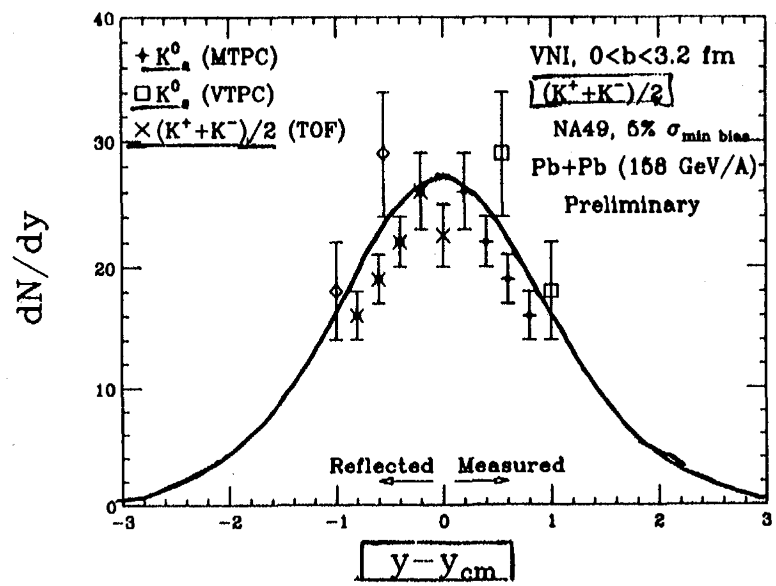
1/4

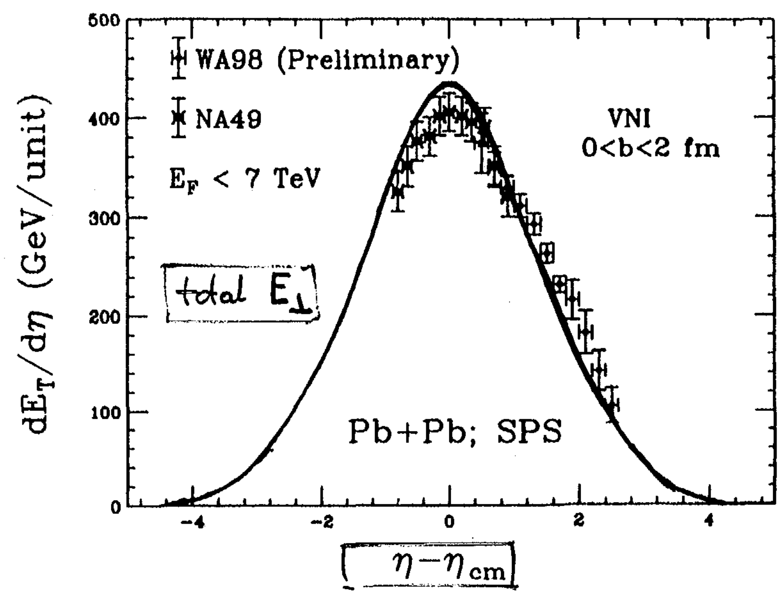
CERN SPS Pb+Pb @ 158 A GeV

Srivastava & KKG '97

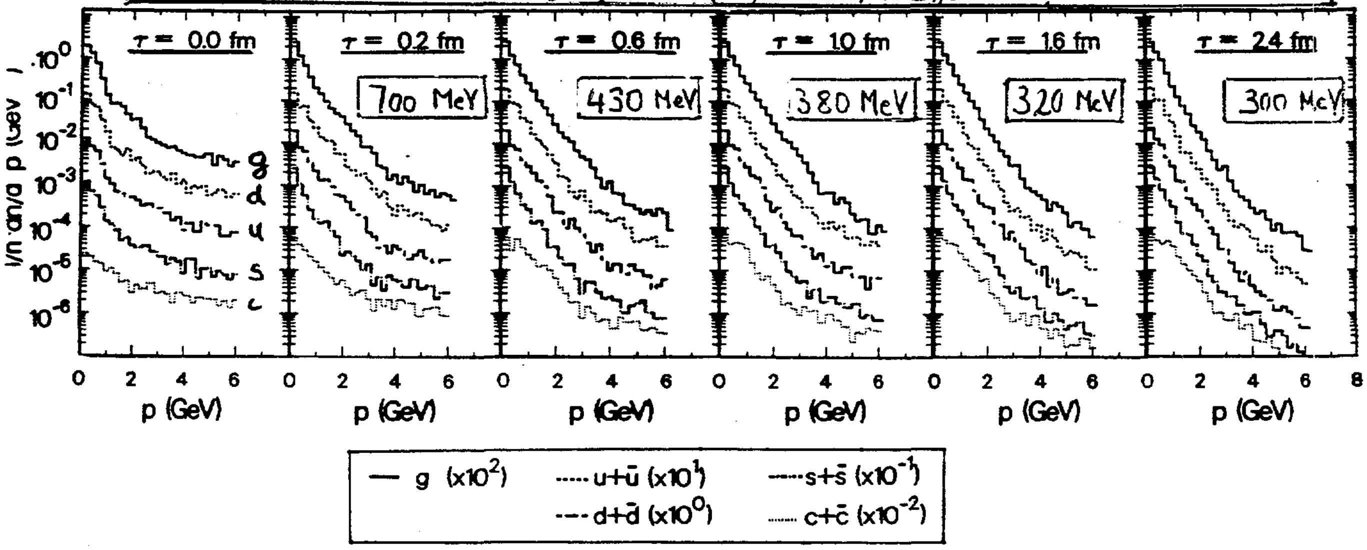




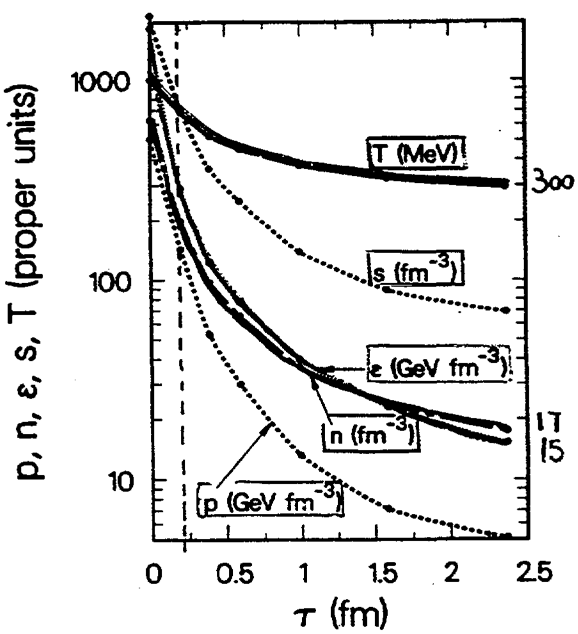


