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**Abstract**

Using the conventional Young's experiment with two pinholes, we observe the shift effect of the Fresnel's phase on the interference patterns, by measuring its intensity on the optical axis. It allows us to propose a criterion for distinguishing between Fraunhofer and Fresnel diffraction. Indeed, in the Fraunhofer domain the Young's patterns will be centered on the optical axis. As a consequence, only constructive interference will occur at this point. But in the Fresnel domain, the Young's patterns will be laterally shifted in such a way that constructive and destructive interference will occur alternatively on the optical axis, and the intensity of the diffraction pattern at this point will oscillate. Extended diffracting apertures can be analyzed as ensembles of Young's pairs of wavelet sources. From this point of view, the intensity distribution they produce on the detector plane results from the superposition of all their interference patterns.

## 1. PHYSICAL ANALYSIS OF THE FRESNEL'S ZONES

Fig.1 depicts the well-known diffraction of light waves by an aperture in a homogeneous and isotropic medium. According to Huygens-Fresnel principle [1], each point inside the aperture will be a source of spherical wavelets. Furthermore, all the sources must emit in phase. The superposition of these wavelets at a specific point on the observation plane yields the amplitude of the perturbation there.

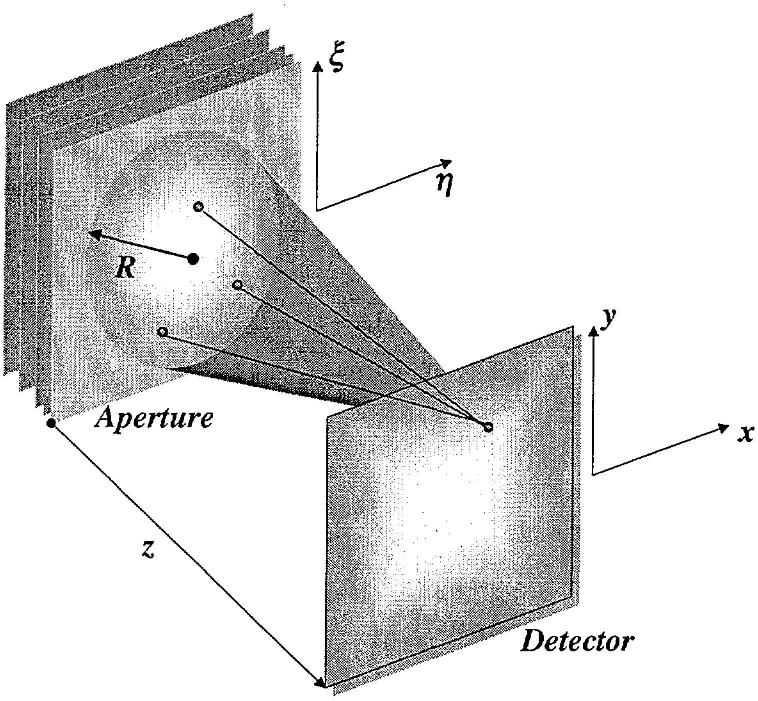


**Fig. 1: Diffraction of light waves**

This behavior is mathematically described by the diffraction integral [1]. In parabolic approach, it takes the form

$$u(x,y,z) = -\frac{i}{\lambda}\frac{e^{ikz}}{z} e^{i\frac{k}{2z}\left(x^2+y^2\right)} \iint t(\eta,\xi)\, e^{i\frac{k}{2z}\left(\eta^2+\xi^2\right)}\, e^{-i\frac{k}{z}(\eta\, x+\xi\, y)}\, d\eta\, d\xi\,, \quad (1)$$

where $u(x,y,z)$ is the amplitude of the light wave at the point of coordinates $(x,y,z)$ on the observation plane, $i=\sqrt{-1}$, $\lambda$ is the wavelength of the light, $k=\frac{2\pi}{\lambda}$ and $t(\eta,\xi)$ is amplitude distribution of the optical field that emerges from the aperture. It is proportional to the transmission of the aperture if it is illuminated by a plane wave.

