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ON THE FERMI-SURFACE DYNAMICS OF ROTATING NUCLEI

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Generalized virial theorems are written for rotating nuclear systems with intrinsic currents. A set of dynamical equations of motion for angular momentum, inertia and pressure tensors is obtained to study the collective vortical modes (e.g., modes including the Kelvin circulation) in nuclear excitation and reaction processes.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR and Centre d'Études Nucléaires de Bordeaux-Gradignan, CNRS-IN2P3, Gradignan, France.

О динамике ферми-поверхности вращающихся атомных ядер

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Сформулированы обобщенные вириальные теоремы для вращающихся ядерных систем при наличии внутренних вихревых потоков. Получена система динамических уравнений движения для тензоров углового момента, момента инерции и давления с целью изучения коллективных вихревых мод возбуждения (в том числе с ненулевой циркуляцией Кельвина) в ядерных реакциях и процессах возбуждения атомных ядер.

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Recent studies of collective nuclear motion (see, e.g., [1], [2]) show the limitations of theoretical models dealing only with the coordinates describing the distribution of nuclear matter in space (such as electric multipole moments). The motion of nuclear surface is of course accompanied by the currents of matter, i.e., by a rearrangement of the particles momenta. However, the role of the quantities determining the distribution of particles in momentum space depends on the dynamical conditions. It means that some of them must be acknowledged as generalized coordinates kinematically independent of the coordinates of a geometrical nature. One possible way to incorporate into the theory such quantities was proposed in [3] in which the method of "virial theorems" initiated by Chandrasekhar [4] was suggested for the study of nuclear multipole giant resonances and then generalized to the motion of large amplitude for the study of nuclear fusion reaction [5].

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In the above-quoted references, the rotational motion was not considered. For a rotating system of nucleons the single-particle Wigner distribution function looses its spherical symmetry in phase space due to the implementation of collective currents. Not only the shape of a composite nuclear system, and its density distribution, but also the pressure tensor become spherically asymmetrical. The importance of an explicit dynamical treatment of the latter anisotropy has been pointed out in papers dealing with the fission [6] and fusion reactions [2].

Nuclear collective vortical motion may differ drastically from a traditional case of a uniform rigid rotation with a constant angular velocity $\vec{\Omega}$. Namely a local vorticity within the rotating frame $\vec{\zeta}(\vec{r},t) \equiv rot\vec{v}(\vec{r},t) \neq 2\vec{\Omega}$ may appear. This naturally leads to an intrinsic vorticity concept. Its usefulness in nuclear physics has been pointed out by various authors [7] and some connections of these modes with current research in mesoscopic systems [8] and nonlinear excitations [9] have been drawn.

In this paper we suggest the following way to analyze possible dynamical effects associated with the intrinsic vorticity. Within the mean-field approximation, we analyze the evolution of one-body Wigner phase-space distribution function of the full many-body wave function. We will follow the well-developed scheme using as the starting point the Vlasov equation for the Wigner phase-space distribution function [10]

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \cdot \frac{\partial f}{\partial \vec{r}} - \frac{\partial V}{\partial \vec{r}} \cdot \frac{\partial f}{\partial \vec{p}} = I_{\rm rel}, \tag{1}$$

with a "relaxation term" $I_{\rm rel}$ added to the kinetic equation to describe dissipation effects. The quantity $V(\vec{r},t)$ is the self–consistent single-particle potential which is assumed here to be local, m is the mass of nucleon.

