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Interacting boson model for exotic nuclei at low isospin

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Abstract :

With Wigner's $SU(4)$ supermultiplet symmetry [and its generalization to pseudo-$SU(4)$] as a starting point, a boson-model space is constructed that includes $T=0$ as well as $T=1$ bosons (IBM-4). The boson Hamiltonian is derived microscopically from a realistic shell-model interaction through a mapping that relies on the existence of approximate shell-model symmetries. Applications are presented for odd-odd $N=Z$ nuclei from $^{58}_{29}Cu_{29}$ to $^{70}_{35}Br_{35}$.

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The Interacting Boson Model (IBM) of Arima and Iachello [1] has achieved an impressive success in the phenomenological description of collective motion in the medium-mass and heavy even-even nuclei at low excitation energies. Originally, this approach makes use of a U(6) spectrum generating algebra to quantify the five-dimensional motion of a liquid drop with vibrational and rotational excitations. The explicit realization involves two kinds of bosons $b^{+}_{j,m}$ carrying angular momentum $j = 0,2$, to which later a microscopic interpretation was given in terms of correlated pairs of nucleons [2]. Away from stability, an extension of the model, which would involve a limited number of parameters that can be derived from nucleon degrees of freedom and which would be rich enough to be realistic, is of course highly desirable. The purpose of this Letter is precisely to present a first step in this direction concerning exotic nuclei with roughly equal numbers of neutrons and protons. These systems are currently the object of intense experimental and theoretical study because of the possibility that they might exhibit new collective behaviors induced by the neutron-proton exchange symmetry. In particular, they are the only nuclei that might display some evidence for an isoscalar superfluidity characterized by proton-neutron Cooper pairs with total spin $S = 1$. From a theoretical point of view, many frameworks have in fact already been developed to investigate the structure of heavy $N \approx Z$ nuclei. For example, variational approximations (like BCS or HFB), have been extended to include both $T = 1$ and $T = 0$ pairing correlations [3]. However, schematic models seem to show that the validity of this mean-field approach along the $N = Z$ line is limited by the lack of particle-number projection [4]. Shell-model calculations have also been performed either by a direct diagonalization [5] of the effective hamiltonian or by using Monte-Carlo techniques at non-zero temperature [6]. In the first case, however, it is not possible to deal with large numbers of nucleons in a big space whereas non-negligible uncertainties are generated in the second method by the use of an extrapolation procedure to avoid sign problems. Nevertheless, an alternative approach, called QMCD and proposed by Otsuka & al [7], seems to be in a position to obtain detailed spectroscopy of all the $N=Z$ nuclei of the 28-50 shell, even if applications to odd-odd systems have not been done at present. The work reported here represents a first step towards the use of the isospin invariant interacting boson model IBM-4 in a detailed spectroscopic analysis of exotic $N \approx Z$ nuclei in the $pf_{5/2}g_{9/2}$ space with the aim to develop an alternative to shell model or mean-field approximations that is simpler and computationally less intensive. Simultaneously, the microscopic foundation of a boson calculation in this mass region is investigated by the derivation of the hamiltonian from a realistic shell model interaction through a mapping that relies on the existence of appproximate fermionic symmetries.