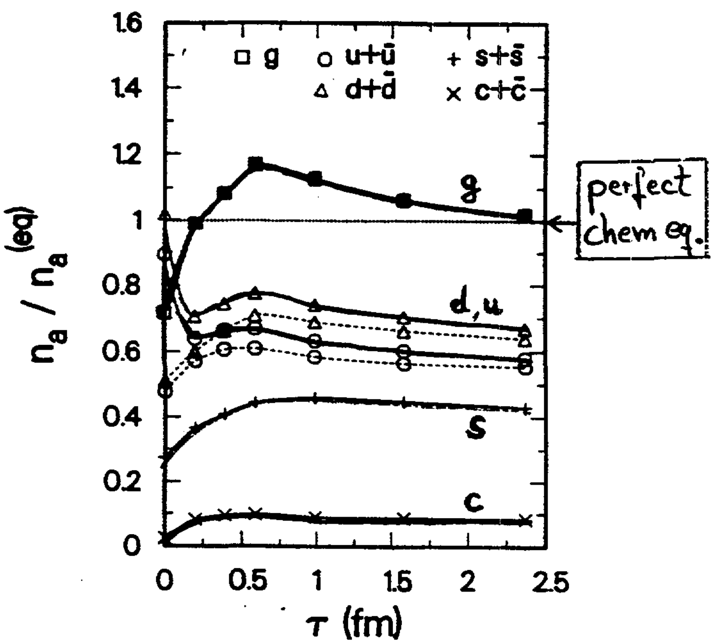


Momentum distributions $\frac{1}{\bar{n}_a}\frac{d\bar{n}_a}{d^3p}(\tau) = \frac{1}{(2\pi)^3}\frac{1}{\pi R^2 \Delta\eta}\int_{-\Delta\eta/2}^{\Delta\eta/2} d\eta \int_0^R d^2r_\perp\, F_a(\tau,\eta,r_\perp;y,m_\perp)$