Most of the theoretical and experimental developments in diffraction optics concern the solution of eq.(1). The main difficulty is to deal with the square phase factor of the integrand. Because the integral limits are determined by the support of the aperture transmission, the amplitude of the solid angle subtended by the aperture as seen from the point $(x,y,z)$ will determine the impact of such factor on the integral values. This solid angle is proportional to $\frac{\pi R^2}{z^2}$, with $\pi R^2$ the circle that inscribes the support of the aperture transmission.

Enough small solid angles allow us to approach $e^{i\frac{k}{2z}(\eta^2+\xi^2)} \approx 1$. So, eq.(1) reduces to a Fourier transform, which is usually solved by both analytical and numerical methods. This range of solid angles determines the well-known *Fraunhofer domain* [1-4]. The chapter of optics devoted to the specific solutions and applications of diffraction in Fraunhofer domain is called *Fourier Optics*, and plays an important role in optical engineering [2-4]. Fourier optics simulations are a popular tool for research and education, because of the wide use and popularity of two-dimensional Fast Fourier Transform (FFT) algorithms for image analysis and processing [5].

However, the solution of eq.(1) when the Fraunhofer approach is no longer valid becomes an important task in modern optical design, because great apertures and small distances between apertures and observation planes (like in integrated optics devices for instance [6]) introduce large solid angles.

Apertures with relative simple functions allow analytical solutions for eq.(1), but in most interesting applications, this integral should be solved numerically. Nowadays computers are used for this aim, but the beautiful Cornu algorithm [1] was developed in the last century to perform it by hand. Fresnel has also introduced his useful concept of *zones* as a mathematical attempt to make this calculation easier [1].

Independently of the solution procedures, it is also important (especially for educational purposes) to consider the physics involved in eq.(1). Usual detectors like phototubes, photographic plates or CCD arrays cannot detect the amplitude $u(x,y,z)$ of the light wave. Instead of that, they record its intensity that is proportional to $|u(x,y,z)|^2$, i.e.

$$I(x,y,z) = C\left(\frac{1}{\lambda z}\right)^2 \iint t(\eta_1,\xi_1)\, e^{i\frac{k}{2z}(\eta_1^2+\xi_1^2)}\, e^{-i\frac{k}{z}(\eta_1 x+\xi_1 y)}\, d\eta_1\, d\xi_1 \iint t^*(\eta_2,\xi_2)\, e^{-i\frac{k}{2z}(\eta_2^2+\xi_2^2)}\, e^{i\frac{k}{z}(\eta_2 x+\xi_2 y)}\, d\eta_2\, d\xi_2 , \quad (2)$$

where $C$ is a constant and $*$ denotes complex conjugate. To understand this expression properly, let us consider it as composed by two types of contributions, i.e.:

- Intensity contributions from the individual sources of wavelets inside the aperture, determined by the condition $(\eta_1,\xi_1) \equiv (\eta_2,\xi_2)$. This component is described by the integral $I_s(x,y,z) = C\left(\frac{1}{\lambda z}\right)^2 \iint |t(\eta,\xi)|^2 \, d\eta \, d\xi$. The phase factors in the integrand of eq.(2) do not have any impact on its value.

- Modulations due the interference of the wavelets emitted by all the source pairs inside the aperture. This component is described by eq.(2) provided that the condition $(\eta_1,\xi_1) \neq (\eta_2,\xi_2)$ is satisfied.

From this point of view, the wavelet sources inside the aperture constitute an ensemble of interfering pairs, and the intensity distribution recorded by the detector will result from the superposition of all interference patterns they produce.

