CONDITIONS FOR CONVERGENCE AND COMPARISON

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Comparison theorems, proven under different conditions for different types of matrix splittings representing a large class of applications, play an essential role in the convergence analysis of iterative methods for solving linear systems.

The analysis presented in the paper [1] is based on condition implications derived from the properties of regular splittings.

The decomposition $A = M - N$ is called a regular splitting of A, if M is a nonsingular matrix with $M^{-1} \geq 0$ and $N \geq 0$.

It is easy to verify that for regular splittings of a monotoe matrix A (i.e., $A^{-1} \geq 0$),

$$A = M_1 - N_1 = M_2 - N_2 \quad (1)$$

the assumption

$$N_2 \geq N_1 \geq 0 \quad (2)$$

implies the equivalent condition

$$M_2 \geq M_1 \quad (3)$$

but the last inequality implies the condition

$$M_1^{-1} \geq M_2^{-1} \geq 0 \quad (4)$$

This condition can be expressed, as follows

$$(I + A^{-1}N_1)^{-1} A^{-1} \geq A^{-1} (I + N_2A^{-1})^{-1} \quad (5)$$

which, after relevant multiplications, is equivalent to

$$A^{-1} N_2A^{-1} \geq A^{-1} N_1A^{-1} \geq 0. \quad (6)$$

From the above inequality, one obtains

$$A^{-1} N_2A^{-1} N_1 \geq (A^{-1} N_1)^2 \geq 0 \quad (7)$$

and

$$(A^{-1}N_2)^2 \geq A^{-1}N_1A^{-1}N_2 \geq 0. \quad (8)$$

Hence,

$$\rho^2(A^{-1}N_2) \geq \rho(A^{-1} N_1A^{-1} N_2) = = \rho(A^{-1} N_2A^{-1} N_1) \geq \rho^2(A^{-1}N_1) \quad (9)$$

which gives us

$$\rho (A^{-1}N_2) \geq \rho(A^{-1}N_1). \quad (10)$$

Since $\rho(M^{-1}N) = \rho(A^{-1}N /(I+ \rho (A^{-1} N))$, the inequality

$$\rho (M_1^{-1}N_1) \leq \rho(M_2^{-1}N_2) \quad (11)$$

can be deduced.

In the case of the strict inequality in (4), similar considerations lead to the strict inequality in (11).

On the other hand, from the inequality (2), one obtains

$$A^{-1}N_2 \geq A^{-1}N_1 \geq 0. \quad (12)$$

which implies the inequalities (6), (7) and (8), and additionally

$$A^{-1} N_1A^{-1}N_2 \geq (A^{-1}N_1)^2 \geq 0, \quad (13)$$

and

$$(A^{-1}N_2)^2 \geq A^{-1} N_2A^{-1}N_1 \geq 0. \quad (14)$$

The inequality (3) gives us that

$$A^{-1}M_2 \geq A^{-1}M_1 \geq 0, \quad (15)$$

since for each regular splitting of A

$$A^{-1}M = I + A^{-1}N, \quad (16)$$

hence, it is evident that both conditions (12) and (15) are equivalent.

Each of the above conditions, except (8) and (14), leads to the inequality (11). However, as can be shown on simple examples of regular splittings the reverse implications is not true. Thus, the above inequalities are progressively weaker conditions which used as hypotheses in comparison theorems provide successive generalizations of results.

REFERENCES:

[1]. Z.I. Woźnicki: Conditions for Convergence and Comparison. Proc. 15th IMACS World Congress on Scientific Computation, Modelling and Applied Mathematics, Berlin, August 24-29 (1997), Vol. 2 *Numerical Analysis,* pp.291-296, Edited by Achim Sydow, Wissenschaft & Technik Verlag (1997).

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REMARKS ON SOME RESULTS FOR MATRIX SPLITTINGS

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The paper [1] is an extension of the former version. The subject of the paper is devoted to the discussion of aspects related mainly to the use of proper conditions in splitting definitons in order to avoid a confusion in the interpretation of comparison theorems. For instance, one of such questions is a confusion caused by the use of different definitions of weak regular splittings. The orginal definition of weak regular splitting of $A = M - N$, introduced by Ortega and Rheinboldt [2], is based on three conditions: $N \geq 0$, $M^{-1}N \geq 0$ and $NM^{-1} \geq 0$. Some authors ignore the last condition which implies weakening this definition and some comparison theorems, proven for regular splittings, do not carry over.

The definitions of splittings, with progressively weakening conditions and consistent from the viewpoint of names, are here collected.