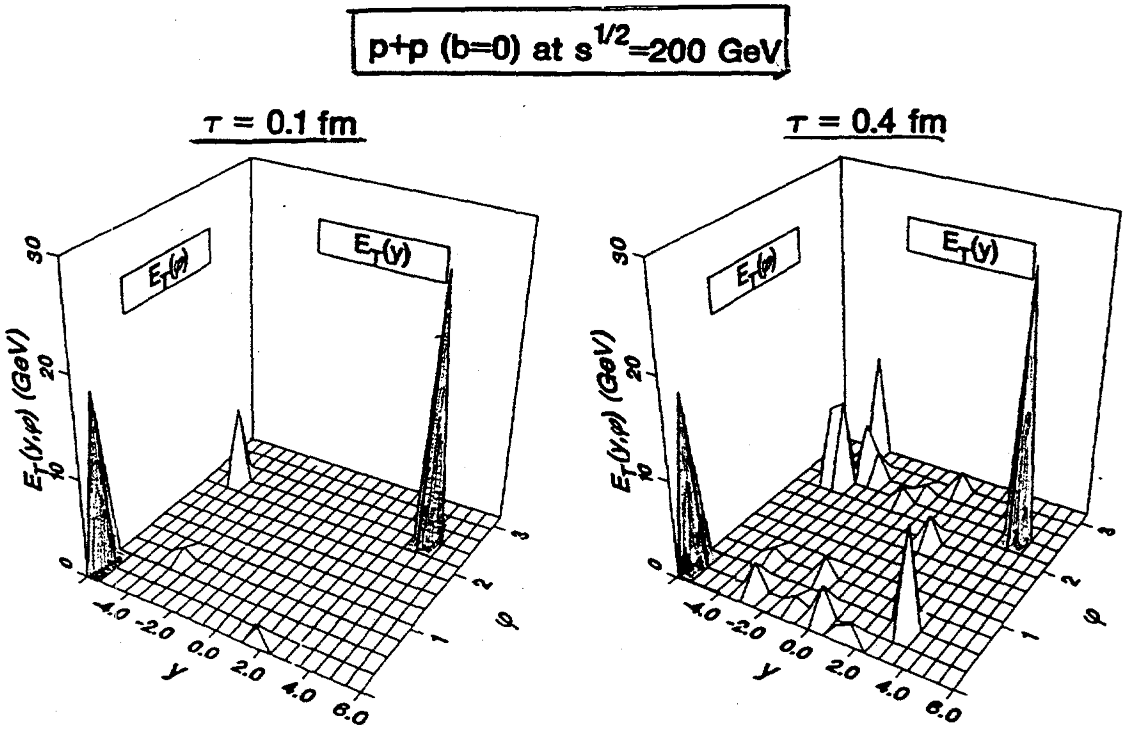
Central Au+Au at $s^{1/2} = 200$ A GeV

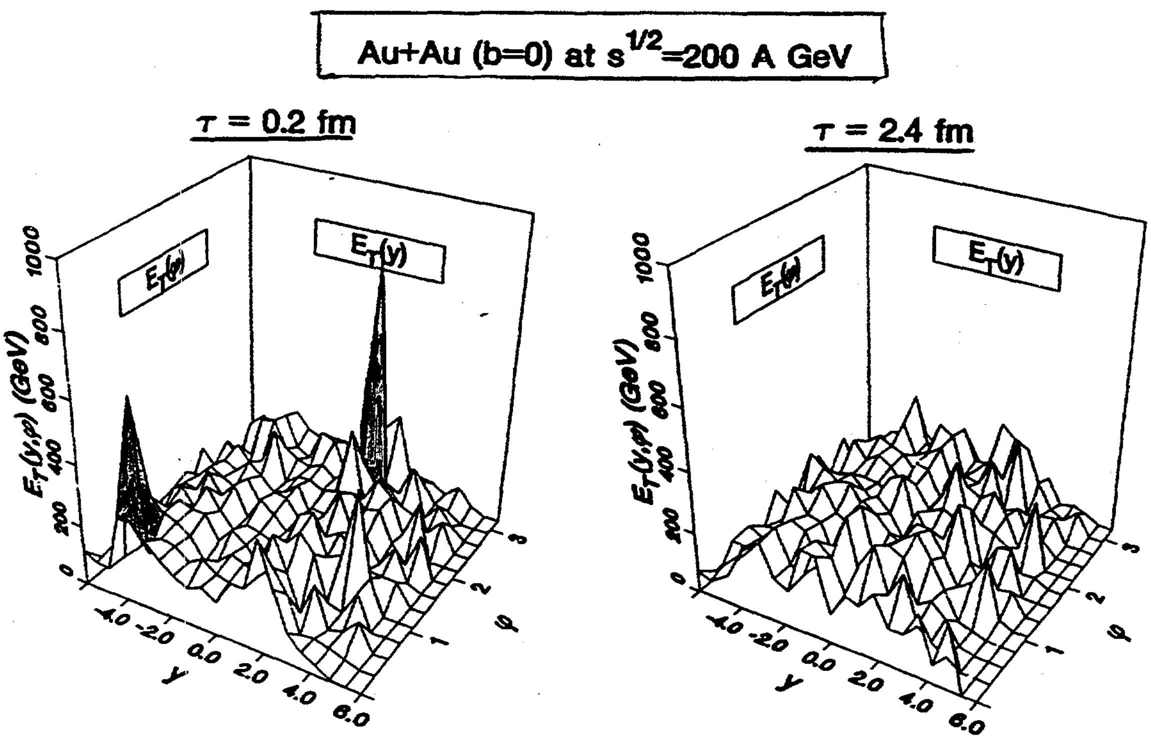




CENTRAL REGION:

$-1 \leq \eta \leq 1$ $\qquad$ $0 \leq r_\perp \leq 5$ fm





"JET QUENCHING" (Wang; Gyulassy)