2.4 Accurate Perturbation Calculation for Nodal Diffusion Method

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Nodal diffusion methods[1] have widely been used in reactor physics analyses. However, two problems remain for the methods in relation to adjoint calculations. The first one is the treatment of a discontinuity factor[2] in an adjoint problem. Conventional treatment[1), 2)] of the discontinuity factor for a forward problem can not be applied to the adjoint one; meaningful adjoint fluxes can not be obtained and eigenvalues for both problems don't coincide. To overcome this problem, Kobayashi presented a consistent theory[3] for the treatment of the discontinuity factor in adjoint problems, and he demonstrated analytically that an accurate perturbation calculation could be done in a simple one-group and one-dimensional problem by introducing his theory. On the other hand, the second problem appears in multi-dimensional problems. In conventional nodal methods, a three-dimensional flux distribution in a node is not directly calculated, while a one-dimensional transverse integrated flux in each direction is obtained. In the case of an exact perturbation calculation, the following two terms appear;

$$\begin{aligned}\left\langle \phi_g^{\dagger} \cdot \phi_g^{*} \right\rangle &\equiv \int_{Node} \left\{ \phi_g^{\dagger}(\boldsymbol{r}) \cdot \phi_g^{*}(\boldsymbol{r}) \right\} d\boldsymbol{r} \\ &\approx \int_{Node} \phi_g^{\dagger}(\boldsymbol{r}) d\boldsymbol{r} \cdot \int_{Node} \phi_g^{*}(\boldsymbol{r}) d\boldsymbol{r} \; / \; \int_{Node} d\boldsymbol{r} \,,\end{aligned} \tag{1}$$

$$\begin{aligned}\left\langle \nabla \phi_g^{\dagger} \cdot \nabla \phi_g^{*} \right\rangle &\equiv \int_{Node} \left\{ \nabla \phi_g^{\dagger}(\boldsymbol{r}) \cdot \nabla \phi_g^{*}(\boldsymbol{r}) \right\} d\boldsymbol{r} \\ &\approx \int_{Node} \nabla \phi_g^{\dagger}(\boldsymbol{r}) d\boldsymbol{r} \cdot \int_{Node} \nabla \phi_g^{*}(\boldsymbol{r}) d\boldsymbol{r} \; / \; \int_{Node} d\boldsymbol{r} \,,\end{aligned} \tag{2}$$

where, $\phi_g^{\dagger}(\boldsymbol{r})$ and $\phi_g^{*}(\boldsymbol{r})$ is an adjoint flux and a perturbed forward flux in a node, respectively. The approximations in Eqs. (1) and (2) are appropriate in the finite difference method, but not always valid in the nodal methods that use coarse meshes. This problem doesn't appear in a one-dimensional problem, because an intra-nodal flux distribution can be given explicitly by analytical or polynomial functions.

To overcome the second problem, a kind of flux reconstruction technique[4] was introduced. In the case of the 4th order polynomial nodal expansion method (NEM)[1], a three-dimensional flux distribution in each node is approximated as

$$\phi_g(\boldsymbol{r}) \approx \phi_{g,0} + \sum_{u=x,y,z} \left\{ C_{1u,g} u + C_{2u,g} u^2 + C_{2u,g} u^3 + C_{2u,g} u^4 \right\}, \tag{3}$$

where $\phi_{g,0}$ is the node averaged flux, and $C_{1u,g} \sim C_{4u,g}$ are expansion coefficients of a transverse integrated flux in each direction. This function is consistent with all nodal parameters like volume or surface averaged fluxes and surface averaged currents obtained in a core calculation by NEM. In addition, no additional computation cost is necessary, because all of the coefficients are obtained in the consequence of core calculation. It was confirmed that Eq. (3) always give sufficient accuracy for the integrated parameters in Eqs. (1) and (2), in spite of the simple reconstruction as compared with that used in a pin power reconstruction[4].

