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The Hurst exponent and long-time correlation

G. Wang, G. Antar, P. Devynck

Association Euratom-CEA sur la fusion contolee, CEA Cadarache,
13108 Saint Paul lez Durance, France

ABSTRACT

The rescaled range statistics (R/S) method is applied to the ion saturation current fluctuations measured by Langmuir probe at edge on Tore Supra to evaluate the Hurst exponent. Data block randomization is carried out to the data sets in order to investigate the relationship between the Hurst exponent and long-time correlation. It is observed that H is well above 0.5 in the long-time self-similar range. However, it is found that the information which leads to $H > 0.5$ is totally contained in the short-time correlation and no link to long times is found.

Recently some of the observed phenomena suggest that self-organization-criticality (SOC) [1] mechanism may play an important role in plasma transport in magnetically confined plasmas. [2, 3] Some plasma turbulence models have also shown some features of SOC systems: [4, 5] transport by avalanches, $1/f$ behavior of the frequency spectrum, etc.

To identify the existence of transport by avalanches, we can detect the long-time correlation induced by avalanche-type transport. This long-range dependence should appear as an algebraic decay of auto-correlation function (ACF) for long time lags. However, because of short available data sets, it is hard to accurately determine the long tail of auto-correlation in magnetic fusion devices. A rescaled range statistics (R/S) method has been proposed [6] to evaluate Hurst exponent (H) to determine this long-time dependence. For a time series of length n, $X = \{X_t : t = 1,2,\cdots,n\}$, the R/S ratio is defined as the ratio of the maximal range of the integrated signal normalized to the standard deviation:

$$\frac{R(n)}{S(n)} = \frac{\max(0, W_1, W_2, \cdots, W_n) - \min(0, W_1, W_2, \cdots, W_n)}{\sqrt{S^2(n)}}$$

Here $W_k = X_1 + X_2 + \cdots + X_k - k\overline{X}(n)$, where $\overline{X}(n)$ and $S^2(n)$ are respectively the mean and variance of the signal. The expected value of R/S scales like cn^H as $n \to \infty$, where H is called Hurst exponent. Hurst *et al* have found [7] that $H = 0.5$ for random data, and $H > 0.5$ for biased random data. However, it is not clear to what extend Hurst exponents $H > 0.5$ obtained from observations by the R/S method imply persistence. In Ref. 6, it is believed that when $1 > H > 0.5$, there are long-time correlation (persistence) and when $0.5 > H > 0$, the series has long-time anti-correlation (anti-persistence). In several devices [6], H has been calculated to range from 0.62 to 0.72, which is assumed to indicate the existence of algebraic decay of ACF for long time lags, i.e., the existence of long-time correlation in the plasma edge ion saturation current fluctuations. The result was supposed to be in accordance with plasma transport characterized by SOC.

In the present study, we apply the R/S method to ion saturation current fluctuations measured by Langmuir probe at the edge on Tore Supra. It is observed that H is well above 0.5 in the long time-range. However, it is found that the information which leads to $H > 0.5$ is totally contained in the short-time correlation and no link to long times is found.

Experiments have been done in ohmic helium discharges on Tore Supra. The Langmuir probe used in the experiments is the standard reciprocating probe used on Tore Supra as a Mach probe which measures routinely the density and temperature profiles at the plasma edge and in the scrape-off layer. The probe consists of 6 composite carbon tips of 6 mm diameter poloidally and toroidally separated. Three probe tips poloidally separated by 0.68 cm are facing the plasma in the upstream current direction and three others in the downstream direction. They are shielded from the plasma by a 4 cm diameter cylinder with holes drilled through it. The surface collection of the probe is then defined by the size of the holes which is 4 mm in diameter. The probe tips are biased relatively to machine ground at −100 V to measure the ion saturation current. A fast acquisition with a sampling rate of 1 MHz is triggered at selected probe penetrations. For each probe plunge, typically 8 kbytes are recorded for each probe tip. Because the probe movement can introduce a low frequency part into the fluctuation data, the currently studied data are chosen with the probe nearly motionless.

Shown in figure 1 is R/S as a function of the time lag. H is defined in the self-similar range 0.1-7 ms and equal to 0.8. The upper time limit (7 ms) is set by the length of the sampled data. For this data set, the typical decorrelation time is about 0.02 ms, and the plasma energy confinement time is about 200 ms, so the time lag in the self-similar range is much longer than the turbulence decorrelation time, but less than the confinement time, similar to H calculations in other devices. [6] For several data sets with probe located at edge, it is found that H ranges from 0.6 to 0.94, well above 0.5.