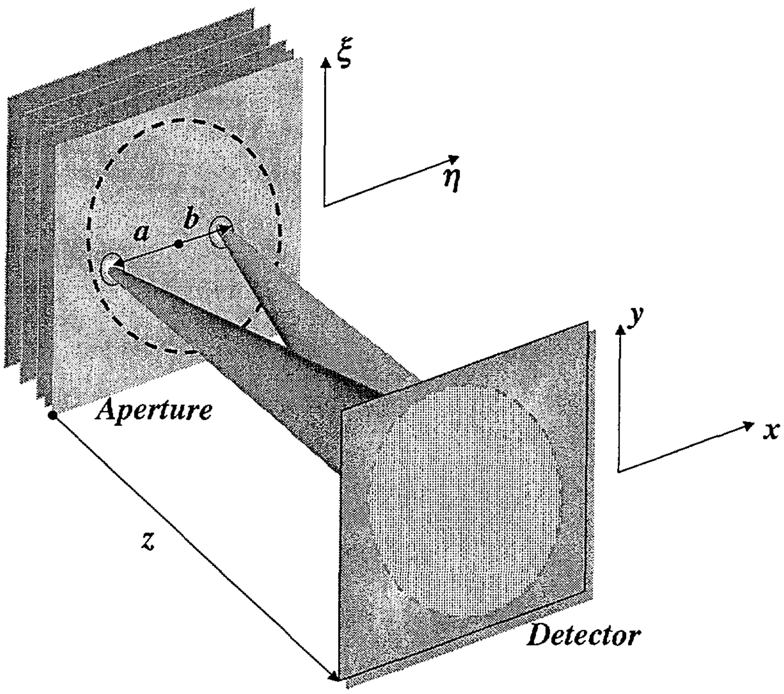


**Fig. 2: The pinhole experiment**

To see that more clearly and without loss of generality, let us consider an aperture conformed by two pinholes located on the $\eta$-axis at the positions $(a,0,0)$ and $(b,0,0)$, uniformly illuminated by a plane wave (Fig.2). The amplitude distribution that emerges from the pinholes will be

$$t(\eta,\xi) = \delta(\eta - a, \xi) + \delta(\eta - b, \xi), \tag{3}$$

where $\delta(\eta,\xi)$ is Dirac delta function [3]. Thus, the amplitude and intensity distributions at the detector plane will be given by

$$u(x,y,z) = -\frac{i}{\lambda}\frac{e^{ikz}}{z} e^{i\frac{k}{2z}\left(x^2+y^2\right)} \left[ e^{i\frac{k}{2z}a^2} e^{-i\frac{k}{z}a\,x} + e^{i\frac{k}{2z}b^2} e^{-i\frac{k}{z}b\,x} \right] \tag{4}$$

and

$$I(x,y,z) = C\left(\frac{1}{\lambda\,z}\right)^2 \cos^2\left(\frac{k}{2\,z}\left[(a-b)\,x - \frac{a^2-b^2}{2}\right]\right) \tag{5}$$

respectively. So, the intensity distribution produced by this aperture at the detector plane will be a cosine fringe pattern, whose fringes are parallel to the *y*-axis and whose period is determined by the pinhole separation *a-b*, i.e. a Young's interference pattern. The quantity $\frac{a^2-b^2}{2}$ is a number introduced by the quadratic phase factor of the integrand. It denotes a shift of the Young's pattern due the specific locations of the pinholes. It does not appear when the Fraunhofer approach is fulfilled. As a consequence, there are no shifts of the interference patterns at all if the diffraction geometry obeys the Fourier Optics. That is, the optical realisation of the Fourier spectrum invariance under function displacements [2-4].

It also means that constructive interference always occurs on the optical axis at the detector plane under the Fraunhofer approach. But constructive interference also appears there if the pinholes are equidistant from the optical axis too, i.e. $a^2 - b^2 = 0$, no matter the validity of the Fraunhofer approach. Such pinholes will be located on the same equi-phase curve of $e^{i\frac{k}{2z}\left(\eta^2+\xi^2\right)}$.

Therefore, the pattern shifts predicted by eq.(5) occurs if simultaneously the Fraunhofer approach is no longer valid and the pinholes are not equidistant from the optical axis. These shifts can be determined by measuring the intensity on the optical axis (*x*=0), which is given by

$$I(0,y,z) = I_0 \cos^2\left(\frac{\pi}{2\lambda\,z}\left(a^2-b^2\right)\right), \tag{6}$$

where $I_0 = C\left(\frac{1}{\lambda\,z}\right)^2$ is the maximum intensity on the optical axis when the Fraunhofer approach is fulfilled. The experimental situation can be analyzed in a simpler way by considering one of the pinholes attached directly on the optical axis, say $b = 0$. Thus, eqs.(5) and (6) reduce to

$$I(x;a) = I_0 \cos^2\left(\frac{k}{2\,z}\left[a\,x - \frac{a^2}{2}\right]\right) \tag{7}$$

and

$$I(0;a) = I_0 \cos^2\left(\frac{\pi}{2\lambda z} a^2\right) \quad (8)$$

respectively, for all values of $y$ and a fixed value of $z$. In the following we consider pinholes with variable separation $a$ attached to a fixed distance $z$ from the detector.