Out of the Wigner distribution function, virials at different orders can help one to extract useful physical information from the total phase space dynamics. Integrating the initial kinetic equation (1) over the momentum space with different polynomial weighting functions of the $\vec{p}$ variable one comes, as is well known [3], [4], [11], to an infinite chain of equations for local collective observables including the density, collective velocity, pressure and an infinite set of tensorial functions of the time and space coordinates, which are defined as moments of the distribution function in the momentum space:

- the particle $n(\vec{r},t) \equiv \int d\vec{p}\, f(\vec{r},\vec{p},t)$, and the mass $\rho(\vec{r},t) = m\, n(\vec{r},t)$ densities,
- the collective current and velocity of nuclear matter

$$\rho(\vec{r},t)\vec{u}(\vec{r},t) = \int d\vec{p}\,\vec{p} f(\vec{r},\vec{p},t),$$

- the pressure tensor and the energy and momentum transfer tensors of different orders

$$\begin{aligned}
\mathbb{P}_{ij}(\vec{r},t) &= \frac{1}{m}\int d\vec{p}\, q_i q_j f(\vec{r},\vec{p},t), \qquad q_i \equiv p_i - m u_i, \\
\mathbb{P}_{\underbrace{ij..k}_{n}}(\vec{r},t) &= \frac{1}{m^{n-1}}\int d\vec{p}\, \underbrace{q_i q_j .. q_k}_{n} f(\vec{r},\vec{p},t),
\end{aligned}$$

- and the integrals related to relaxation terms

$$\int d\vec{p}\, I_{\rm rel} = 0, \qquad \int d\vec{p}\,\vec{p} I_{\rm rel} = 0,$$
$$\mathbb{R}_{ij} \equiv \frac{1}{m}\int d\vec{p}\, q_i q_j I_{\rm rel}, \qquad \ldots$$

Truncating this chain at order two in $\vec{q}$ one arrives at the "fluid dynamical" level of description of nuclear processes.

$$\frac{\partial \rho}{\partial t} + \sum_k \frac{\partial}{\partial x_k}(u_k \rho) = 0, \tag{2}$$

$$\begin{aligned}\rho\frac{Du_i}{Dt} &+ \sum_k \frac{\partial \mathbb{P}_{ik}}{\partial x_k} + \frac{\rho}{m}\frac{\partial V}{\partial x_i} + \rho(\Omega_i \sum_k \Omega_k x_k - \Omega^2 x_i) \\ &+ \rho \sum_{s,j} \varepsilon_{isj}(2\Omega_s u_j + \frac{d\Omega_s}{dt} x_j) = 0,\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{D\mathbb{P}_{ij}}{Dt} &+ \sum_k \left(\mathbb{P}_{ik}\frac{\partial u_j}{\partial x_k} + \mathbb{P}_{jk}\frac{\partial u_i}{\partial x_k} + \mathbb{P}_{ij}\frac{\partial u_k}{\partial x_k}\right) \\ &+ 2\sum_{s,m} \Omega_m(\varepsilon_{jms}\mathbb{P}_{is} + \varepsilon_{ims}\mathbb{P}_{js}) \\ &+ \sum_k \frac{\partial}{\partial x_k}\mathbb{P}_{ijk} = \left(\frac{\partial \mathbb{P}_{ij}}{\partial t}\right)_{\rm rel},\end{aligned} \tag{4}$$
$$\left(\frac{\partial \mathbb{P}_{ij}}{\partial t}\right)_{\rm rel} \equiv \frac{1}{m}\int d\vec{p}\, q_i q_j I_{\rm rel},$$

where the usual notation $\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \sum_k u_k \frac{\partial}{\partial x_k}$ is introduced for the operator giving the material derivative, or the rate of change at a point moving locally with the fluid. The hydrodynamical set of Eqs. (2-5) describes an evolution of a rotating nuclear system. We consider two frames of reference with a common origin: an inertial frame, (X_1, X_2, X_3), and a moving frame, (x_1, x_2, x_3). Let $x_i = \sum_{j=1}^{3} \mathbb{T}_{ij} X_j$ be the linear transformation that relates the coordinates, $\vec{X}$ and $\vec{x}$, of a point in two frames. The orientation of the moving frame, with respect to the inertial frame, will be assumed to be time dependent. Since $\mathbb{T}_{ij}(t)$ must represent an orthogonal transformation, the vector

$$\Omega_i = \frac{1}{2}\sum_{j,k,m} \varepsilon_{ijk}\left(\frac{d\mathbb{T}}{dt}\right)_{jm} \mathbb{T}^{+}_{mk},$$

represents a general time-dependent rotation of the $\vec{x}$ frame with respect to the inertial frame.