In order to examine how far the H exponent reflects the long-time correlation, we divide the original time series into non-overlapping blocks with equal length, then randomize the blocks in order to decorrelate time scales longer than the block size, and recalculate the H value in the same self-similar range (0.1-7 ms). Typically an average over 20 times of randomization is done to get good statistics. Figure 2 shows the normalized ACFs for the original data (the same as in figure 1) and randomized data with the length of blocks $\Delta N = 30, 50,$ and 80 (corresponding time scale is $T = 0.03, 0.05$ and 0.08 ms respectively). It is found that the normalized ACFs for the randomized data decrease to zero at T, showing that randomization of the blocks does in fact decorrelate scales longer than T. Correspondingly, the R/S versus time lag for randomized time series with different ΔN are shown in figure 3 where $H = 0.61, 0.67,$ and 0.71 is evaluated. These values are obtained with no long-time correlation because of the randomization, as indicated by the normalized ACFs in figure 2. The procedure has been done to a variety of shots on Tore Supra, and the same result is obtained. This shows that a time series with H exponent well above 0.5 does not necessarily reflect long-time correlation.

Figure 4 illustrates H values for randomized data versus ΔN for two plunges of the reciprocating probe as well as the normalized ACFs for the original data. The standard deviation, deduced from 20 times of randomization, is indicated by the error bar in figure 4(b). Figure 4(b) shows that H at first increases with ΔN, then it saturates to a constant value $H_0 \approx 0.8$ which is the value estimated for the original data as illustrated in figure 1. The result that H saturates to H_0 when ΔN is larger than a critical value ΔN_c is verified for all studied data sets with probe located at edge. For figures 4(b) and 4(d), ΔN_c is about 110 and 60 respectively, i.e. $T_c = 0.11$ ms and 0.06 ms correspondingly. This indicates that the H value of the original data is totally determined by times less than T_c, and that $T > T_c$ does not bring any new information. Note that the normalized ACFs in figures 4(a) and 4(c) decrease to zero around T_c, so T_c is associated with the macro-scale of turbulence, which is shorter than times in the self-similar range. Therefore figure 4 illustrates a clear relationship between the H value and short-time correlation. In order to test whether this is related to probe movement, we also did the same calculation for fixed probe data. Shown in figure 5 is an example with the probe located in the scrape-off layer. We get the same result as in figure 4, indicating the robustness of the relationship between the H value and short-time correlation.

In conclusion, applying the rescaled range statistics method to the ion saturation current fluctuations measured by Langmuir probe, it is found that H is well above 0.5 in the long-time self-similar range. However, randomization of the data blocks indicates that this does not necessarily mean long-time correlation. In fact, it is found to be connected with short-time correlation which is manifested by the fact that H saturates to H_0 for ΔN larger than ΔN_c. These results suggest that the R/S method is not appropriate for evaluating long-time correlation in fusion devices. More research is needed to clarify the relationship, when it exists, between the H value and long-time correlation.

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Figure captions

Figure 1 R/S as a function of time lag for shot 25141, probe at $r/a = 1.04$. Plasma conditions: toroidal magnetic field $B_t = 3.9\,\mathrm{T}$, plasma current $I_p = 1\,\mathrm{MA}$, averaged plasma density $\bar{n}_e = 3.78 \times 10^{19}$ m^{-3}, and edge safety factor $q_a = 5.62$.

Figure 2 Normalized ACF for the original data and randomized data with block length $\Delta N = 30, 50$ and 80.

Figure 3 R/S as a function of time lag for randomized data with block length $\Delta N = 30, 50$ and 80.

Figure 4 (a) Normalized ACF and (b) H versus block length for shot 25141, (c) normalized ACF and (d) H versus block length for shot 25164 with probe at $r/a = 0.99$, $B_t = 3.77\,\mathrm{T}$, $I_p = 0.81\,\mathrm{MA}$, $\bar{n}_e = 1.28 \times 10^{19}$ m^{-3}, $q_a = 6.89$.

Figure 5 (a) Normalized ACF and (b) H value versus block length for shot 22253 with probe fixed in the SOL with $r/a = 1.1$ in an ergodic divertor module, $B_t = 3.14\,\mathrm{T}$, $I_p = 0.9\,\mathrm{MA}$, $\bar{n}_e = 3.24 \times 10^{19}$ m^{-3}, $q_a = 4.93$.

