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30. バリオンを考慮した(3＋1)次元流体模型に基づく QGP流体の時空発展の解析

Three dimensional relativistic hydrodynamical model for QGP gas

C. Nonaka[1], S. Muroya[*2], O. Miyamura[3]

Dep. of Physics, Hiroshima Univ., Higashi-hiroshima, Hiroshima, 739, Japan
*Tokuyama Women's College, Tokuyama, Yamaguchi, 745, Japan**

Abstract

We numerically solved fully (3+1)-dimensional relativistic hydrodynamical equation coupled with the baryon number conservation law without spatial symmetry. We discuss the effect of transverse expansion based on the deviation our numerical result from Bjorken's scaling solution. We analyze the space-time evolution of the QGP gas in the case of non cylindrical initial conditions.

1 Introduction

The various kinds of collective flow phenomena such as directed flow, elliptic flow and radial flow has been observed in recent experiments at AGS [1] and SPS [2]. It is a matter of interest that such flow are results of hydrodynamical motion of hadronic fluid. Our first trial to tackle the problem is to develop (3+1)-dimensional hydrodynamical model. Assuming the local thermal equilibrium for hot and dense fire ball produced in ultra relativistic nuclear collisions, we analyze the evolution of the fire ball based on the (3+1)-dimensional hydrodynamical model. The hydrodynamical model for Quark-Gluon Plasma (QGP) fluid has already been discussed in many papers since Bjorken first introduced the simple scaling model based on (1+1)-dimensional expansion picture [3]. For simplicity, cylindrical symmetry is assumed in usual hydrodynamical [4, 5, 6, 7] analysis, but this assumption disables us from discussing the anisotropic collective flow. In this paper, in order to investigate collective flow not only in the central collisions but also in the non-central ones we numerically solve the (3+1)-dimensional relativistic hydrodynamical equation coupled with the baryon number conservation law.

2 The relativistic hydrodynamical model

The relativistic hydrodynamical equation for perfect fluid is given as

$$\partial_\mu T^{\mu\nu} = 0, \tag{1}$$

where $T^{\mu\nu}$ is energy momentum tensor,

$$T^{\mu\nu} = \epsilon U^\mu U^\nu - P(g^{\mu\nu} - U^\mu U^\nu). \tag{2}$$

Here, ϵ is energy density, P is pressure, metric tensor is $g^{\mu\nu} = \text{diag.}(1, -1, -1, -1)$ and local velocity is $U^\mu = (1, v_x, v_y, v_z)/\gamma$ respectively. In order to take account of the finite baryon number density, we must solve the baryon number conservation law,

$$\partial_\nu \{n_B(T, \mu) U^\nu\} = 0, \tag{3}$$

also, where $n_B(T, \mu)$ being baryon number density. Through the time like component of Eq.(1), $U_\nu \partial_\mu T^{\mu\nu} = 0$, Eq.(3) and thermodynamical relation, $\epsilon + P = TS + \mu n_B$, we can obtain the conservation law of entropy density current, $S^\mu = SU^\mu$,

$$\partial_\mu S^\mu = 0. \tag{4}$$

[1] E-mail:nonaka@butsuri.sci.hiroshima-u.ac.jp
[2] E-mail:muroya@yukawa.kyoto-u.ac.jp
[3] E-mail:miyamura@fusion.aci.hiroshima-u.ac.jp

Our numerical algorithm solving the hydrodynamical equation is based on the entropy conservation law Eq.(4).

In order to solve the hydrodynamical equation, the equation of state is needed. Though we consider the QGP gas and the hadron gas(excluded volume model [9]) for the realistic model equation of states, in this paper we adopt the QGP gas for numerical simplicity. The QGP gas model of massless N_f flavor quarks is given by,

$$P = \frac{32 + 21N_f}{180}\pi^2 T^4 + \frac{N_f}{2}\mu_q^2 T^2 + \frac{N_f}{4\pi^2}\mu_q^4,$$ (5)

where the number of flavor being $N_f = 3$ and chemical potential for quarks being $\mu_q = \mu/3$.

