CONDITIONS FOR CONVERGENCE AND COMPARISON

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Comparison theorems, proven under different conditions for different types of matrix splittings representing a large class of applications, play an essential role in the convergence analysis of iterative methods for solving linear systems.

The analysis presented in the paper [1] is based on condition implications derived from the properties of regular splittings.

The decomposition $A = M - N$ is called a regular splitting of A, if M is a nonsingular matrix with $M^{-1} \geq 0$ and $N \geq 0$.

It is easy to verify that for regular splittings of a monotoe matrix A (i.e., $A^{-1} \geq 0$),

$$A = M_1 - N_1 = M_2 - N_2 \quad (1)$$

the assumption

$$N_2 \geq N_1 \geq 0 \quad (2)$$

implies the equivalent condition

$$M_2 \geq M_1 \quad (3)$$

but the last inequality implies the condition

$$M_1^{-1} \geq M_2^{-1} \geq 0 \quad (4)$$

This condition can be expressed, as follows

$$(I + A^{-1}N_1)^{-1} A^{-1} \geq A^{-1} (I + N_2A^{-1})^{-1} \quad (5)$$

which, after relevant multiplications, is equivalent to

$$A^{-1} N_2A^{-1} \geq A^{-1} N_1A^{-1} \geq 0. \quad (6)$$

From the above inequality, one obtains

$$A^{-1} N_2A^{-1} N_1 \geq (A^{-1} N_1)^2 \geq 0 \quad (7)$$

and

$$(A^{-1}N_2)^2 \geq A^{-1}N_1A^{-1}N_2 \geq 0. \quad (8)$$

Hence,

$$\rho^2(A^{-1}N_2) \geq \rho(A^{-1} N_1A^{-1} N_2) = = \rho(A^{-1} N_2A^{-1} N_1) \geq \rho^2(A^{-1}N_1) \quad (9)$$

which gives us

$$\rho (A^{-1}N_2) \geq \rho(A^{-1}N_1). \quad (10)$$

Since $\rho(M^{-1}N) = \rho(A^{-1}N /(I+ \rho (A^{-1} N))$, the inequality

$$\rho (M_1^{-1}N_1) \leq \rho(M_2^{-1}N_2) \quad (11)$$

can be deduced.

In the case of the strict inequality in (4), similar considerations lead to the strict inequality in (11).

On the other hand, from the inequality (2), one obtains

$$A^{-1}N_2 \geq A^{-1}N_1 \geq 0. \quad (12)$$

which implies the inequalities (6), (7) and (8), and additionally

$$A^{-1} N_1A^{-1}N_2 \geq (A^{-1}N_1)^2 \geq 0, \quad (13)$$

and

$$(A^{-1}N_2)^2 \geq A^{-1} N_2A^{-1}N_1 \geq 0. \quad (14)$$

The inequality (3) gives us that

$$A^{-1}M_2 \geq A^{-1}M_1 \geq 0, \quad (15)$$

since for each regular splitting of A

$$A^{-1}M = I + A^{-1}N, \quad (16)$$

hence, it is evident that both conditions (12) and (15) are equivalent.

Each of the above conditions, except (8) and (14), leads to the inequality (11). However, as can be shown on simple examples of regular splittings the reverse implications is not true. Thus, the above inequalities are progressively weaker conditions which used as hypotheses in comparison theorems provide successive generalizations of results.

REFERENCES:

[1]. Z.I. Woźnicki: Conditions for Convergence and Comparison. Proc. 15th IMACS World Congress on Scientific Computation, Modelling and Applied Mathematics, Berlin, August 24-29 (1997), Vol. 2 *Numerical Analysis*, pp.291-296, Edited by Achim Sydow, Wissenschaft & Technik Verlag (1997).

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REMARKS ON SOME RESULTS FOR MATRIX SPLITTINGS

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The paper [1] is an extension of the former version. The subject of the paper is devoted to the discussion of aspects related mainly to the use of proper conditions in splitting definitons in order to avoid a confusion in the interpretation of comparison theorems. For instance, one of such questions is a confusion caused by the use of different definitions of weak regular splittings. The orginal definition of weak regular splitting of $A = M - N$, introduced by Ortega and Rheinboldt [2], is based on three conditions: $N \geq 0$, $M^{-1}N \geq 0$ and $NM^{-1} \geq 0$. Some authors ignore the last condition which implies weakening this definition and some comparison theorems, proven for regular splittings, do not carry over.