A FINITE-VOLUME COMPUTATIONAL MECHANICS FRAMEWORK FOR MULTI-PHYSICS COUPLED FLUID-STRESS PROBLEMS

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Abstract

Where there is a strong interaction between fluid flow, heat transfer and stress induced deformation, it may not be sufficient to solve each problem separately (i.e. fluid vs. stress, using different techniques or even different computer codes). This may be acceptable where the interaction is static, but less so, if it is dynamic. It is desirable for this reason to develop software that can accommodate both requirements (i.e. that of fluid flow and that of solid mechanics) in a seamless environment. This is accomplished in the University of Greenwich code PHYSICA, which solves both the fluid flow problem and the stress-strain equations in a unified Finite-Volume environment, using an unstructured computational mesh that can deform dynamically. Example applications are given of the work of the group in the metals casting process (where thermal stresses cause elasto-visco-plastic distortion).

Introduction

Two main computational mechanics methods have developed over the years. Finite element methods (FEM) to tackle stress-strain problems, e.g. [1] and Finite Volume (FV) methods to solve fluid flow, e.g. Patankar[2].

Because even the simplest fluid flow problems are non linear, efficient segregated iterative techniques are the mainstay of FV-type algorithms, whilst in contrast, the mostly linear structural problems encountered by FEM codes led to the development of efficient direct solvers. The nature of the problems addressed also governed the type of mesh topologies used. So, FV type codes tend to be topologically cartesian and either structured of at least block-structured, whilst FEM codes are usually unstructured.

The authors, in an attempt to address the compatibility divide generated the PHYSICA code [3], which uses a hybrid Unstructured Mesh Finite Volume (FVUM) scheme, based on the ideas of Baliga et al.[4], but extended to address both the fluid and stress problems e.g. [5,6]. The versatility of this approach is such that it is now possible to tackle truly multi-physics problems, ranging from the problem of metal castings, where fluid flow and heat transfer in the presence of a free surface are coupled to solidification/re-melting, stress development in the solidified part of the metal and the mould at points of contact, and consequent distortion. The non-linearities present due to the elasto-visco-plastic behaviour of metal at high temperature cause no problems to the algorithm, since it was designed in the first place to handle non-linear (albeit fluid) problems.

In the problem of castings, the flow structure interaction can be treated as a static change in geometry of the problem at different timesteps. However, the method can also be applied to problems where the dynamic behaviour of the coupled system is important. Examples include flutter of aircraft wings, the oscillation of slender structures in high winds or in water-immersed structures due to wave action. The work of Slone et al. [7] shows this to be possible, using a modification of standard FEM schemes e.g. Farhat[8], in the FV environment.

In this paper the authors outline the salient features of the techniques adopted in PHYSICA, and show typical applications of the type indicated above.

Basic Equations

Fluid flow is characterised by transport equations for all the conserved quantities of the problem, i.e. mass, momentum and energy, together with closure models for turbulence, and phase change as appropriate:

Mass

$$\partial\rho/\partial t + \nabla(\rho\bar{u}) = 0, \qquad (1)$$

Momentum

$$\frac{\partial}{\partial t}(\rho\bar{u}) + \nabla(\rho\bar{u}\bar{u}) = \nabla.(\mu\nabla\bar{u}) - \nabla p + \bar{S} \qquad (2)$$

Enthalpy

$$\frac{\partial}{\partial t}(\rho h) + \nabla(\rho\bar{u}h) = \nabla.(k\nabla T) + \bar{S}_h \qquad (3)$$

Where u is the velocity vector, ρ is the material density, μ is the fluid effective viscosity and p the pressure. S, the source term in (2), contains the buoyancy and Darcy terms representing the semi-fluid "mushy" zone in alloys.

Heat transfer is represented by the enthalpy, h, in (3), with T being the temperature and k the thermal conductivity. Latent heat effects appear in the source term,

$$S_h = \frac{\partial(L\rho f_l)}{\partial t} - \nabla.(L\rho\bar{u}f_l) \qquad (4)$$

where L is the latent heat and f_l is the liquid fraction.

Solid mechanics equations are represented by the incremental stress σ and strain ε.

Stress Equilibrium

$$\Delta\sigma_{ij,j} = 0 \qquad (5)$$

Hooke's Law

$$\{\Delta\sigma\} = [D]\{\Delta\varepsilon_e\} \qquad (6)$$

relates incremental elastic strain to stress, with $[D]$ being the elasticity matrix. For metals, the von-Mises yield criterion is used and the total $\{\Delta\varepsilon\}$, thermal $\{\Delta\varepsilon_t\}$ and viscoplastic $\{\Delta\varepsilon_{vp}\}$, strains are related to the elastic strain by,

$$\{\Delta\varepsilon_e\} = \{\Delta\varepsilon\}-\{\Delta\varepsilon_t\}-\{\Delta\varepsilon_{vp}\} \qquad (7)$$

The Perzyna model is used for the viscoplastic strain-rate. The incremental strain is then related to incremental displacement $\{\Delta d\}$ through, $\{\Delta\varepsilon\}=[A]\{\Delta d\}$ (8)

$[A]$ being the differential operator matrix.

In dynamic response problems a slightly different approach is adopted as explained in Slone et al. [8].

FVUM Discretisation Procedure

The equations given above are discretised on unstructured meshes using FV procedures. For the flow equations the finite control volumes are cell-centred and for the CSM equations they are vertex-based. So, although the mesh is common, slightly different procedures are used to assemble the FV equations for the fluid velocity and solid displacement. The resulting system is of the form, $[A]\{\phi\} = \{b\}$ (9)

where $[A]$ is the coefficient matrix, $\{b\}$ the source vector, and $\{\phi\}$ any dependent variable.

