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EVOLUTION OF TUNNELLING, CAUSALITY AND THE "HARTMAN-FLETCHER EFFECT"

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Abstract: A new concept of the macroscopic tunnelling time is added to our previous definition of the microscopic tunnelling time. The formally acausal jump of a time advance near the forward barrier wall is interpreted as a result of the superposition and interference of incoming and reflected waves. The reality " H.-F. effect" is confirmed.

1.In [1-3] a new definition of microscopic tunnelling times, which are self-consistent within conventional quantum mechanics, was introduced. There were used some formulae which imply integrations over time of $j_\pm(x,t)$ as functions of time t and penetration depth x. We recall that generally speaking the total flux $j(x,t)$ consists of two components, $j_+(x,t)$ and $j_-(x,t)$, associated with the motion along the positive and the negative x-direction, respectively. One particular prediction in [1] of the reality of the Hartman-Fletcher effect [4,5] in tunnelling processes has received (due to the analogy [6] between tunnelling particles and photons) certain experimental verifications [7 - 10].

Here we add the definition of the macroscopic tunnelling time and examine the relation between it and the microscopic tunnelling time. Then we explain the origin of the advance jump near the forward barrier wall. At the end we briefly discuss the following directions of time analysis of tunnelling processes: microscopic advance and reshaping analysis, instanton-tachyon formalism and evolution of multiple successive internal reflections inside barriers.

2.For the study of the microscopic tunnelling times for barriers which are descibed by the expression $V(x) \equiv V(x)\theta(x)\theta(a-x)$ in [1-3] we have introduced the following expression for the mean transmission duration $<\tau_{Transm}(x_i, x_f)>$:

$$<\tau_{Transm}(x_i, x_f)> = <t_+(x_f)> - <t_+(x_i)> \quad (1)$$

$$\text{with} \quad -\infty < x_i < a, \quad a < x_f < \infty \quad \text{and}$$

$$<t_+(x)> = \int_{-\infty}^{+\infty} t\, j_+(x,t)\, dt \;\Big/\; \int_{-\infty}^{+\infty} j_+(x,t)\, dt \quad (2)$$

$$j_+(x,t) = j(t)\theta(j), \quad j = (\hbar/m)\,\mathrm{Re}[\Psi(x,t)\, i\, \partial/\partial x\, \Psi^*(x,t)] \quad (3)$$

$$\Psi(x,t) = \int_0^{\infty} G(k-\bar{k})\,\Psi(k,x)\exp(-iEt/\hbar)dE, \quad (4)$$

$$E = \hbar^2k^2/2\mu = \mu v^2/2, \quad \int_0^{\infty} |G(k-\bar{k})|^2 dE = 1, \quad G(0) = G(\infty) = 0, \quad k>0, \quad \Psi(x,t)$$

being the solution of the one-dimensional stationary Schrodinger equation (with a potential barrier) having the asymptotic form $\Psi_i(k;x) = \exp(ikx) + A_R\exp(-ikx)$ on the left of the barrier and $\Psi_T(k,x) = A_T\exp(ikx)$ on the right of the barrier. The expression (1) was defined for points x_i and x_f near and inside a barrier and hence it has a microscopic meaning although it can be principally measured, at least in "gedanken experiments" with detectors which registrate all particles (incoming in detectors from both sides - with resulted fluxes $j_+(x,t)$).

In reality one has to deal often with "macroscopically measured" times

$$< \tau^{macr}_{Transm} (x_i ,x_f)> = <t_{fin} (x_f)> - <t_{in} (x_i)> \quad (5)$$

with $x_f \geq a$ and $|x_i| \gg a$ where $<t_{in} (x_i)>$ and $<t_{fin} (x_f)>$ are defined not by (2) but by the expressions formally similar to (2), in which $j_+(x,t)$ is substituted by $j(x,t)$ with $\Psi_{in} (k,x)=\exp(ikx)$, instead of $\Psi (k,x)$ for $<t_{in} (x_i)>$, and with

$\Psi_T (k,t)=A_T \exp(ikx)$ instead of $\Psi (k,x)$ for $<t_{fin} (x_f)>$:

