On linear and nonlinear stability of coaxial pipe flow.

I. Shapiro, L. Shtilman* and A. Tumin
Department of Fluid Mechanics and Heat Transfer, Tel-Aviv University
*Conference Lecturer

ABSTRACT

Linear and nonlinear stability of the flow between coaxial cylinders is investigated. The linear analysis is carried out within scope of temporal linear stability theory. The eigenvalue map is obtained with the collocation method based on Chebyshev polynomials. The nonlinear analysis is based on a novel parallel code for DNS of a coaxial pipe flow. It is shown that the major source of finite amplitude instability is associated with the interaction of three-dimensional disturbances and streamwise rolls.

INTRODUCTION

"Classical" shear flows as pipe Poiseuille flow, plane Couette flow have received much considerations for the recent years. The active interest in the flows is associated with the problem of bypass in laminar-turbulent transition. The flows are stable with respect to infinitesimal disturbances, while experiments revealed that there are critical Reynolds numbers above which the flows might be turbulent. Besides the understanding of specific flows, analysis of the bypass transition may be also important for other types of flows when level of perturbations is sufficiently high. Various "transition scenarios" have been proposed for bypass mechanisms of transition. A brief discussion of them are presented in[1, 2, 3] where a conception of self-sustaining process (SSP) is also proposed and illustrated with plane Couette flow. The idea is in the following. No matter what is the initial source, weak streamwise rolls are excited in the flow and they 'redistribute the streamwise momentum, and spanwise fluctuations occur in streamwise velocity. The flow becomes unstable with respect to three-dimensional wave-like type of disturbances, that, in turn, sustain the streamwise rolls. In the case of a pipe flow the SSP includes a generation of circumferential fluctuations in streamwise velocity, which are analogous to the spanwise variations in plane Couette flow. Recently, the idea of SSP found a support in experiments[4] with pipe Poiseuille flow subjected to controlled disturbances. The main result of the experiments indicates that the laminar-turbulent transition occurs only when a mean velocity distortion by the longitudinal vortices is developed.

In present paper we study a stability of flow between two concentric cylinders, r_1 and r_2 are radii of external and internal cylinders, respectively. When the gap width is relatively small with respect to the radii ($r_1 - r_2 \ll r_2$), the flow corresponds to plane Poiseuille flow. If the gap width is comparable with the biggest radius ($r_1 - r_2 \simeq r_1$), the flow is similar to the pipe Poiseuille flow. We are interested in an intermediate case, when $r_2/\ r_1 \simeq 0.5 - 0.7$ and the goal is to find out the SSP in this type of flow.

The basic tool of the present study is the direct numerical simulation of incompressible fluid in a coaxial cylinder. The flow is assumed to be periodic in the streamwise direction, periodic boundary conditions for the velocity are also used in axial direction. We consider an incompressible flow governed by the Navier Stokes equations. Reynolds number is based on distance between two coaxial cylinders. We use a cylindrical coordinates system (r, θ, z). The velocity vector is of the form $\mathbf{V} = \mathbf{u}\mathbf{e_r} + \mathbf{v}\mathbf{e}_\theta + \mathbf{w}\mathbf{e_z}$, where u, v, w are the velocity components, and e_r, e_θ, e_z are unity vectors in the radial,circumferential and axial directions. No slip and impermeable boundary conditions are taken on the internal walls of cylinders.

LINEAR ANALYSIS

We begin our analysis with the eigenvalue problem by using the temporal theory. We consider a laminar incompressible flow in a framework of the linearized Navier-Stokes equations for velocity components and pressure disturbance. The disturbances for velocities u, v, w, and pressure disturbance p are proportional to $e^{i(m\theta + n\frac{2\pi}{L}z - \omega t)}$, where parameter m is an integer azimuthal index, n is an integer streamwise index, L -is the tube length and ω is a frequency. We obtain the governing equations as a system of ordinary differential equations with boundary conditions and obtain the eigenvalue problem for ω. The frequency ω is a complex one and, according to the temporal theory, the disturbance grows when $Im(\omega) > 0$. It was verified that it is enough to choose 60 Chebyshev polynomials for the first few tens of eigenmodes. The map of complex frequency ω of modes with $n = 1$, $m = 1$ is shown in Fig.1

for $Re = 20000$ and various relations r_2/r_1. It is shown that for $r_2/r_1 > 0.6$ there is an unstable mode. The maps for $m = 0, 2$ and 3 are similar.