In order to complete the perturbation calculation by the nodal method, Kobayashi's theory and the presented method were incorporated in our nodal diffusion code MOSRA-Light[5], which was based on the 4th order NEM. The method was verified in the two-dimensional and two-group DVP-BWR benchmark problem[2] shown in Fig.2.4.1. Six kinds of discontinuity factors are considered in the problem for the surfaces with wide or narrow gap of fuel assemblies with three types. As an extension of the original benchmark problem, a perturbation by withdrawal of all control rods was considered as shown in Fig.2.4.1. The perturbed reactivities were calculated on the basis of the exact perturbation theory by using the unperturbed adjoint flux and the perturbed forward one obtained with MOSRA-Light. In the core calculation, one node per fuel assembly was taken as a coarse mesh. The following three cases of calculations were performed.

Case 1: without Kobayashi's theory in the adjoint core calculation and no flux reconstruction in perturbation calculation (by Eqs. (1) and (2)),

Case 2: with Kobayashi's theory and no flux reconstruction,

Case 3: with Kobayashi's theory and the flux reconstruction by Eq. (3).

The perturbed reactivities obtained for the above three conditions were compared with the reference solution that was directly obtained from two forward solutions of eigenvalues for the unperturbed and perturbed cores. As shown in Table 2.4.1, a very accurate reactivity was

obtained by the condition of the Case 3. Thus, Kobayashi's theory was validated in a multi-dimensional and multi-group problem. The contribution of the flux reconstruction by Eq. (3) was relatively small in this problem. However, it should not be neglected when smaller perturbation reactivity (~20 pcm) is evaluated.

The presented method is easily applicable to the calculations of one-point kinetics parameters or space dependent kinetics analyses by the improved quasi-static method, which is based on the exact perturbation theory.

References

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3) Kobayashi K. : J. Nucl. Sci. Technol., 35, 20 (1998).

4) Koebke K. and Hetzelt L. : Nucl. Sci. Eng. 91, 123 (1985).

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Table 2.4.1 Comparison of perturbed reactivities ($\Delta k / kk'$) in DVP-BWR benchmark

Case 1	Case 2	Case 3	Reference
2.58E-2	2.31E-2	2.30E-2	2.30E-2

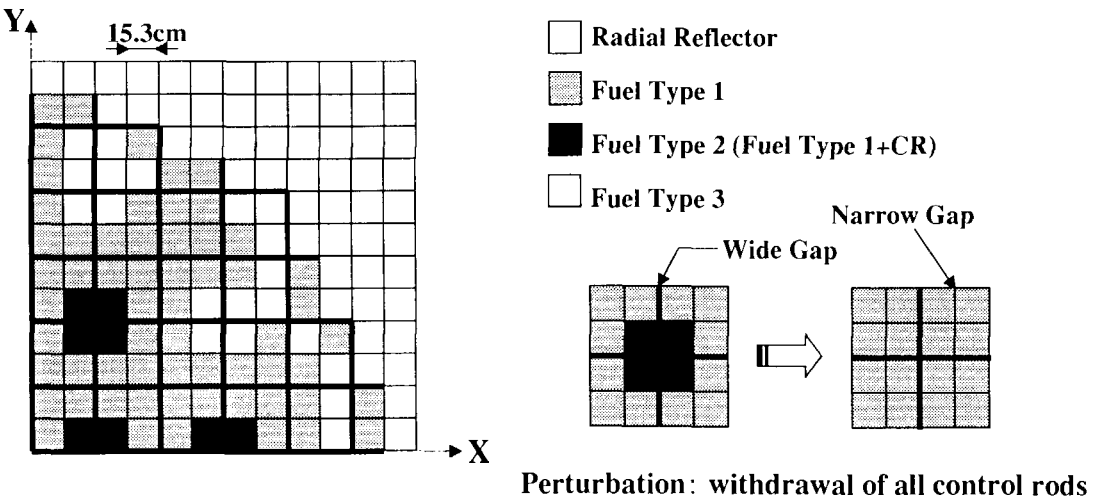

Fig. 2.4.1 Extended DVP-BWR benchmark problem for perturbation investigation