The IBM-4 [8], the most elaborate version of the interacting boson model, describes each nucleus as a symmetric representation of the unitary spectrum generating algebra $U(36)$. The bosons are labeled by an orbital angular momentum $l=0,2$, and by an intrinsic spin s and an isospin t for which only the combinations $(s,t)=(0,1)$ and $(s,t)=(1,0)$ are retained. The rationale behind this choice is that for two nucleons in the same harmonic oscillator shell interacting via a delta force and without spin-orbit coupling, the low-energy eigenstates will be characterized by (L,S,T) quantum numbers identical to those of the IBM-4 bosons. Another justification is provided by the existence of a classification of IBM-4 states which contains an $SU(4)$ algebra that can be connected to Wigner's supermultiplet algebra [9] that is known to have physical significance in light nuclei. A crucial aspect of the IBM-4 is that it does not require an exact or even an approximate validity of the fermionic (L,S) coupling scheme : if the shell-model effective hamiltonian favours two-nucleon correlations in (J,T) channels which are contained in IBM-4, one can use this boson approximation even if an important spin-orbit coupling breaks the (L,S) labeling. In this case, the l and s quantum numbers associated with a single boson should be thought of as effective labels in the shell model which acquire an exact significance only in the boson space. This, for example, occurs just beyond ${}^{56}_{28}Ni_{28}$ where the active orbits are $\left(p_{3/2}, f_{5/2}, p_{1/2}, g_{9/2}\right)$. Using a realistic G-matrix interaction for this valence space and after a phenomenological adjustment of the monopole part [10], the two-nucleon states $|\alpha, J, M_J, T, M_T\rangle_F$ of table 1 can be found. The six first levels have a total angular momentum J and an isospin T that correspond to those of the IBM-4 and so a connection between these two-fermion eigenstates and the bosons can be established :

$$|\alpha, J, M_J, T, M_T\rangle_F \leftrightarrow \left| \, l_\alpha s_\alpha, j = J, m_j = M_J, t = T, m_t = M_T \right\rangle_B , \tag{1}$$

where l_α and s_α are arbitrarily chosen among the values allowed in IBM-4 since the (L,S) classification is badly broken by the strong spin-orbit term in the nuclear mean field.

With the mapping (1), the one-body boson hamiltonian can be determined with the usual OAI procedure [11] which consists in the calculation of the matrix elements of the shell-model hamiltonian between orthogonal fermionic images of the boson states. In the present case, only diagonal values, identical to the energies of the collective pairs $|\alpha, J, M_J, T, M_T\rangle_F$, are obtained. The determination of the two-body part of the boson hamiltonian is considerably more involved. It is nevertheless possible to adopt a simple solution using two-pair states with good total $JM_J \; TM_T$ quantum numbers. Such vectors $\left\|\alpha_1 J_1 T_1 \, ; \, \alpha_2 J_2 T_2 \, ; \, JM_J \; TM_T\right\rangle\!\rangle_F$ are in fact non-orthogonal in the labels $\alpha_1 J_1 T_1 \, ; \, \alpha_2 J_2 T_2$ because of Pauli effects induced by the internal structure of the pairs. In contrast, the boson analogue vectors $\left| \, l_{\alpha_1} s_{\alpha_1} j_1 \, t_1 \, ; \, l_{\alpha_1} s_{\alpha_1} j_2 \, t_2 \, ; \, JM_J \; TM_T \right\rangle_B$ form an orthonormal basis and so a correspondence can only be established after having applied an orthonormalization operator O in the fermionic space :

$$O\left\|\alpha_1 J_1 T_1 \, ; \, \alpha_2 J_2 T_2 \, ; \, JM_J \; TM_T\right\rangle\!\rangle_F \leftrightarrow \left| \, l_{\alpha_1} s_{\alpha_1} j_1 \, t_1 \, ; \, l_{\alpha_1} s_{\alpha_1} j_2 \, t_2 \, ; \, JM_J \; TM_T \right\rangle_B \tag{2}$$

As pointed out by Ginocchio and Johnson [12], the choice of this orthogonalization procedure is crucial since it decides whether two-body interactions between the bosons provide an adequate mapping or whether, instead, higher-order interactions are needed. In the case of the $j-j$ basis used here, there are so many possible ways to order the states that a Gram-Schmidt method is not unambiguous and only the use of the overlap matrix eigensystem (a so-called 'democratic' mapping [13]) remains. Unfortunately, direct application of this algorithm gives an IBM-4 hamiltonian whose spectroscopy is very poor compared to the fermionic results : for example, the ground state of ${}^{62}_{31}Ga_{31}$ is isoscalar whereas it is isovector in the experimental spectra and in the $pf_{5/2}g_{9/2}$ shell-model calculation [14].

