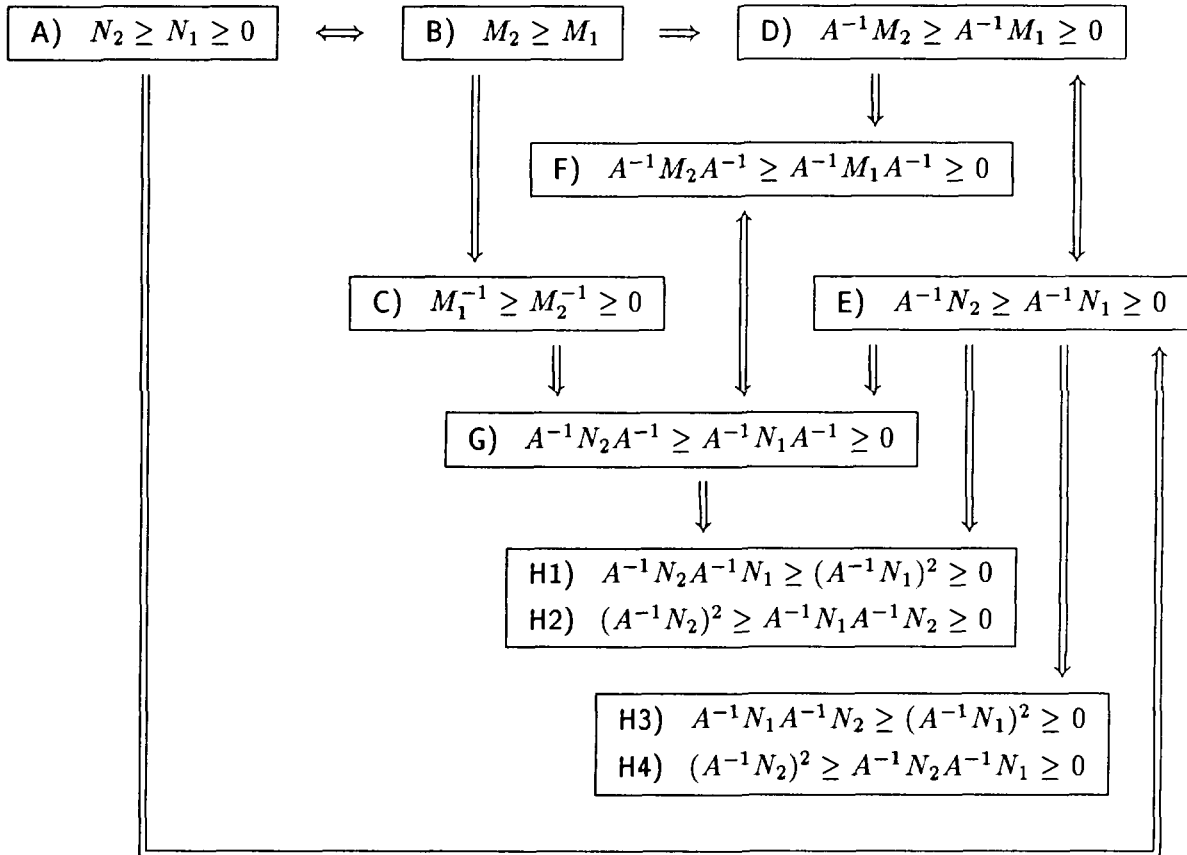# MATRIX SPLITTING PRINCIPLES

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In the paper [1] the systematic analysis of convergence conditions, used in comparison theorems proven for a few types of matrix splittings representing a large class of applications, is presented.

The central idea of this analysis is the scheme of condition implications, derived from the properties of regular splittings of a monotone matrix $A = M_1 - N_1 = M_2 - N_2$ [2], shown below.

A) $N_2 \geq N_1 \geq 0$ $\iff$ B) $M_2 \geq M_1$ $\implies$ D) $A^{-1}M_2 \geq A^{-1}M_1 \geq 0$

F) $A^{-1}M_2A^{-1} \geq A^{-1}M_1A^{-1} \geq 0$

C) $M_1^{-1} \geq M_2^{-1} \geq 0$

E) $A^{-1}N_2 \geq A^{-1}N_1 \geq 0$

G) $A^{-1}N_2A^{-1} \geq A^{-1}N_1A^{-1} \geq 0$

H1) $A^{-1}N_2A^{-1}N_1 \geq (A^{-1}N_1)^2 \geq 0$

H2) $(A^{-1}N_2)^2 \geq A^{-1}N_1A^{-1}N_2 \geq 0$

H3) $A^{-1}N_1A^{-1}N_2 \geq (A^{-1}N_1)^2 \geq 0$

H4) $(A^{-1}N_2)^2 \geq A^{-1}N_2A^{-1}N_1 \geq 0$

An equivalence of some conditions as well as an autonomous character of the conditions $M_1^{-1} \geq M_2^{-1} \geq 0$ and $A^{-1}N_2 \geq A^{-1}N_1 \geq 0$ are pointed out.

Conditions ensuring that a splitting of a nonsingular matrix $A = M - N$ will be convergent are unkown in a general case. However, general properties of a spilitting of A (not necessary convergent), useful in proving many comparison theorems, are given in the following lemma.

Lemma. Let $A = M - N$ be a splitting of A. If A and M are nonsingular matrices, then

$M^{-1}NA^{-1} = A^{-1}NM^{-1}$,

the matrices $M^{-1}N$ and $A^{-1}N$ commute, and the matrices NM^{-1} and NA^{-1} also commute.

REFERENCES:

[1]. Z.I. Woźnicki: Matrix Splitting Principles. Submitted to *Linear Algebra and its Aplications*. (The main results of the paper have been presented in the invited paper in XII Conference on Applied Mathematics, Palić, Yugoslavia, September 8-12 1997).

[2]. Z.I. Woźnicki: Conditions for convergence and comparison. *This Annual Report*.