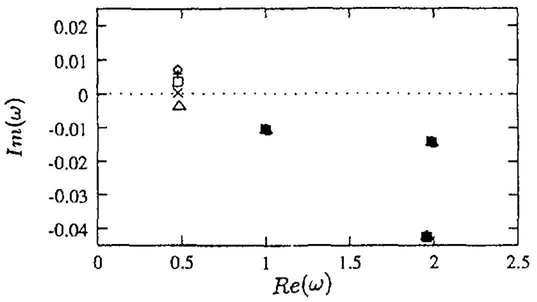

Fig. 1 *Eigenvalues in the complex plane ω. ◇ - $r_2/r_1 = 0.9$, $+ - r_2/r_1 = 0.8$, □ $- r_2/r_1 = 0.7$, $\times - r_2/r_1 = 0.06$, $\Delta - r_2/r_1 = 0.5$.*

DIRECT NUMERICAL SIMULATIONS

1. Linear Theory and DNS

We have compared results of numerical simulations with the presented above results of linear temporal theory. We have chosen initial velocity field as a sum of the basic flow and a small amplitude first unstable eigenmode. A development of the velocity field in space and time has been considered. The solutions are presented in dimensionless form with distance between coaxial cylinders as length scale and with the maximum of the mean axial velocity as a velocity scale. We have checked two cases: small axisymmetric and non axisymmetric perturbations of the basic velocity profile. For both cases the Reynolds number $Re = 20000, r_2/r_1 = 0.7$. The amplitude of the axial velocity disturbance was chosen equal to 0.007. A comparison between the dynamics of the perturbation obtained from the numerical simulation and from the stability analysis is done by space comparing in Fig. 2 and by time comparing in Fig. 3. Analogous results were obtained for case non-axisymmetric flow with $m = 1, n = 2$.

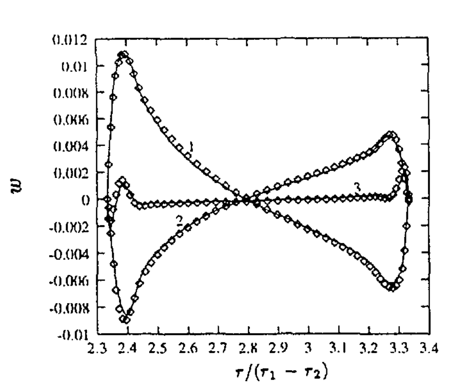

Fig. 2 *The solid curves 1, 2, 3 show the solution according to the linear theory for $z = L$, $z = 0.47L$, $z = 0.3L$ correspondingly, ◇ - numerical solution at the same conditions, at time moment t=6T, $T = 2\pi/Re(\omega)$.*

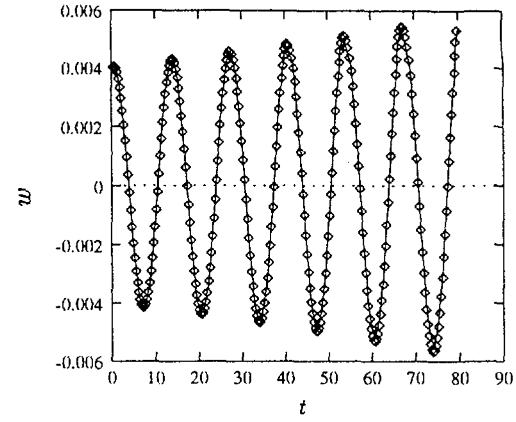

Fig. 3 *The solid line shows the dynamics of axial velocity perturbation $w(t)$ for the unstable linear eigenmode. ◇ – numerical solution.*

2. Finite amplitude analysis

We carried out numerical experiment with the initial disturbance consists of two eigenmodes with equal streamwise numbers $n = 1$. They have azimuthal numbers equal by value but of opposite sing : $m = \pm 1$. Amplitude of axial velocity component for each mode was chosen equal to 0.025. In these simulations $Re = 15500$, $r_2/r_1 = 0.7$. The dynamics of axial velocity perturbation is shown on Fig. 4. One can see a fast build up of initial disturbances and then their destruction. The Fourier analysis of the velocity field for this case shows that from $t > 0$ a strong growth of nonlinear harmonics with $n = 0$ and $m = \pm 2$ takes place.

To examine an importance of the modes with $n = 0$, $m = \pm 2$, corresponding to counterrotating rolls, for an origin of turbulence we carried out several experiments. In those simulations $Re = 13000$, $r_2/r_1 = 0.7$ and linear unstable eigenmode was absent. We present below just some typical results.

Experiment 1. The eigenvalue for modes with $n = 1, m = \pm 1$ and maximal value of $Im(\omega)$ is $\omega_1 = 0.51414 - i0.00249$. There are small decreasing harmonics: $Re(\omega_1)/Im(\omega_1) = -206$. These two modes have been used as initial disturbances of the basic laminar flow in the first of the simulation. The maximal value of axial velocity component for each mode was equal to 0.025. The dynamics of axial velocity perturbation is shown on Fig. 5. It is similar to dynamics of unstable modes, which is shown on Fig. 4. There is a fast build up of disturbances at $t \leq 50$ and the Fourier analysis of velocity field show a strong growth of harmonics with $n = 0$ and $m = \pm 2$ - corresponding to counterrotating rolls. At $t > 50$ numerical scheme becomes unstable. The longer time of development of instability is associated with decreasing of initial modes in this case.