The oscillating variation of eq.(8) as a function of $a^2$ (Fig.3) reveals that constructive but also destructive interference will non-periodically alternate on the optical axis, depending on the pinhole separation. That is the cause of the pattern shift denoted by eq.(7).

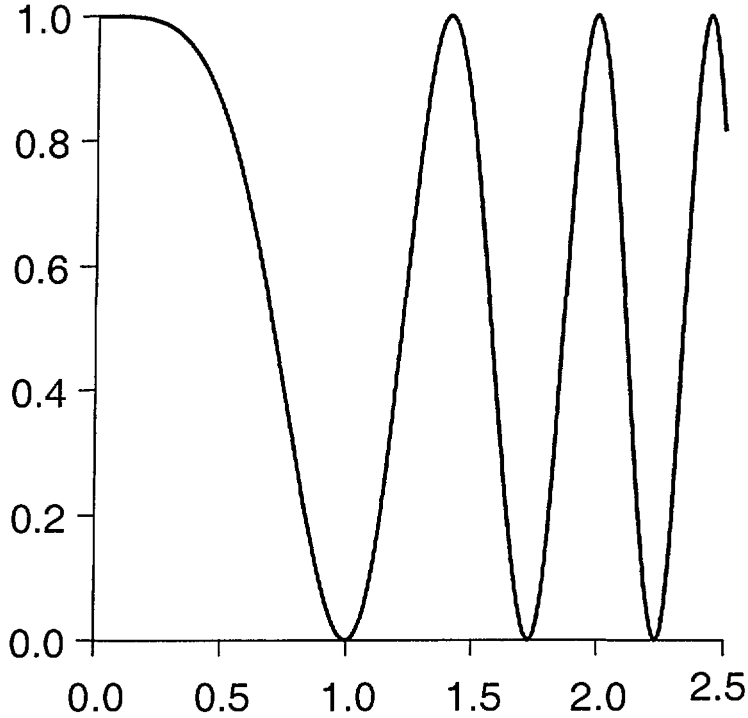


**Fig. 3: $\frac{I(0;a)}{I_0}$ vs $\frac{a}{\sqrt{\lambda z}}$. $I_0, \lambda, z$ will have fixed values. This graph describes the variation of the intensity on the optical axis at the detector plane, $I(0;a)$, as a function of the pinhole separation $a$.**

Constructive interference will be fully observed if the condition $a_{MAX} = \sqrt{2\,n\,\lambda\,z}$ ($n$=1,2,3…) is fulfilled. These values determine the locations of the maxims in Fig.3, except obviously for $a$=0. The registered intensity on the optical axis will be $I(0;a) = I_0$, and the Young's pattern will have the form $I(x;n) = I_0 \cos^2\left(\pi \sqrt{\frac{2\,n}{\lambda\,z}}\, x\right)$.

On the other hand, full destructive interference, i.e. $I(0;a)=0$, will be observed under the condition $a_0 = \sqrt{(2n-1)\lambda\, z}$. They determine the locations of the minims in Fig.3. Thus, the Young's pattern will be given by $I(x;n) = I_0 \sin^2\left(\pi \sqrt{\frac{2\,n-1}{\lambda\, z}}\; x\right)$.

