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COMPARISON OF FOURIER TRANSFORM AND CONTINUOUS WAVELET TRANSFORM TO STUDY ECHO-PLANAR IMAGING FLOW MAPS

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Abstract

Velocity maps were studied combining Doyle and Mansfield's method (1986) with each of the following transforms: Fourier, window Fourier and wavelet (Mexican hat). Continuous wavelet transform was compared against the two Fourier transforms to determine which technique is best suited to study blood maps generated by Half Fourier Echo-Planar Imaging. Coefficient images were calculated and plots of the pixel intensity variation are presented. Finally, contour maps are shown to visualise the behaviour of the blood flow in the cardiac chambers for the wavelet technique.

Introduction

Blood flow visualisation has always been a subject of great interest for many scientists during centuries. It is until the present time that we have available more than one single imaging method (US, MRI, SPECT, PET), which allow us to visualise blood flow. We are interested to apply the Fourier, windowed Fourier and wavelet transforms to investigate the dynamic properties of blood flow in the heart.

Fourier analysis and ultrasonic techniques have been traditionally used to study periodic motion like pulsatile blood flow in humans. However, Fourier analysis has a mayor drawback: in transforming from the frequency domain into the time domain, it is not possible to know when a particular event took place.

In order to correct this deficiency of the formula of the Fourier transform, Gabor (1) introduced a time-localised window function, g(t-b), where b is used to translate the window to cover the entire time-domain, and to acquire local information of the Fourier transform signal. This function is also called the windowed Fourier transform. It seems to be a logical step to produce an efficient multi-scale method, which is capable of extracting microscopic information about the scaling attributes of an object. The analytical tool satisfying these conditions is the wavelet transform (2-4).

The wavelet transform of a signal can be thought of as a decomposition into a set of frequency channels of equal bandwith on a logarithmic scale: it is an analysis of the signal into a family of functions. The complete decomposition characterises the signals studied (4). Thus, the wavelet transform can be regarded as a mathematical microscope; increasing the magnification one wins insight into the heamodynamics, and could separate out those details, which are of no interest, as well as to show the information in a more appropriate manner (2-3).

The wavelet transform has been used to study turbulent flow in the past (5). As far as we know, wavelet analysis has not yet been used to study the human haemodynamics. It is important to stress that the wavelet transform is not being used to study blood flow because it is currently fashionable, but rather because it allows us to study human heamodynamics in both space and scale simultaneously.

Method

It has previously been shown that velocity maps generated with Half Fourier Echo-Planar Imaging (HF-EPI) (6) are very difficult to analyse by simple inspection. Therefore, we have applied three different analytical tools: Fourier analysis, windowed Fourier and wavelet analysis (Mexican hat wavelet: the second derivative of the Gaussian probability density function) to study blood flow in the great arteries and cardiac chambers. The properties of these particular transforms are thoroughly discussed in (2-3).

Velocity maps were analysed with the method proposed by Doyle and Mansfield in 1986 (7), which is capable of studying the heart periodic motion by acquiring a number of images in one single cardiac cycle using a cine technique. Also, MATLAB™ programmes were specially written to compute all the coefficients of 14 cardiac flow maps, taken at every 50 ms in one single cardiac cycle (8). Hence, the corresponding coefficients for each type of transform were calculated pixel by pixel from the corresponding flow maps over the whole image.

Results

The coefficient images showed a great contrast between the Fourier methods and the wavelet approach, see fig. 1. Whereas, the Fourier and windowed Fourier images can only yield one image with a very good signal-to-noise ratio (SNR), the wavelet transform can generate as many images with an excellent SNR, as flow maps are available. As a result of this, all the flow and anatomical information is gathered in one single image.

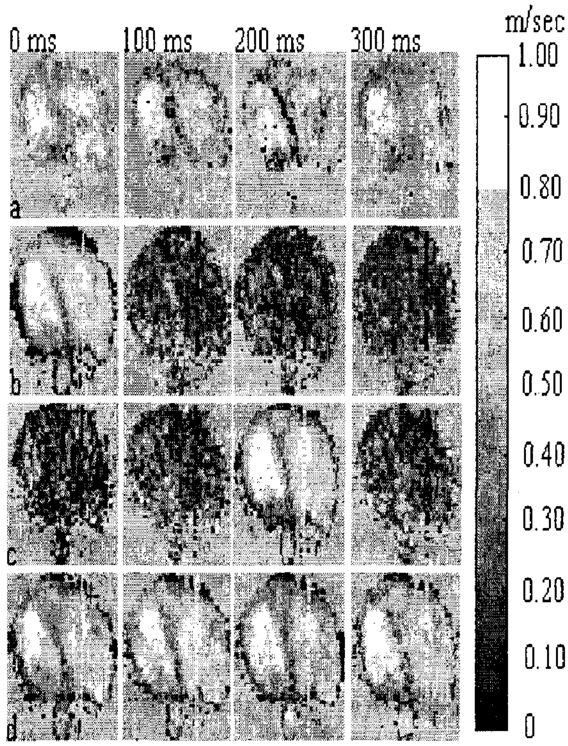

Figure 1. Comparison of coefficient images:
a) original flow maps, b) Fourier maps, c) windowed Fourier maps and d) wavelet maps.