The following procedure is then used to solve the equations:-

1. Solve momentum and continuity;
2. Solve energy equation;
3. Evaluate properties and auxiliary variables;
4. Repeat 1-3 to convergence;
5. Solve CSM equations;
6. Update total stress variables;
7. Recalculate mesh and geometry;
8. Repeat 1-7 for time-step advancement.

In castings parts of the mesh are allowed to split, to model the formation of gaps between cast and mould. In dynamic applications the fluid mesh surrounding the solid obstacle is distorted elastically, by assigning and low Young's modulus value to it.

Results

The procedures outlined above have been applied successfully to a large number of casting problems and also recently to aero-elastic applications [7]. Space permits only a small sample of simulations here. The reader is referred to the publications in the reference section for many detailed examples.

The first case concerns a benchmark test case, of an aluminium plate casting which appeared in [9]. Of interest in this situation is the behaviour of the free surface during filling, the solidification process and development of defects, i.e macro-porosity and mould and cast distortion. Figure 1. shows a filling sequence, where air (shown dark) is seen to be trapped in areas of recirculation. The sequence agrees well with x-ray photographs. Figure 2, shows the development of stresses in both the mould and plate casting which result in deformation, which is apparent in Figure 3.

Figure 4, shows a geometrically more complex example, which is nevertheless typical of what industry requires and indicative of the capability of the FVUM technique.

Conclusions

Increasingly there is a requirement to model *multiphysical* phenomena. A brief account of solid-fluid modelling in castings was presented here, using an unstructured mesh finite volume technique, within the code PHYSICA.

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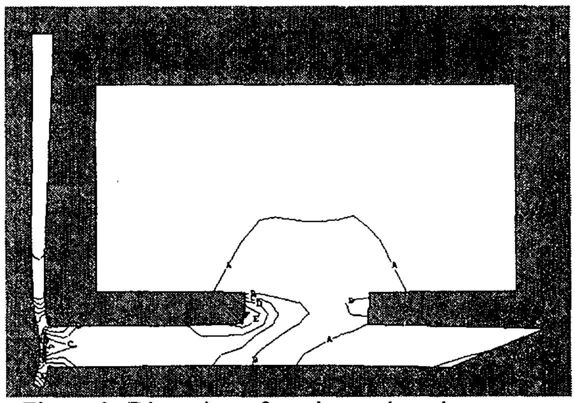

Figure 3: Distortion of casting and strain contours

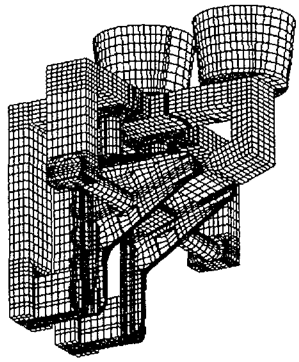

Figure 4: Typical Casting

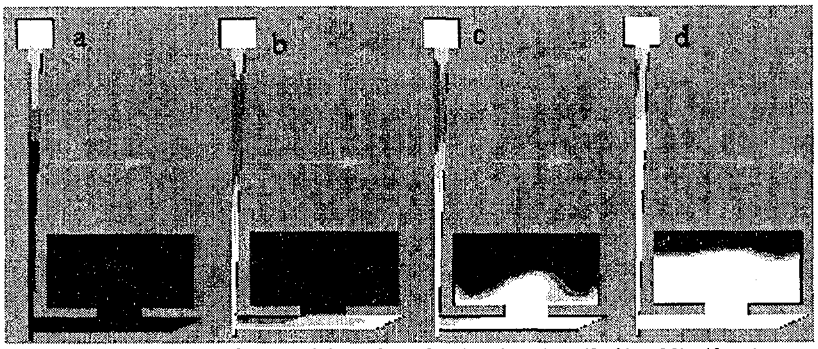

Figure 1: Filling sequence of an aluminium plate, showing air regions (dark) and liquid regions (white). Sequence left, to right

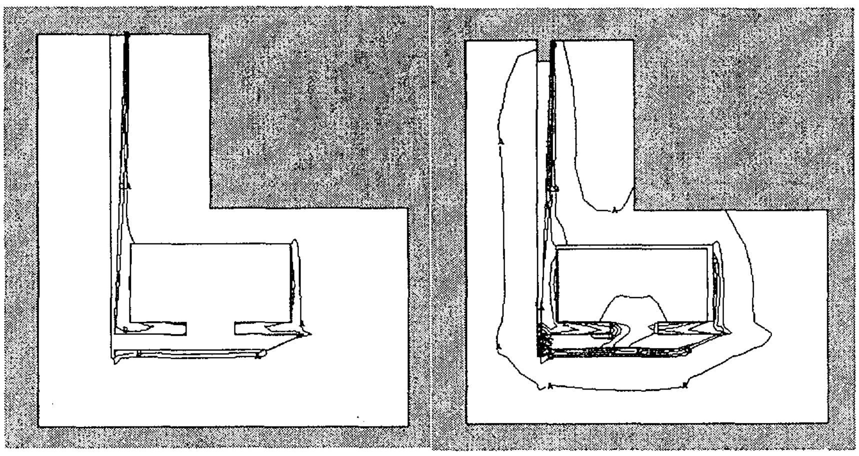

Figure 2: Stresses in mould and casting after (a) 18s, (b) 200s.