



# GDH Sum Rule Measurement at low $Q^2$

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The Gerasimov-Drell-Hearn (GDH) sum rule is derived starting from a general dispersive relation for the forward Compton scattering, which follows causality, crossing symmetry and unitarity principles:

$$\int_0^{\infty} \frac{d\nu}{\nu} [\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)] = -\frac{2\pi^2\alpha}{m_p^2} \kappa^2 \quad (1)$$

where  $\nu$  is the photon energy,  $\sigma_{1/2}$  and  $\sigma_{3/2}$  are the absorption cross sections for total helicity 1/2 and 3/2, and  $\kappa$  is the anomalous magnetic moment of the target nucleon. The sum rule defined in eq. (1) is based on the additional assumptions of the low energy theorems and the no-subtraction hypothesis for the spin-flip part of the Compton scattering amplitude.

The sum rule can be evaluated for the proton, the neutron and for the proton-neutron difference. Multipole analysis suggested the possible violation of the sum rule, so that recently further arguments have been proposed in order to modify the original GDH expression. Among these the Extended Current Algebra approach (ECA) introduced a symmetric term in the charge density commutator added to the anomalous magnetic moment term.

Due to the availability of high quality circularly polarized photon beams, obtained from backscattered Compton laser processes or from bremsstrahlung off longitudinally polarized electrons, and due to the developments of new polarized targets, made of frozen

butanol or solid HD, a big effort is now being performed in several laboratories. Five experiments are planned in order to test the GDH sum rule from the pion threshold up to about 3 GeV where, due to the  $1/\nu$  weighting factor, it is expected a convergence. Therefore at the moment no improvement in the sum rule estimate is foreseen with a new high energy photon beam.

Furthermore the GDH integral can be naturally extended to the case of the absorption of circularly polarized virtual photons deduced from inclusive inelastic scattering of longitudinally polarized electrons off polarized nucleons:

$$I(Q^2) = \frac{m_p^2}{2\pi e^2} \int_{\frac{Q^2}{2m_p}}^{\infty} \frac{d\nu}{\nu} [\sigma_{1/2}(Q^2) - \sigma_{3/2}(Q^2)] \quad (2)$$

which is related to the experimental asymmetries  $A_1$  and  $A_2$  and to the polarized structure functions  $g_1$  and  $g_2$ . It is easy to show that for the high energy limit (DIS), when  $\gamma = \sqrt{Q^2}/\nu \ll 1$ , or for the real photon case, when  $Q^2 = 0$ , the integral is:

$$I(Q^2) = -\frac{\kappa}{4} \quad (3)$$

for  $Q^2 = 0$  [GDH value]

$$I(Q^2) = \frac{2m_p^2}{Q^2} \int_0^1 g_1(x) dx = \frac{2m_p^2}{Q^2} \Gamma \quad (4)$$

for  $\gamma \ll 1$ , where  $\Gamma$  is the first moment of  $g_1(x)$ . Thus the integral  $I(Q^2)$  connects the GDH expectation for real photon with the Ellis-Jaffe integral  $\Gamma$  (for p and n) and the Bjorken integral (for the p-n difference) in the asymptotic region, with an  $1/Q^2$  scaling factor. In fig. 1 are shown the  $1/Q^2$ -scaling of the integrals measured in the  $\langle Q^2 \rangle \sim 2 \div 10 \text{ GeV}^2$  range by different DIS experiments and the real photon point predictions: it is shown that, in order to be connected with the GDH-expectation at  $Q^2 = 0$ , a dramatic  $Q^2$ -dependence of  $I_p$  with a change of sign is required

below  $2\text{GeV}^2$ . Moreover if the ECA predictions are correct, the  $Q^2$ -dependence of  $I_p$  should be even stronger and also the Bjorken integral  $I_{p-n}$  should change sign in the low  $Q^2$  region.

Several phenomenological attempts to connect the two different regions were performed in the framework of VDM, with the inclusion of the  $\Delta$ , the Roper and other nucleon-resonances excitation, by using the Schwinger sum rule, from chiral perturbation theory and from relativistic quark models. Very different  $Q^2$ -dependences are predicted.

In order to evaluate the GDH sum rule in the low  $Q^2$ -region it will be necessary to provide a direct measurement of  $\sigma_{1/2} - \sigma_{3/2}$  or  $A_1$ , through a Rosenbluth plot (TJNAF planned experiments) or by comparing measurements with parallel and perpendicular polarization targets (HERMES experiment). Then due to the fact that  $A_1$  seems to be  $Q^2$ -dependent at low  $Q^2$ , a fine  $Q^2$ -binning and therefore high statistics and a wide and continuous kinematical plane will be required. Finally, contrary to the real photon and the asymptotic cases, both the resonant and the non resonant region will give similar contributions to the sum rule.

In the  $Q^2$ -region around  $1\text{ GeV}^2$  the GDH integral measurement will be provided by a combination of data coming from different experiments at SLAC, DESY and TJNAF in order to put together the resonance and the DIS regions (see fig. 2). In the very low  $Q^2$ -region close to the photon point (less than about  $0.5\text{ GeV}^2$ ) the TJNAF proposed experiments seem to be not sufficient to fulfill the sum rule due to their kinematical restriction just above the resonance region. Thus the study in this interesting region will be completed only by the measurements that will be performed with a higher energy upgrade of TJNAF or with the proposed new European electron machine ELFE. In fig. 2 is shown the kinematical region that could be covered by the three passes 5 GeV LINAC of original ELFE design in combination with the acceptance of the FAST spectrometer.