To obtain a more valid mapping procedure, an elegant solution can be proposed using the approximate pseudo-$SU(4)$ shell-model symmetry which has been found to be relevant at the beginning of the 28-50 shell [15]. In such an approach, shell-model states in the $pf_{5/2}g_{9/2}$ space only involve correlations between the three orbits $(p_{3/2}, f_{5/2}, p_{1/2})$ which are treated as a pseudo-sd shell $(\bar{d}_{3/2}, \bar{d}_{5/2}, \bar{s}_{1/2})$. In addition, the pseudo-$SU(4)$ classification implies a labeling by the total pseudo-orbital angular momentum $\bar{L}$, the total pseudo-spin $\bar{S}$ and by an irreducible representation $\overline{(\lambda\mu\nu)}$ of the pseudo-spin isospin algebra $SU(4)$ in direct analogy with Wigner's supermultiplet model [9]. To illustrate the validity of this scheme in nuclei of mass $A \approx 60$, some results of Ref [15] have been reported in table 1 concerning the pseudo-$SU(4)$ decomposition of the realistic wavefunctions $|\alpha, J, M_J, T, M_T\rangle_F$ of two nucleons in the $pf_{5/2}g_{9/2}$ space. It is seen that all lowest states have small $g_{9/2}$ admixtures and that they carry a large component in a subspace $\overline{(\lambda_\alpha\mu_\alpha\nu_\alpha)}\,\bar{L}_\alpha\,\bar{S}_\alpha$ with $\overline{(\lambda_\alpha\mu_\alpha\nu_\alpha)} \equiv \overline{(010)}$. Therefore, each of them can be associated to a pseudo-$SU(4)$ vector $|\overline{\alpha, JM_J, TM_T}\rangle = |\overline{(010)}\,\bar{L}_\alpha\bar{S}_\alpha, JM_J, TM_T\rangle_F$ in such a way that its structure is as close as possible to the realistic state. The main advantage of these new collective pairs is their aptitude to be coupled to good total $\overline{(\lambda\mu\nu)}\,\bar{L}\,\bar{S}\,JM_JTM_T$

quantum numbers by diagonalizing a linear combination of Casimir operators $C_2[SU_{\bar{S}T}(4)], \bar{L}^2, \bar{S}^2, J^2$ in each uncoupled subspace spanned by all the vectors $|\overline{(010)}\,\bar{L}_{\alpha_1}\bar{S}_{\alpha_1}, J_1M_{J_1}, T_1M_{T_1}\rangle_F \otimes |\overline{(010)}\,\bar{L}_{\alpha_2}\bar{S}_{\alpha_2}, J_2M_{J_2}, T_2M_{T_2}\rangle_F$ with fixed $\bar{L}_{\alpha_1}\bar{L}_{\alpha_2}$ values. The basis thus obtained can be denoted as :

$$\left\|\bar{L}_{\alpha_1}\bar{L}_{\alpha_2}\,;\,\left(\overline{\lambda\mu\nu}\right)\bar{L}\,\bar{S}\,;JM_J\;TM_T\right\rangle\!\!\Big\rangle_F \tag{3}$$

and it has the nice property to be orthogonal in all labels except $\bar{L}_{\alpha_1}\bar{L}_{\alpha_2}$. In the present application, it is important to specify that the construction of the basis (3) implies the use of a set of pseudo-$SU(4)$ pairs that is larger than those associated with the six first states that are mapped to IBM-4. Since these realistic levels contains correlations in the $\bar{L}_\alpha = 0,2,4$ channels, we can only define the two-pair pseudo-$SU(4)$ basis (3) if we consider all the allowed vectors $|\overline{(010)}\,\bar{L}_\alpha\bar{S}_\alpha, JM_J, TM_T\rangle_F$ for which, in particular, $(\bar{S}_\alpha, T)$ can have either the values $(0,1)$ or $(1,0)$. In the boson space, a basis similar to (3) can be constructed through known coupling coefficients, as those for $SU_{ST}(4) \supset SU_S(2)\otimes SU_T(2)$ [16], immediately leading to an orthonormal set :