Now, it is interesting to note that the pinhole pair is completely included in a circular area centered on the optical axis and of radius *a*. This area takes the form

$$\pi\, a^2 = \begin{cases} 2\,n\,\pi\,\lambda\, z & \text{for constructive interference} \\ (2n-1)\pi\,\lambda\, z & \text{for destructive interference} \end{cases} \quad , n=1, 2, 3, \ldots. \qquad (9)$$

Let us remember the Fresnel's algorithm. He developed a geometrical representation of $e^{i\frac{k}{2z}\left(\eta^2+\xi^2\right)}$ by sectoring the diffracting aperture in zones with the following properties [1]:

- All of them are circular sectors centred on the optical axis. It exploits the rotational symmetry of the quadratic phase factor, i.e. all the circumferences centred on the optical axis will be equi-phase curves.
- Starting on the optical axis, a change of the phase sign will occur at radii $\sqrt{n\,\lambda\, z}$. As a consequence, the phase inside zones with minor radius $\sqrt{(n-1)\lambda\, z}$ and major radius $\sqrt{n\,\lambda\, z}$ will have the same sign. Therefore, consecutive zones will have contrary phase signs.
- All the zones will have the same area $\pi\lambda z$. Then, we can characterize the area of an aperture at a given distance $z$ from the detector by the quantity $N\,\pi\lambda z$. $N$ is a real number that denotes the number of inscribed Fresnel's zones inside the aperture when it is observed from the intercept of the optical axis with the detector plane. The observation solid angle will be $N\,\pi\,\frac{\lambda}{z}$.

Nowadays, these zones are called *Fresnel's zones* and the range of solid angles for which they have impact on the diffraction phenomena is called *Fresnel domain* [1-4].

According to the properties of the Fresnel's zones, eq.(9) shows that:

- If the area determined by the pinholes is an even multiple of the Fresnel's area (pinhole separations $a = \sqrt{2n\,\lambda\, z}$), the corresponding wavelets will interfere constructively, because the pinholes are actually located on equal signed zones.
- On the contrary, if the area determined by the pinholes is an odd multiple of the Fresnel's area (pinhole separations $a = \sqrt{(2n-1)\lambda\, z}$), they will be located at reversed-signed zones and their wavelets will interfere destructively (Fig.4).

In summary, the oscillating behavior of the intensity on the optical axis observed by the detector [eq.(8)] and depicted in Fig.3, is a consequence of the square phase distribution of the Fresnel's zones, i.e. the *Fresnel's phase.*

Furthermore, the graph in Fig.3 shows a region for relative small pinhole separations in which the intensity variations are not significant. In other words, the shifts of the Young's patterns produced by such pinholes will be negligible. This region is corresponding to the Fraunhofer domain (Fourier Optics). Outside of it, the effect of the Fresnel's phase must be taken into account.

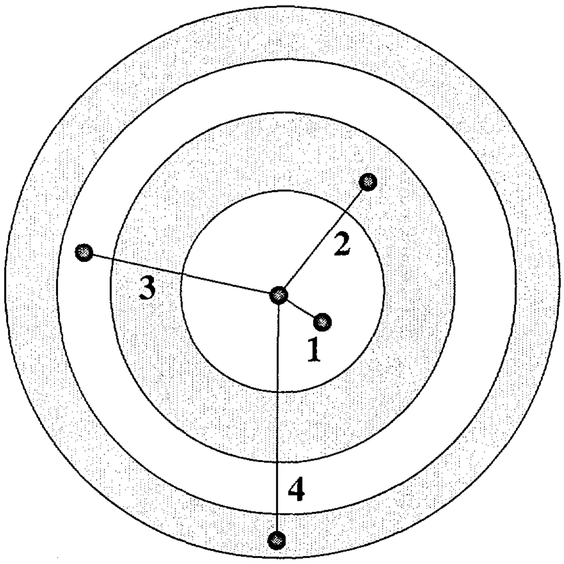


**Fig. 4: Illustrating the relationship between the pinhole separations and the Fresnel's zones. 1) Pinhole separation less than $a_{FF}$ (Fraunhofer domain). Their wavelets will always interfere constructively. 2) Pinhole separation about $\sqrt{\lambda z}$. They are located at consecutive zones (reversed phase signs), so that their wavelets will interfere destructively. 3) Pinhole separation about $\sqrt{2\lambda z}$. They are located in zones with equal sign Fresnel's phases, so that their wavelets will interfere constructively. 4) Pinhole separation about $\sqrt{3\lambda z}$. They are located in zones with reversed sign Fresnel's phases. Their wavelets will interfere destructively.**