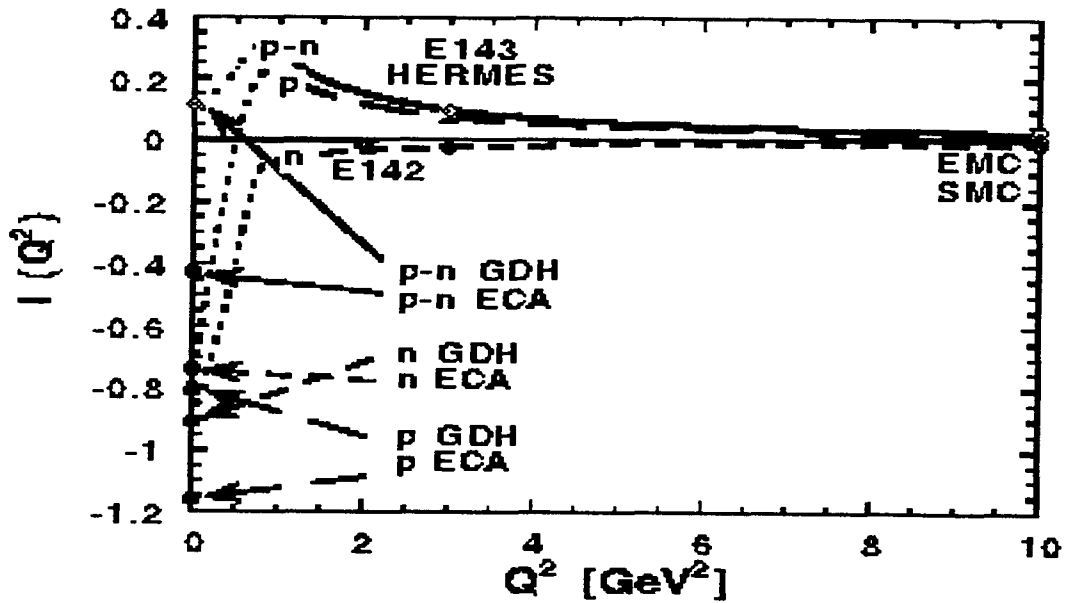


Fig. 1.  $1/Q^2$  evolution of the integrals  $I(Q^2)$  for the proton (long dashed line and open triangles), the neutron (short dashed line and open diamonds) and the proton neutron difference (solid line and open circles) derived from DIS experiments. Dotted lines are simple visual connections with the GDH expectation values (open symbols at  $Q^2 = 0$ ); also shown are the ECA predictions (close symbols).

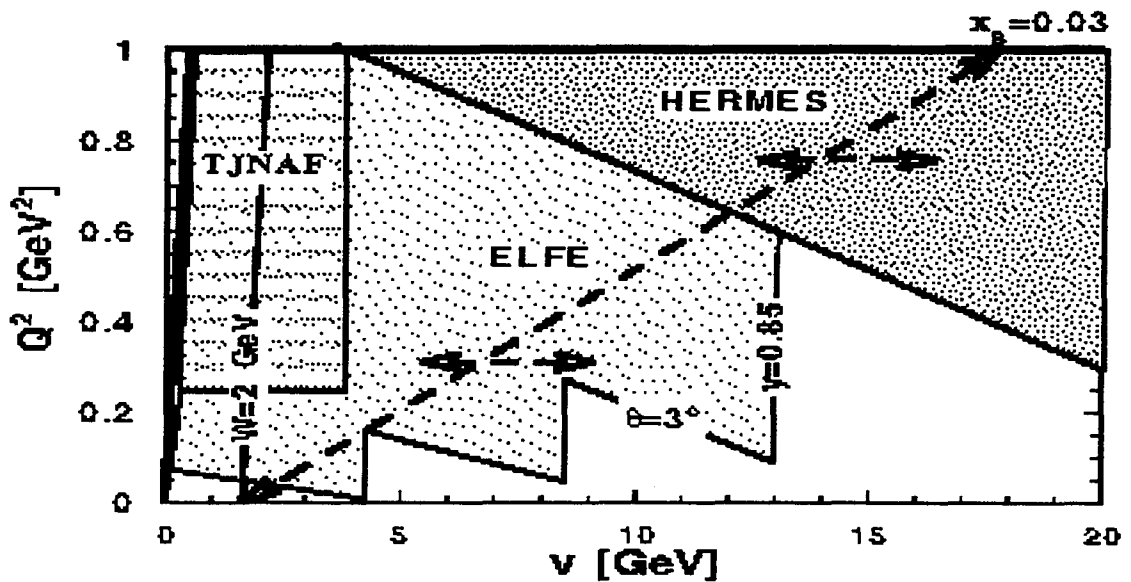


Fig. 2. Kinematical region covered by the TJNAF, HERMES and ELFE measurements. The dashed line connects the minimal upper- $\nu$  limits for the GDH evaluation for real photon ( $W = 2\text{GeV}$ ) and for DIS ( $x = 0.003$  at  $Q^2 \simeq 1.5\text{GeV}^2$ ).