So, following [1], we obtain

$$< \tau^{macr}_{Transm} (x_i ,x_f)> =(x_f -x_i) < V^{-1} >_E + < \Delta\tau^{phase}_{Tunn} >_E \quad (6)$$

where
$$< V^{-1} >_E = \int_0^\infty dE\, |G(k-\bar{k})|^2 \Big/ \int_0^\infty dE\, V|G(k-\bar{k})|^2$$

and
$$< \Delta\tau^{phase}_{Tunn} >_E = \int_0^\infty V|G(k-\bar{k})|^2\, \hbar\, d(\arg A_T)/dE \Big/ \int_0^\infty dE\, V|G(k-\bar{k})|^2$$

In particular,
$$< \tau^{macr}_{Transm} (x_i ,a)> =|x_i| <V^{-1} >_E + < \Delta\tau^{phase}_{Tunn} >_E \quad (6a)$$

$$< \Delta\tau^{phase}_{Tunn} >_E = \int_0^\infty dE\, V|G|^2 [aV^{-1} + \hbar\, d(\arg A_T)/dE] \Big/ \int_0^\infty dE\, V|G|^2$$

for $V(x) =V(x)\theta(x)\theta(a-x)$

In (6) and (6a) we have used approximations

$$\int_0^\infty dE\, V^n |GA_T|^2 \cong \int_0^\infty dE\, V^n |G|^2 ,\ n=0,1,$$

for sufficiently small energy (momentum) spreads in initial wave packets. Comparing (1) with (5) and taking into account that $<t_+(x_f)>=<t_{fin}(x_f)>$ for $x_f \geq a$ when $V(x) =V(x)\theta(x)\theta(a-x)$, one does immediately obtain:

$$< \tau^{macr}_{Transm} (x_i ,x_f)> = < \tau^{micr}_{Transm} (x_i ,x_f)> + <t_+ (x_i)> - <t_{in} (x_i)> \quad (7)$$

For real weight amplitudes $G(k-\bar{k})$ in wave packet (4) we have: $<t_{in} (0)>=0$ and hence

$$< \tau^{micr}_{Tunn} > = < \tau^{micr}_{Transm} (0 ,a)> =< \tau^{macr}_{Tunn}>_E - <t_+(0)> \quad (8)$$

The preliminary calculations of $< \tau^{micr}_{Tunn} >$,which have been performed by A.Zaichenko for electronic wave packets and rectangular potential barriers with the same parameters as in [3], have manifested the negative values of $<t_+(0)>$ when weight amplitudes $G(k-\bar{k})$ in (4) *are real and* hence $<t_{in}(0)>=0$. These "*acausal*" advance jumps can be interpreted as a result of the superposition and interference of incoming and reflected waves: the reflected-wave packet does extinguish the back edge of the incoming-wave packet, and the larger is the barrier width the larger is the part of the back edge of the incoming-wave packet which is extinguished by the superimposing reflected-wave packet, up to the saturation when a contribution of the reflected-wave packet becomes almost constant, independently from a . Since all $<t_+(0)>$ are negative and $< \Delta\tau^{phase}_{Tunn}>$ are positive , the quantities $<\tau^{micr}_{Tunn}>$ are always positive and moreover have larger values than $< \Delta\tau^{phase}_{Tunn}>$, *relative* to (8).

In connection with this it is relevant to note that the simple example ,presented in [11] for a classical ensemble of two particles (one with a large superbarrier energy and other with a small subbarrier energy) does contradict to our conclusion firstly because of that that tunnelling is a pure quantistic phenomenon without the direct classical limit and secondly because of in [11] it is overlooked the fact that the values of $<t_{+}(0)>$ are negative.

3. Even more exotic, "acausal" phenomena take place in photon and evaneshent-mode wave packet tunnelling [7-10] and also in propagation of electromagnetic wave packets in media with abnormal dispersive properties [12,13]. So we have an interesting open problem of constructing a causal time-dependent theory for nonrelativistic and relativistic waves propagating in media with barrier-like and abnormal dispersive properties. We intend in the future to try to elaborate two adequate approaches to describe such phenomena: (i) one with detailed analysis of advance jumps and (ii) one with utilizing instanton and tachyon forma lisms. And then to compare both approaches which can be principally equivalent.

There is one more interesting problem which we are beginning to study: time analysis of multiple successive internal reflections of tunnelling wave packets (inside a barrier).

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