Fig. 1

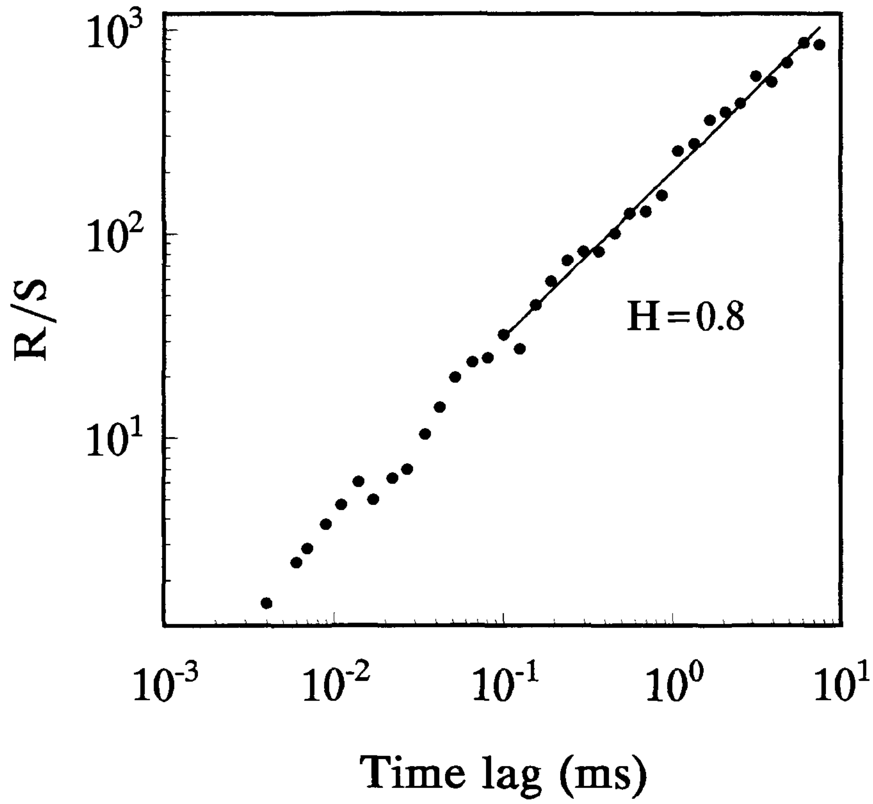
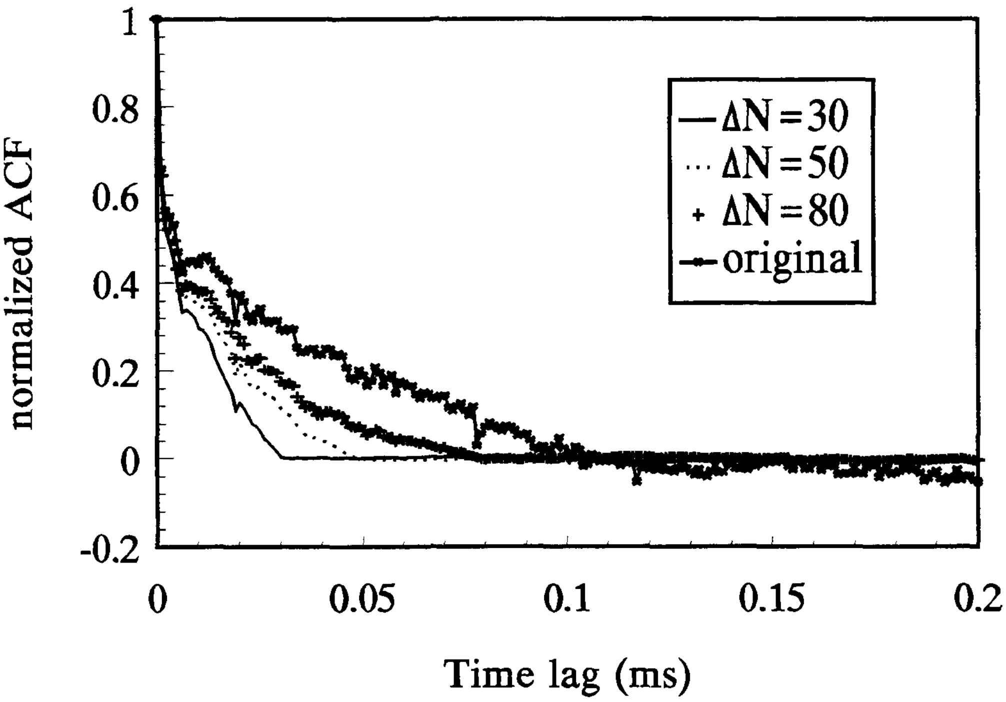
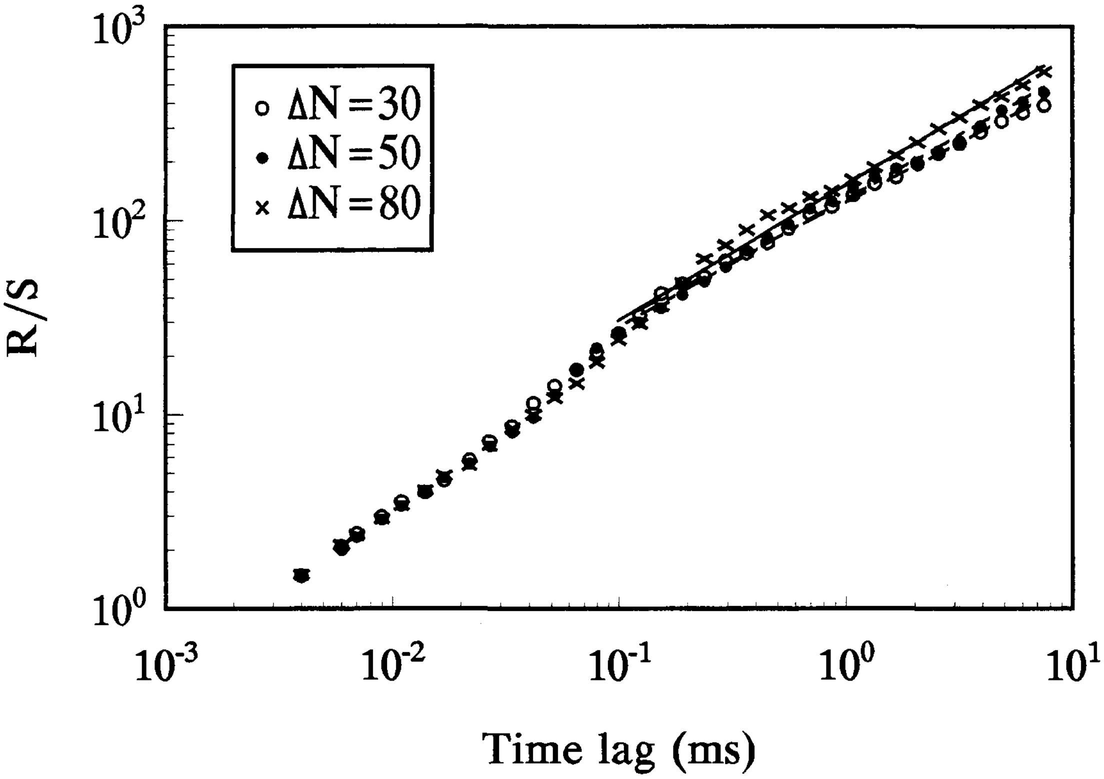