Let M, N $\in \mathrm{IR}^{n \times n}$. Then the decomposition $A = M - N$ is called:

(a) a regular splitting of A if $M^{-1} \geq 0$ and $N \geq 0$,

(b) a nonnegative splitting of A if $M^{-1} \geq 0$, $M^{-1}N \geq 0$, and $NM^{-1} \geq 0$,

(c) a weak nonnegative splitting of A if $M^{-1} \geq 0$ and either $M^{-1}N \geq 0$ (the first type) or $NM^{-1} \geq 0$ (the second type),

(d) a weak splitting of A if M is nonsingular and either $M^{-1}N \geq 0$ (the first type) or $NM^{-1} \geq 0$ (the second type). In particular a given weak splitting can be both types,

(e) a convergent splitting of A if $\rho(M^{-1}N) < 1$.

Another question discussed in the paper is the analysis of additional conditions for the hypotheses used by Beauwens, which ensure the correctness of his results [3].

The following result represents the modified version of Beauwens' theorem.

Theorem. Let $A = M_1 - N_1 = M_2 - N_2$ be two convergent weak splittings of A the same type, that is, either $M_1^{-1}N_1$ and $M_2^{-1}N_2$ or $N_1M_1^{-1}$ and $N_2M_2^{-1}$ are nonnegative matrices. Then anyone of the following assumptions

(a) $(A^{-1}N_2 - A^{-1}N_1)\,A^{-1}N_1 \geq 0$

(b) $(A^{-1}N_2 - A^{-1}N_1)\,A^{-1}N_2 \geq 0$

N_2 - nonsingular

(c) $A^{-1}N_1(A^{-1}N_2 - A^{-1}N_1) \geq 0$

(d) $A^{-1}N_2(A^{-1}N_2 - A^{-1}N_1) \geq 0$

N_2 - nonsingular

(a') $(N_2A^{-1} - N_1A^{-1})\,N_1A^{-1} \geq 0$

(b') $(N_2A^{-1} - N_1A^{-1})\,N_2A^{-1} \geq 0$

N_2 - nonsingular

(c') $N_1A^{-1}(N_2A^{-1} - N_1A^{-1}) \geq 0$

(d') $N_2A^{-1}(N_2A^{-1} - N_1A^{-1}) \geq 0$

N_2 - nonsingular

implies

$$\rho(M_1^{-1}N_1) \leq \rho(M_2^{-1}N_2).$$

Thus, Beauwens' results, as they are given in [3], are correct if the matrix N_2 is nonsingular in the case of the assumptions (b) and (d) of his theorem.

REFERENCES:

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[3]. R. Beauwens: Factorization Iterative Methods, M-operators and H-operators. *Numer. Math.*, **31**, pp.335-357 (1979).

THERMAL PROBLEMS WHILE IRRADIATING THE TARGET MATERIALS IN MARIA REACTOR

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One of the major goals of MARIA reactor is production of radioisotopes for medical and industrial use. Possibility of neutron flux increase due to its upgrading allows to obtain the higher specific activities of the irradiated target materials. The neutron flux increase is accompanied by an augmentation of heat generation in these materials caused mainly by emission of gamma radiation. The basic materials to be considered are sulphur (S), tellurium dioxide (TeO_2) and iridium (Ir).

Preparation of sulphur for irradiation by melting it and pouring into an aluminium can of an outer diameter of d = 15 mm and wall thickness of σ = 0,6 mm. On closing the can and checking its tightness it is placed in one out of eight vertical isotope channels of diameter 18 mm made in a special aluminium block. In the course of irradiation the cooling water is flowing through the annular gap and removing the heat generated in cans and aluminium block. Fig. 1 presents the radial distribution of temperature in the can for heat generation rate $q_m \approx 5$ W/g, water at the inlet of the isotope channel $T_{in} \approx 50°C$ and a flow velocity in annular gap $v \approx 1,5$ m/s [1].

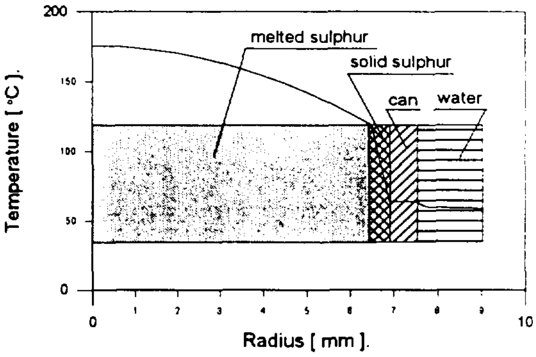

Fig. 1. Radial distribution of temperature in the can