Let us define integral collective "observables" (the integrals over the whole phase space of one nucleon containing the distribution function appropriately weighted), namely an inertia tensor $\mathbb{J}_{ij}(t)$, the dynamical part of the angular momentum $L_i(t)$, the integral pressure tensor $\Pi_{ij}(t)$ defined as

$$\begin{aligned}
\mathbb{J}_{ij} &\equiv \int d\vec{x}\, x_i x_j \rho, \qquad \Pi_{ij} \equiv \int d\vec{x}\, \mathbb{P}_{ij}, \\
L_k &\equiv \sum_{i,j} \varepsilon_{kij} \int d\vec{x}\, \rho x_i u_j.
\end{aligned}$$

The dynamics in terms of the latter "observables" is expressed by a set of virial equations in the rotating frame

$$\begin{aligned}
\frac{d^2}{dt^2}\mathbb{J}_{ij} &+ \sum_k \Omega_k(\Omega_i \mathbb{J}_{jk} + \Omega_j \mathbb{J}_{ik}) - 2\Omega^2 \mathbb{J}_{ij} \\
&+ 2\sum_{s,k} \Omega_s \int d\vec{r}\, \rho u_k(\varepsilon_{isk} x_j + \varepsilon_{jsk} x_i) \\
&+ 2\mathbb{W}_{ij} - 2\mathbb{K}_{ij} - 2\Pi_{ij} \\
&+ \sum_{s,k} \frac{d\Omega_s}{dt}(\varepsilon_{isk}\mathbb{J}_{kj} + \varepsilon_{jsk}\mathbb{J}_{ki}) = 0, \\
\frac{dL_k}{dt} &+ \sum_{i,j,m} \varepsilon_{kji}\Omega_i\Omega_m \mathbb{J}_{jm} - 2\sum_s \Omega_s \int d\vec{r}\, \rho u_k x_s \\
&- \sum_s \frac{d\Omega_s}{dt}\mathbb{J}_{ks} + \frac{d}{dt}(\Omega_k \sum_j \mathbb{J}_{jj}) = 0, \\
\frac{d}{dt}\Pi_{ij} &+ \mathbb{F}_{ij} + 2\sum_{s,k} \Omega_s(\varepsilon_{isk}\Pi_{kj} + \varepsilon_{jsk}\Pi_{ki}) = \mathbb{R}_{ij}, \\
\mathbb{F}_{ij} &\equiv \sum_k \int d\vec{r} \left(\mathbb{P}_{ik}\frac{\partial u_j}{\partial x_k} + \mathbb{P}_{jk}\frac{\partial u_i}{\partial x_k}\right),
\end{aligned}$$

where the tensors of collective kinetic and potential energies, and the relaxation tensor are

$$\begin{aligned}
\mathbb{K}_{ij} &= \int d\vec{x}\, u_i u_j \rho, \qquad \mathbb{W}_{ij} = \int d\vec{x}\, x_j \frac{\partial V}{\partial x_i} n, \\
\mathbb{R}_{ij} &\equiv \int d\vec{x} \left(\frac{\partial \mathbb{P}_{ij}}{\partial t}\right)_{\mathrm{rel}} x_i x_j.
\end{aligned}$$

The above equations constitute a starting point for the study of the stationarity conditions and dynamical properties of rotating nuclear systems. They provide a formal framework within which the coupling of the deformations in the $\vec{r}$ space and in the $\vec{p}$ space can be explicitly worked out. The development of a collective model on the basis of such equations is currently in progress.

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