3 The numerical calculation

We solve the (3+1)-dimensional hydrodynamical equation without symmetrical conditions by using an algorithm in which lattice points of volume element is moved along local velocity and the entropy conservation law Eq.(4) is adopted explicitly. D. H. Rischke et al. discuss relativistic hydrodynamics in (3+1)-dimensional situation and collective behavior by using Eulerian hydrodynamics [8]. Our numerical calculation is explained briefly as follows:

In the first step the coordinates $x^m = X^m(t,i,j,k)$ $(m = 1,2,3)$ of lattice points at time $t + \Delta t$ are replaced by

$$X^m(t + \Delta t, i, j, k) = X^m(t, i, j, k) + \frac{U^m(t, i, j, k)}{U^t(t, i, j, k)}\Delta t$$ (6)

In the determination of lattice points in Eq.(6), the coordinates move in parallel with $n_B U^\mu$, $S U^\mu$.

In the next step the local velocity is determined by,

$$\begin{aligned} v^m(t + \Delta t, i, j, k) &= v^m(t, i, j, k) + \partial_t v^t(i, j, k, t)\Delta t \\ &+ \sum_{m=1}^{3} \partial_m v^m(i, j, k, t)(X^m(t + \Delta t, i, j, k) - X^m(t, i, j, k)) \end{aligned}$$ (7)

where $\partial_\mu v^\mu$ obtained from Eq.(1), Eq.(3) is used.

In the final step the temperature and chemical potential of lattice points is calculated by using Eq.(3), Eq.(4).

4 Comparison with Bjorken's solution

Comparing our numerical solutions with Bjorken's scaling solution $v_z = z/t$, we can easily evaluate the effect of the transverse flow. Based on Bjorken's scaling solution and Eq.(4) entropy density is given as,

$$S(\tau) = S(\tau_0)\frac{\tau_0}{\tau},$$ (8)

where proper time τ, $\tau = \sqrt{t^2 - z^2}$. In order to make comparison clear, in this section the velocity of our model in the longitudinal direction is fixed to the Bjorken's scaling solution. We put the initial temperature distribution and chemical potential distribution respectively as follows:

$$T(t_0, x, y, z) = T_0 \exp\left\{-\frac{(x - x_0)^2}{3 \cdot 2\sigma_x{}^2}\theta(x - x_0) - \frac{(y - y_0)^2}{3 \cdot 2\sigma_y{}^2}\theta(y - y_0) - \frac{(z - z_0)^2}{3 \cdot 2\sigma_z{}^2}\theta(z - z_0)\right\},$$ (9)

$$\mu(t_0, x, y, z) = \mu_0\left\{\exp\left[-\frac{(z - z_B)^2}{2\sigma_B{}^2}\right] + \exp\left[-\frac{(z + z_B)^2}{2\sigma_B{}^2}\right]\right\}$$

$$\times \exp\left\{-\frac{(x - x_0)^2}{3 \cdot 2\sigma_x{}^2}\theta(x - x_0) - \frac{(y - y_0)^2}{3 \cdot 2\sigma_y{}^2}\theta(y - y_0)\right\},$$ (10)
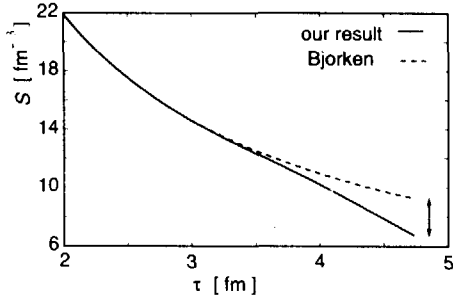
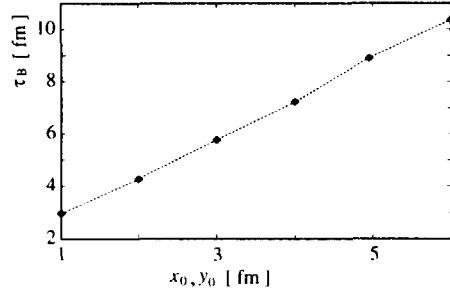Figure 1: the comparison with Bjorken's scaling solution