For estimating the size of the region corresponding to the Fraunhofer domain, let us consider the approach $I(0;a) \approx I_0 \left[1 - \left(\frac{\pi}{2\lambda z} a^2\right)^2\right]^2 \approx I_0 \left[1 - \frac{1}{2}\left(\frac{\pi}{\lambda z} a^2\right)^2\right]$ for small pinhole separations. Now, we regard a decay of 20% in $I(0;a)$ as maximum permissible inside this region. It means that the maximum pinhole separation inside the Fraunhofer domain should be $a_{FF} = \sqrt{0.2\,\lambda z}$, i.e. a small fraction of the first Fresnel's zone. This result constitutes a criterion for distinguishing Fresnel and Fraunhofer diffraction, which is based on the pinhole separation. Such a criterion is complementary of criteria that have been formulated before [7].

## 2. EXPERIMENTAL RESULTS

Fig. 5 shows the used experimental set-up. The pinhole mask was composed by a circular stop of 3mm diameter, inside which a semitransparent photographic negative with high transparent pinhole pairs was inserted. The diameter of each pinhole was 0.3mm and the pinhole separations were 1.2, 1.4, 1.5, 1.8 and 2.5mm, respectively.

Two different illumination sources were used, i.e. a HeNe laser and a laser diode (630-675nm bandwidth). The light was collimated so that the pinhole mask was illuminated by a plane wave.

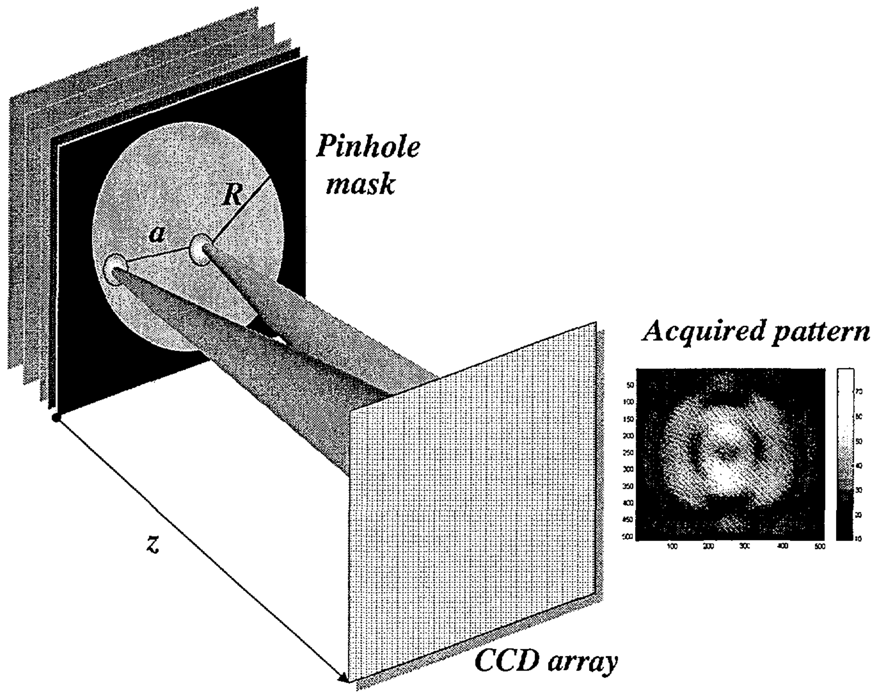


**Fig. 5: Experimental set-up**

The distance $z$ from the pinhole mask and the CCD array was adjusted to assure that only two Fesnel's zones were subtended by the circular stop, seeing it from the optical axis at the CCD plane. This adjustment is very fine and is characterised by a circular symmetric diffraction pattern of concentric rings with a black spot at the centre. This black spot also determines the intercept between the CCD plane and the optical axis.