Fig. 2



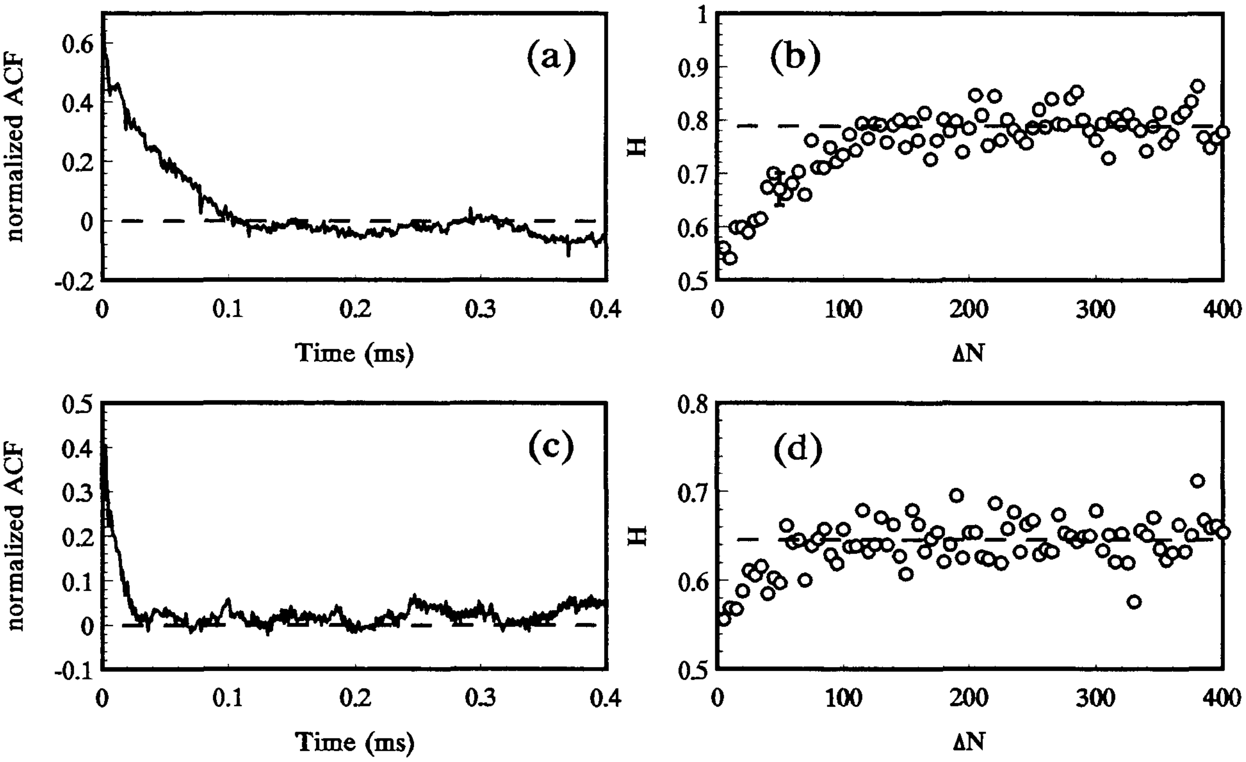
1
0.8
0.6
0.4
0.2
0
-0.2
normalized ACF
ΔN=30
ΔN=50
ΔN=80
original
0
0.05
0.1
0.15
0.2
Time lag (ms)