Figure 2: the difference between our solution and Bjorken's scaling solution by changing x_0, y_0 from 1.0 fm to 6.0 fm
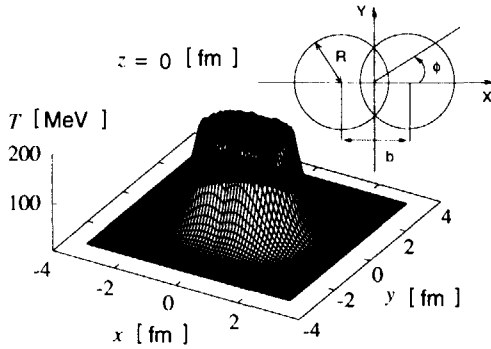

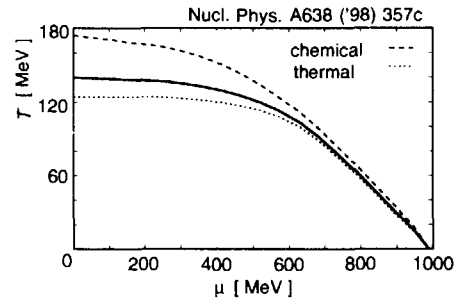Figure 3: the non cylindrical initial condition of temperature distribution at $z = 0$ fm

Figure 4: The solid line stands for freeze-out which we assume from chemical freeze-out (the dashed line) and thermal freeze-out (the dotted line).

where $x_0 = y_0 = z_0 = 1.0$ fm, $z_B = 0.7$ fm, $\sigma_x = \sigma_y = \sigma_z = 1.0$ fm, $\sigma_B = 0.7$ fm, $T_0 = 200$ MeV, $\mu_0 = 210$ MeV and the initial transverse velocity is set to 0. We focus to the volume elements at $(x, y, z) = (0, 0, 0)$ for the comparison of our numerical solution with Bjorken's scaling solution. Figure 1 shows that our numerical calculation is coincident with Bjorken's scaling solution up to $\tau = 3.0$ fm. After $\tau = 3.0$ fm the difference between them increases with the proper time because of the transverse flow. We define a characteristic time τ_B at which instance the difference between two models becomes larger than 0.1 % . Figure 2 indicates that τ_B is almost proportional to the initial x_0, y_0 during 1.0 fm to 6.0 fm, and Bjorken's scaling solution seems to be a proper solution for a large system.

5 Non cylindrical initial conditions

In this section we investigate the space-time evolution of the hydrodynamical flow with non cylindrical initial temperature distribution and chemical distribution. Other parameters are put as the previous section. Figure 3 shows the initial temperature distribution in $b = 0.8$ fm and $R = 1.0$ fm. We evaluate the effect of the flow in particle distributions by giving initial conditions like this, though this condition is not realistic to analyze the experimental data. We use Cooper-Frye formula [10] for particle emission from hadronic fluid.