$$|\,l_{\alpha_1}l_{\alpha_2};\,(\lambda\mu\nu)\,L\,S\,;JM_J\;TM_T\rangle_B = \sum_{\substack{s_{\alpha_1}j_1t_1\\ s_{\alpha_2}j_2t_2}} \hat{L}\,\hat{S}\,\hat{j}_1\,\hat{j}_2 \left\langle\begin{matrix}(010) & (010)\\ s_{\alpha_1}t_1 & s_{\alpha_2}t_2\end{matrix}\right|\left.\begin{matrix}(\lambda\mu\nu)\\ ST\end{matrix}\right\rangle \begin{Bmatrix} l_{\alpha_1} & l_{\alpha_2} & L\\ s_{\alpha_1} & s_{\alpha_2} & S\\ j_1 & j_2 & J\end{Bmatrix}$$

$$|\,l_{\alpha_1}s_{\alpha_1}j_1\,t_1\,;\,l_{\alpha_2}s_{\alpha_2}j_2\,t_2\,;\,JM_J\;TM_T\rangle_B \tag{4}$$

where $\hat{z} = \sqrt{2z+1}$. As a consequence, if we were dealing with bosons associated with the perfect pseudo-$SU(4)$ pairs, an efficient mapping algorithm would be established by the correspondence between the boson basis (4) and its fermionic image (3) orthogonalized by an operator O only in $\bar{L}_{\alpha_1}\bar{L}_{\alpha_2}$:

$$O\left\|\overline{L}_{\alpha_1}\overline{L}_{\alpha_2}\,;\,\left(\overline{\lambda\mu\nu}\right)\overline{L}\,\overline{S}\,;JM_J\;TM_T\right\rangle\!\!\Big\rangle_F \leftrightarrow \left|\,l_{\alpha_1}l_{\alpha_2};\,(\lambda\mu\nu)\,L\,S\,;JM_J\;TM_T\right\rangle_B \tag{5}$$

with $l_{\alpha_1}=\overline{L}_{\alpha_1}, l_{\alpha_2}=\overline{L}_{\alpha_2}, (\lambda\mu\nu)=\left(\overline{\lambda\mu\nu}\right), L=\overline{L}$ and $S=\overline{S}$. The mapping (5) exploits the existence of shell-model symmetries to reduce the non-orthogonality problem of two-pair states. However, the procedure should be modified to cope with realistic pairs where a pseudo-$SU(4)$ classification is only approximate and where the $g_{9/2}$ orbit plays a role. This can be achieved by rewriting the mapping (5) in jj coupling. Technically, it involves inverting the relation (4) between LS and jj coupled bosons, applying the mapping (5), and subsequently relating two-pair states in $\overline{LS}$ coupling with those in jj coupling. Note that these jj coupled vectors are built from two pseudo-$SU(4)$ pairs $\overline{\alpha JT}$ and orthonormalized in a democratic way from the overlap matrix leading to an orthonormalization operator O_D. The resulting mapping establishes a correspondence between jj coupled two-pair and two-boson states and is exactly equivalent to (5) in case of pseudo-$SU(4)$ symmetry. The generalization to approximate pseudo-$SU(4)$ symmetry in the $pf_{5/2}g_{9/2}$ space can now be readily achieved by assuming that the matrix transformation between the jj boson basis and its fermionic image remains valid when the IBM-4 bosons are associated with the realistic pairs . This hypothesis then leads to the following mapping between two-boson and four-fermions states :

$$\left|\,l_{\alpha_1}s_{\alpha_1}j_1\,t_1\,;\,l_{\alpha_2}s_{\alpha_2}j_2\,t_2\,;\,JM_J\;TM_T\right\rangle_B$$