Each pinhole subtends only 0.02 Fresnel's zone at this distance. Thus, the shift of the Young's pattern they produce will be actually an effect of only the Fresnel's phase. Furthermore, because of the semi-transparency of the photographic film, the diffraction pattern produced by the circular stop is always visible in the background. In other words, the acquired patterns are a combination of both the diffraction pattern of the circular stop and the Young's interference patterns of the pinholes, in such a way that the intensity on the optical axis is always very low.

It allows an accurate alignment of the pinholes by means of a micrometer screw, but impedes the direct measurement of the intensity on the optical axis. For this reason, the experimental observation of the effects of the Fresnel's phase is not based on this measurement, but on the analysis of the symmetry of the acquired patterns.

We have considered two experimental situations for each pinhole mask: equidistant pinholes with respect to the optical axis, and one pinhole centred on the optical axis. By the first experimental situation, the Young's pattern will exhibit cosine fringes and the whole-acquired pattern will be symmetrical with respect to the optical axis, denoted by the central black spot. In the second experimental situation, a shift of the Young's pattern with respect to the diffraction pattern is produced and the symmetry of the whole-acquired pattern will be broken.

Table 1 shows the acquired patterns. The symmetry of the patterns for the first experimental situation is apparent by both sources. Patterns produced by the same pinhole mask only differ on the visibility of the structure. A decay of the visibility by the laser diode is due to the low spatial coherence of this device. So, we can conclude that the Young's patterns for all the pinhole mask are conformed by cosine fringes centred at the optical axis, if the pinholes are equidistant with respect to that point no matter their specific positions inside the aperture stop.

Only the pinholes up to 1.5mm of separation can give experimental results for the second situation. Again the corresponding patterns for the two sources only differ in the visibility of the structure. Now, according to the geometry, the radius of the first Fresnel's zone is 1.06mm. So, centred one of the pinholes on the optical axis at the aperture stop plane, i.e. in the centre of the first Fresnel's zone, the other pinhole will be located inside the second Fresnel's zone for the pinhole masks with separations of 1.2 and 1.4 mm. For the last one (1.5mm separation), the second pinhole will be located at the external border of the second zone.

Thus, according to eq.(9) and Fig. 3, effects of destructive interference due the Fresnel's phase should appear on the optical axis at the CCD plane by the two first pinhole masks. In our experiment it implies a break of the pattern symmetry. Indeed, for 1.2mm of pinhole separation $\pi a^2 \approx 1.28\lambda z$ and for 1.4mm of pinhole separation $\pi a^2 \approx 1.74\lambda z$.

By comparing the two patterns produced by the first pinhole mask under HeNe laser illumination, the symmetry loss in the second experimental situation is apparent. A maximum on intensity appears on the horizontal axis close to the position of the first black ring in the first experimental situation. By laser diode illumination an extended bright zone is apparent on the first black ring of the first situation, because of the low visibility. A similar estimation can be performed by the second pinhole mask.

By the third pinhole mask (1.5mm pinhole separation) the constructive interference condition is restored and therefore, the pattern should be again symmetrical, as can be seen.

## 3. CONCLUSION

By using the conventional Young's experiment with two pinholes, we have shown that the Fresnel's zones have a physical meaning beyond their mathematical usefulness for diffraction calculation. Two domains can be distinguished by observing the shifts of the Young's interference patterns at the detector plane.

In the Fraunhofer domain the Young's patterns will be centered on the optical axis. As a consequence, only constructive interference will occur at this point. But in the Fresnel domain, the Young's patterns will be laterally shifted as an effect of the Fresnel's phase. Thus, constructive and destructive interference will occur alternatively on the optical axis, and the intensity of the diffraction pattern at this point will oscillate. We propose as a limit for distinguishing these two domains the pinhole separation $a_{FF} = \sqrt{0.2\,\lambda z}$ .

Extended diffracting apertures can be analyzed as ensembles of Young's pairs of wavelet sources. From this point of view, the intensity distribution they produce on the detector plane results from the superposition of all their interference patterns. The orientations, periods and shifts of the superposed patterns will depend on the orientation, Young's pair separation and specific locations of the wavelet sources with respect to the Fresnel's zones respectively. As a consequence, the whole diffraction pattern will always have a maximum of intensity at the optical axis in the Fraunhofer domain, but can exhibit black spots there in the Fresnel domain.