$$E\frac{dN}{d^3P} = \frac{g_h}{(2\pi)^3} \int_\sigma d\sigma_\mu P^\mu \frac{1}{\exp[(P_\nu U^\nu - \mu)/T_f] \pm 1} \tag{11}$$
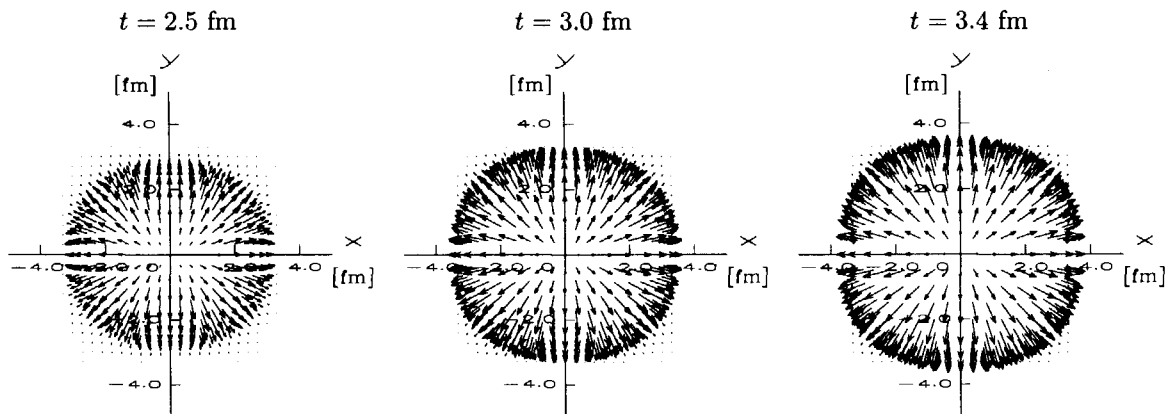
Figure 5: the space-time evolution of the flow at $z = 0$ fm ($b = 0.8$ fm)
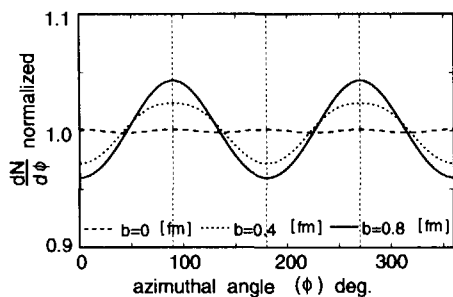
Figure 6: the azimuthal distribution in changing b
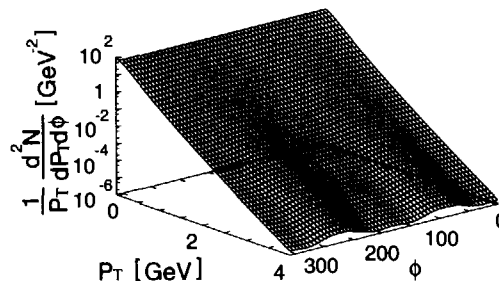
Figure 7: the dependence of the transverse momentum distribution on the flow ($b = 0.8$ fm)

for evaluating one-particle distributions. We assume that hadronization process occurs when the temperature and chemical potential in the volume elements cross the boundary (the solid line) in fig.4. The solid line in fig.4 is so designed that the freeze-out temperature becomes 140 MeV at vanishing chemical potential, based on chemical freeze-out and thermal freeze-out model in ref. [11]. Several calculations are made for different initial conditions. Figure 5 displays the space-time evolution of the flow in $b = 0.8$ fm. Figures 6 and 7 show the azimuthal fluctuation of particle number and P_T distribution which are caused by non-cylindrical properties of transverse expansion. Figure 6 indicates that the variation in the azimuthal distribution increase as separation of two initial blobs increases. Figure 7 indicates the influence of the flow increase with P_T. The yield at $\phi = 90°$, $270°$ is large, because freeze-out hypersurface is large in these directions as fig.5 shows. Furthermore the transverse momentum distribution at $\phi = 90°$, $270°$ is flatter, since the flow is pushed out at $\phi = 90°$, $270°$.

6 Summary

We solved (3+1)-dimensional relativistic hydrodynamical equation without cylindrical symmetry conditions by Lagrangian hydrodynamics. We discussed the effect of the transverse flow and confirmed that Bjorken's scaling solution is a proper solution in a large system by making a comparison with numerical calculation. The effect of the flow to the particle distributions was also investigated. The influence of flow is large at $\phi = 90°$, $270°$. This is because the freeze-out hypersurface is large at $\phi = 90°$, $270°$ and the flow is pushed out in the direction of $\phi = 90°$, $270°$.

We need to use more realistic initial conditions in the analysis of the experimental data. We plan

to adopt the output from the event generator as initial conditions and to use the equation of states including phase transition from the QGP phase to the hadron phase. Investigating the collective flow in experimental data is our next task.

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