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Determining covariances between gamma-ray emission probabilities in very simple decay schemes.

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Assume that the decay scheme represented in figure 1 applies to nucleus M decaying to S. Representing the gamma-ray emission probabilities by P_i and the internal conversion coefficients by α_i, it is obtained :

$P_i = \frac{1}{1+\alpha_i}$ for i=1,2 , with standard deviations $\sigma_{P_i} = P_i \frac{\sigma_{\alpha_i}}{1+\alpha_i}$ and covariance

$$\mathrm{cov}(P_1, P_2) = P_1 P_2 \frac{\mathrm{cov}(\alpha_1, \alpha_2)}{(1+\alpha_1)(1+\alpha_2)}.$$

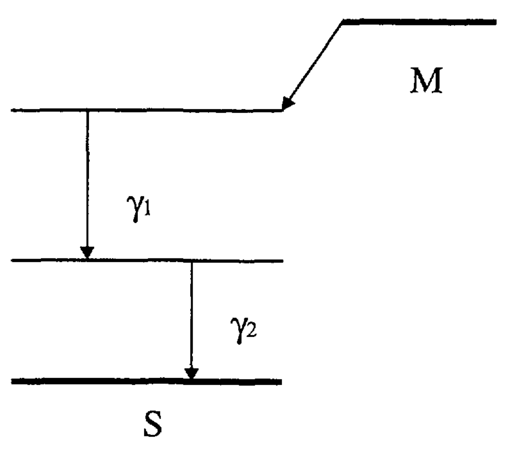

Figure 1. Decay scheme with two γ-rays and a single beta branch.

If the parent nucleus also feeds the ground state of the daughter nucleus as shown in figure 2, there is an additional contribution to the covariance. Representing the probability of feeding the ground state by ε_g: $P_i = \frac{1-\varepsilon_g}{1+\alpha_i}$. Assuming ε_g statistically independent of the α_i, then $\mathrm{cov}(\varepsilon_g, \alpha_i) = 0$ and

$$\sigma_{P_i} = P_i \sqrt{\left(\frac{\sigma_{\alpha_i}}{1+\alpha_i}\right)^2 + \left(\frac{\sigma_{\varepsilon_g}}{1-\varepsilon_g}\right)^2},$$

and

$$\mathrm{cov}(P_1, P_2) = P_1 P_2 \left(\frac{\mathrm{cov}(\alpha_1, \alpha_2)}{(1+\alpha_1)(1+\alpha_2)} + \left(\frac{\sigma_{\varepsilon_g}}{1-\varepsilon_g}\right)^2 \right)$$

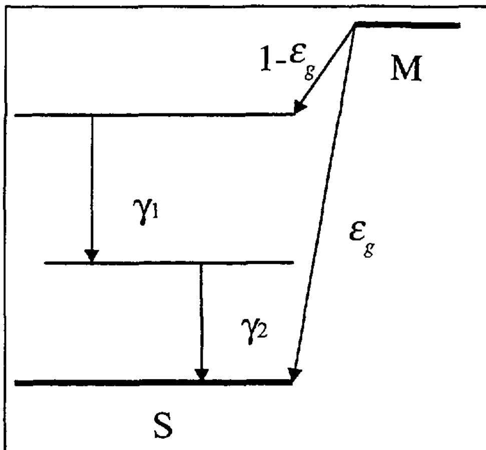

Figure 2. Decay scheme with 2 γ-rays and two beta branches.

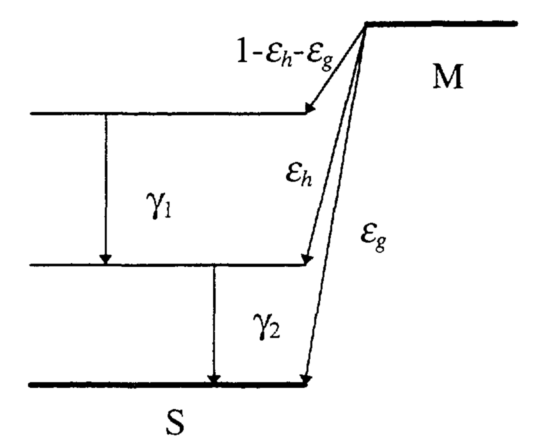

Figure 3. Decay scheme with 2 γ-rays and 3 beta branches.

The formula increase in complexity when there is an additional beta branch to the intermediate level, as implied by the scheme shown in figure 3. The emission probabilities are given by $P_1 = \dfrac{1-\varepsilon_h-\varepsilon_g}{1+\alpha_1}$ and $P_2 = \dfrac{1-\varepsilon_g}{1+\alpha_2}$ with standard deviations $\sigma_{P_2} = P_2\sqrt{\left(\dfrac{\sigma_{\alpha_2}}{1+\alpha_2}\right)^2+\left(\dfrac{\sigma_{\varepsilon_g}}{1-\varepsilon_g}\right)^2}$ and

$$\sigma_{P_1} = P_1\sqrt{\left(\frac{\sigma_{\alpha_1}}{1+\alpha_1}\right)^2+\frac{\sigma_{\varepsilon_h}^2+\sigma_{\varepsilon_g}^2+2\operatorname{cov}(\varepsilon_h,\varepsilon_g)}{(1-\varepsilon_h-\varepsilon_g)^2}}$$

and covariance $\operatorname{cov}(P_1,P_2) = P_1P_2\left(\dfrac{\operatorname{cov}(\alpha_1,\alpha_2)}{(1+\alpha_1)(1+\alpha_2)}+\dfrac{\sigma_{\varepsilon_g}^2+\operatorname{cov}(\varepsilon_h,\varepsilon_g)}{(1-\varepsilon_g)(1-\varepsilon_h-\varepsilon_g)}\right)$, assuming that ε_g and ε_h are statistically independent of the α_i so $\operatorname{cov}(\varepsilon_g,\alpha_i)=0$ and $\operatorname{cov}(\varepsilon_h,\alpha_i)=0$.

When $\operatorname{cov}(\alpha_1,\alpha_2)=\operatorname{cov}(\varepsilon_h,\varepsilon_g)=0$, $\varepsilon_h \ll 1$, and $\varepsilon_g \ll 1$, the covariance between the gamma-ray emission probabilities as described by figures 2 and 3 reduces to $\operatorname{cov}(P_1,P_2) \cong P_1P_2\sigma_{\varepsilon_g}^2$. In the same limit conditions, the covariance is null in the case shown in figure 1.