Experiment 2. Velocity components $u(r), v(r), w(r)$ have been selected by Fourier transform for modes with $n = 1, m = \pm 1$ and modes with $n = 0, m = \pm 2$ at time moment $t = 36$. For this moment modes with $n = 1, m = \pm 1$ are closed to initial eigenmodes but velocity harmonics with $n = 0, m = \pm 2$ have already increased considerably. Maximal value of $w(r)$ for modes with $n = 0, m = \pm 2$ is equal half of one for initial disturbance. Harmonics with $n = 1$ and $m = \pm 3$ have been excited too, but maximal value of $w(r)$ for these harmonics is 6.5 times less then one for harmonics with $n = 0, m = \pm 2$. Two first of selected modes have been used as initial disturbances of the basic flow in first simulation. The result is shown in Fig. 6(curve 1). The two azimuthal modes of the small amplitude do not cause a transition. The modes corresponding rolls have been used as initial disturbances in second simulation. The result is presented in Fig. 6 (curve 2), and we see that the streamwise rolls of the small amplitude also do not cause a transition.

Final step of our study is simulation with the initial disturbance consisted of four selected modes.We observed in this case that the growth of mean value of perturbation at $t < 240$ is produced by a strong growth of the modes with $n = 0, m = \pm 2$. The dynamics of axial velocity perturbation for this case is shown in Fig. 7. One can see a development of instability in this case. The growth of mean value of perturbation at $t < 240$ is also produced by a strong growth of the modes with $n = 0$, $m = \pm 2$ corresponding to two pairs of counterrotating streamwise rolls.

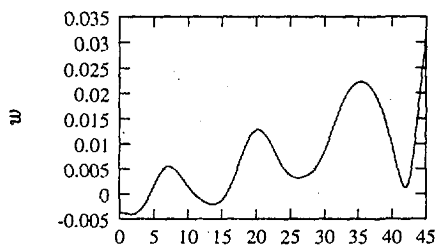

Fig. 4 *The dynamics of the axial velocity perturbation w in fixed point, $Re = 15500$.*

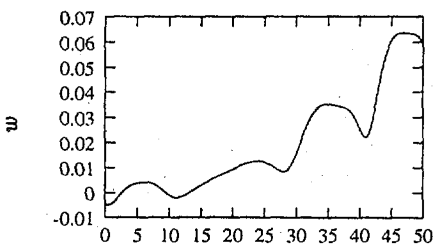

Fig. 5 *The dynamics of the axial velocity perturbation w in a fixed point. $Re = 13000$, an initial disturbance of the flow has been chosen as sum of small decaying eigenmodes with $n = 1$, $m = \pm 1$.*

CONCLUSION

The stability of the flow in a coaxial pipe has been analyzed by means of DNS. We have found a basic mechanism of instability in coaxial pipe - interaction of two counterrotating modes with streamwise rolls.

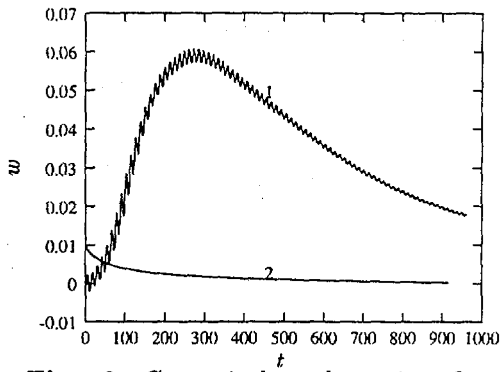

Fig. 6 *Curve 1 show dynamics of w when an initial disturbance have been chosen as sum of nonlinear modes with $n = 1$, $m = \pm 1$. Curve 2 show dynamics of w when an initial disturbance are counterrotating rolls.*

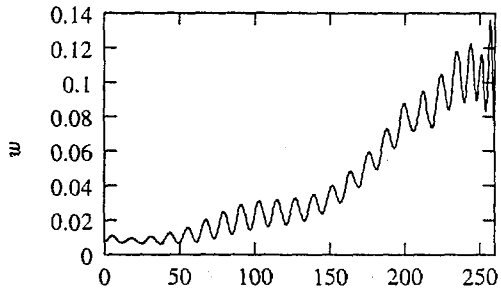

Fig. 7 *The dynamics of w for case when an initial disturbance have been chosen as sum of nonlinear modes with $n = 1$, $m = \pm 1$ and modes with $n = 0$, $m = \pm 2$.*

References

[1] Waleffe, F. "Hydrodynamic stability and turbulence: beyond transients to a self-sustaining progress". Stud. Appl. Math., **95**, 319 (1995).

[2] Waleffe, F. "Transients in shear flows. Nonlinear normality versus non-normal linearity". Phys. Fluids, **7**, 3060 (1995).

[3] Waleffe, F. "On a self-sustaining process in shear flows". Phys. Fluids., **9**, 883 (1997).

[4] Eliahou, S., Tumin, A., and Wygnanski, I. "Laminar-turbulent transition in Poiseuille pipe flow subjected to periodic perturbation emanating from the wall". J. Fluid Mech., Submitted (1996).