Fig. 3



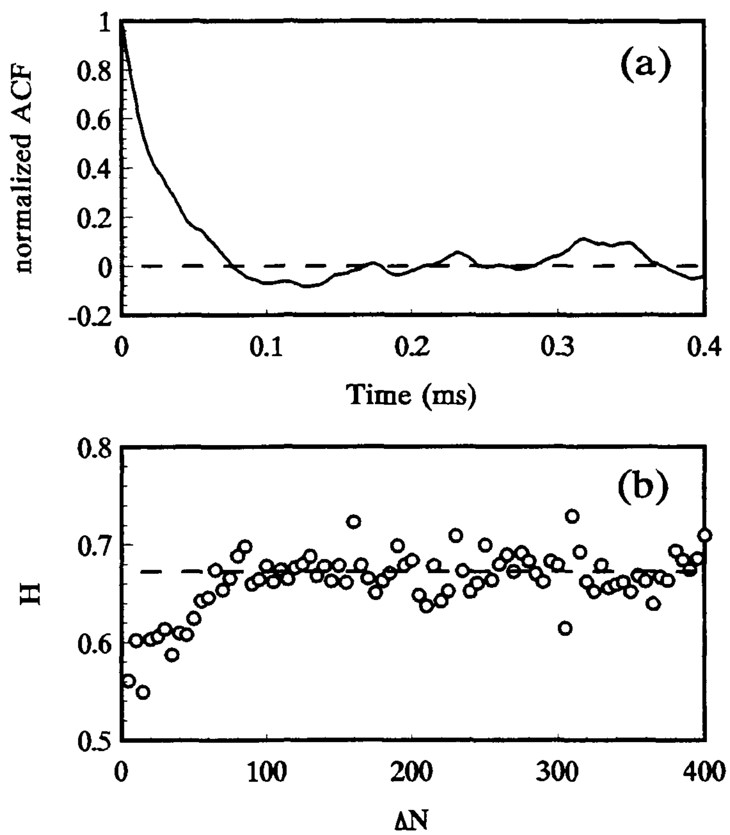
ΔN=30
ΔN=50
ΔN=80
R/S
Time lag (ms)
10^0
10^1
10^2
10^3
10^{-3}
10^{-2}
10^{-1}
10^0
10^1

Fig. 4



(a)
normalized ACF
0.6
0.4
0.2
0
-0.2
0
0.1
0.2
0.3
0.4
Time (ms)
(b)
H
1
0.9
0.8
0.7
0.6
0.5
0
100
200
300
400
ΔN
(c)
normalized ACF
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
0.1
0.2
0.3
0.4
Time (ms)
(d)
H
0.8
0.7
0.6
0.5
0
100
200
300
400
ΔN

Fig. 5



(a)
normalized ACF
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.1
0.2
0.3
0.4
Time (ms)
(b)
H
0.8
0.7
0.6
0.5
0
100
200
300
400
ΔN