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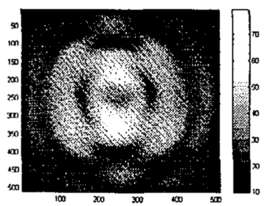
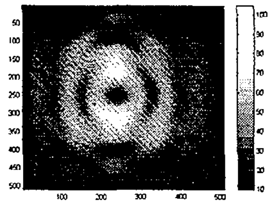
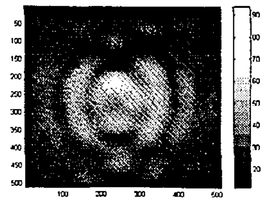
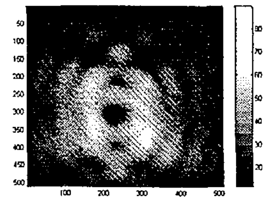
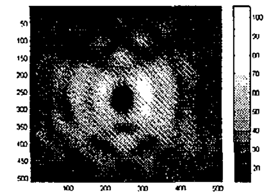
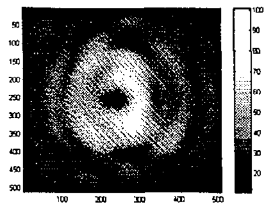
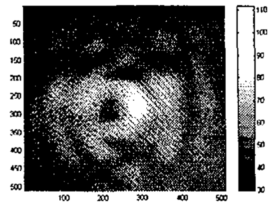
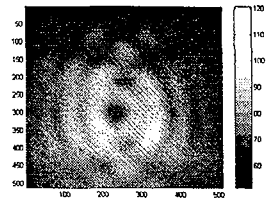
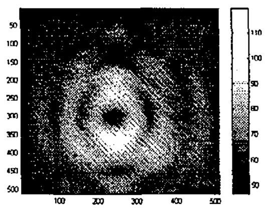
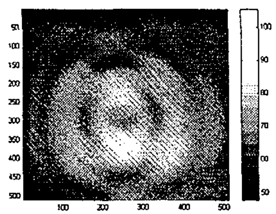
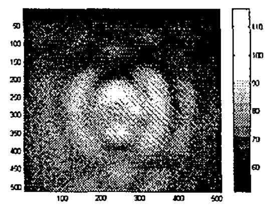
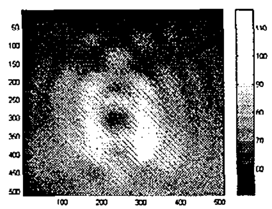
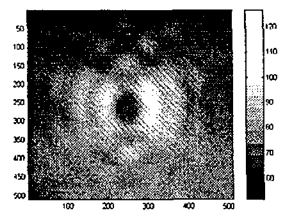
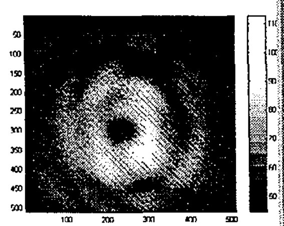
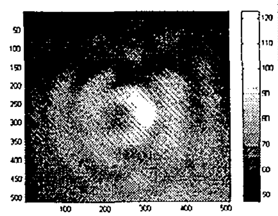
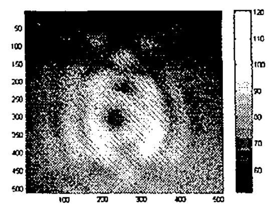
# TABLE 1: EXPERIMENTAL RESULTS

| | | | | | |
|---|---|---|---|---|---|
| | | | *Source: HeNe Laser* | | |
| *Pinhole separation (mm)* | 1.2 | 1.4 | 1.5 | 1.8 | 2.5 |
| *Equidistant pinholes to the optical axis* |  |  |  |  |  |
| *One pinhole centered on the optical axis* |  |  |  | | |
| | | | *Source: Laser Diode* | | |
| *Equidistant pinholes to the optical axis* |  |  |  |  |  |
| *One pinhole centered on the optical axis* |  |  |  | | |