Flow images processed with wavelet analysis are depicted in fig. 2. These wavelet coefficient images can reveal aspects of the flow behaviour in the cardiac chambers and the descending aorta throughout the cardiac cycle.

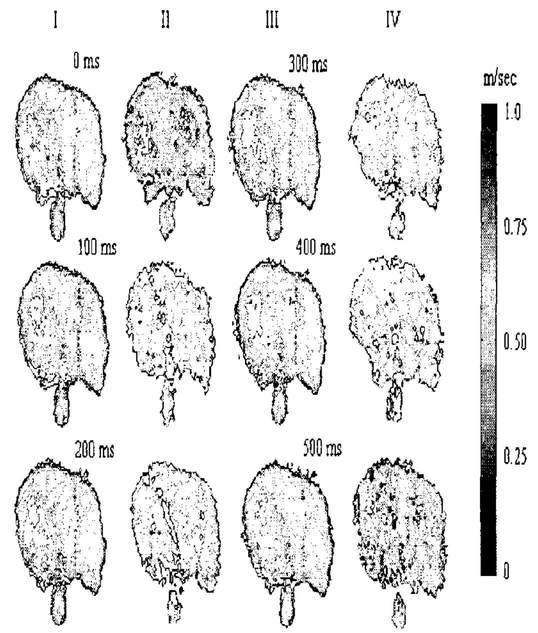

Figure 2. Contour maps of the heart; columns I and III are referred to the flow maps analysed with the Mexican hat wavelet: columns II and IV correspond to original flow maps.

A relevant feature of this analysis is the pixel variation in one single cardiac cycle for each technique. This change of the pixel intensity may contain beneficial information about the behaviour of the blood flow in the heart and great arteries. The shift of pixel intensity in the coefficient has been plotted in fig. 3. Every point forming the graphs, was estimated with the mean value of 176 points: all these points correspond to the squared area in fig. 3(a) The Fourier based methods can not disclose certain characteristics of the flow data at the exact locations in time (fig. 3(b)-(c)). Moreover, wavelet analysis exhibits more details of the functioning of the blood flow in the heart in space and time (fig. 3(d)).

Conclusions

Fourier Methods. It is easily appreciated in fig. 1(b)-(c) that Fourier coefficient images can only generate one image with a very good SNR, compared to the original velocity maps: so, the relevant heamodynamic information is gathered in this enhanced image. The remaining images seem to have just noise. Consequently, Fourier transform and windowed Fourier transform are not recommended for visualisation of blood flow.

Besides, plots of pixel variation (fig. 3(b)-(c)) do not show any drift, trend or abrupt change that might distinguish this flow phenomenon from others. Then, we can not propose this Fourier scheme as a tool to study and visualise our flow maps.

Wavelet method. The Mexican hat wavelet is able to monitor the blood flow in the cardiac chamber, see fig. 2. This wavelet approach can server as microscope to visualise the heamodyamics in the great arteries and the heart, which can also be extended to other veins.

The combination of the wavelet analysis and fast MR imaging methods opens a completely new alternative method to study flow in real time. Flow visualitation with wavelet analysis may become a tool to study the human heamodyamics, since it shows a characteristic variation of flow in the heart (fig. 3(d:i)).

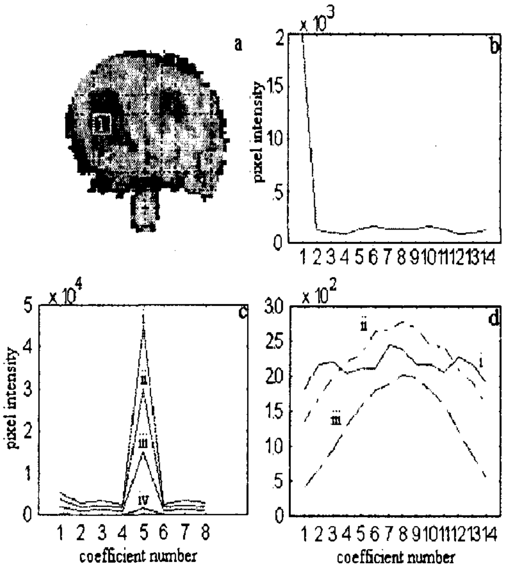

Figure 3. Typical plots of pixel intensity variation versus time from within a cardiac chamber over a cardiac cycle (a), all plots correspond to the same area: b) Fourier images, c) windowed Fourier images with different limits (i: [0, 300], ii: [0, 200], iii: [0, 100], iv: [0,10]) and d) Mexican hat wavelet for various scales (i: 30, ii: 20, iii: 10).

Key words:

EPI, Flow, Half Fourier, Heart, MRI, Wavelet, Windowed Fourier.

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