$$\updownarrow$$

$$\sum_{\substack{\left(\overline{\lambda\mu\nu}\right)\overline{L}\,\overline{S}\\ \overline{\alpha_3J_3T_3}\;\overline{\alpha_4J_4T_4}}} \hat{\overline{L}}\,\hat{\overline{S}}\,\hat{j}_1\,\hat{j}_2 \left\langle\begin{matrix}\left(\overline{010}\right)\\ \overline{S}_{\alpha_1}t_1\end{matrix}\quad\begin{matrix}\left(\overline{010}\right)\\ \overline{S}_{\alpha_2}t_2\end{matrix}\right|\left.\begin{matrix}\left(\overline{\lambda\mu\nu}\right)\\ \overline{S}T\end{matrix}\right\rangle \begin{Bmatrix}\overline{L}_{\alpha_1} & \overline{L}_{\alpha_2} & \overline{L}\\ \overline{S}_{\alpha_1} & \overline{S}_{\alpha_2} & \overline{S}\\ j_1 & j_2 & J\end{Bmatrix}$$

$$_F\Big\langle\!\!\left\langle\overline{\alpha_3J_3T_3}\,;\,\overline{\alpha_4J_4T_4}\,;\,JM_J\;TM_T\right\|O_D^+\,O\left\|\overline{L}_{\alpha_1}\overline{L}_{\alpha_2}\,;\,\left(\overline{\lambda\mu\nu}\right)\overline{L}\,\overline{S}\,;JM_J\;TM_T\right\rangle\!\!\Big\rangle_F$$

$$O_D\left\|\alpha_3J_3T_3\,;\,\alpha_4J_4T_4\;;\,JM_J\;TM_T\right\rangle\!\!\Big\rangle_F \tag{6}$$

It is important to realize that the fermionic labels $\overline{L}_\alpha \, \overline{S}_\alpha$ are not now necessarily identical to the intrinsic quantum numbers $l_\alpha s_\alpha$ of the single bosons. For example, the IBM-4 boson $l_\alpha = 2, s_\alpha = 1, j = 3, t = 0$ is associated with the two-nucleon level $|\alpha = 1, J = 3, T = 0\rangle_F$ which is in fact related to a pseudo-$SU(4)$ pair with $\overline{L}_\alpha = 4$ and $\overline{S}_\alpha = 1$. To be complete, we have again to specify the orthonormalization process O of the pseudo-$SU(4)$ basis (3) in the labels $\overline{L}_{\alpha_1} \overline{L}_{\alpha_2}$. In the work reported here, a Gram-Schmidt procedure has been adopted with an order dictated by seniority as in the OAI mapping [11] where the states are orthogonalized with respect to those with a higher number of pairs characterized by $\overline{L}_\alpha = 0$. Moreover, if for a given seniority several states exist, the remaining non-orthogonality is democratically removed by a diagonalization of the overlap matrix. Once the correspondence (6) is established, the two-boson matrix elements of the hamiltonian can be determined by the usual procedure of equating them to their fermionic equivalent. After subtraction of the one-body contribution, the two-body boson interaction is finally obtained.

If the orthogonalization ordering has been properly chosen, no higher-order boson interactions are needed and one can then make predictions for systems with boson number $N > 2$. Satisfactory results are obtained in this way but the density of levels generally is somewhat low as compared with that in the shell model. This is a truncation effect and can, in principle, be remedied by a renormalization of the hamiltonian due to non-bosonized pair degrees of freedom. Instead of a fully microscopic renormalization, which in the case of IBM-4 is exceedingly difficult, a simple scaling of the entire hamiltonian is adopted here which reproduces the $0^+ - 2^+$ splitting (as obtained in the shell model) in the two-boson nucleus ^{60}Zn. To carry out these calculations, a computer code has been written [17] that diagonalizes a general IBM-4 hamiltonian with one-boson energies and two-boson interactions.

The first test of the IBM-4 Hamiltonian thus derived is the three-boson nucleus ${}^{62}_{31}Ga_{31}$. Figure 1 shows the known experimental levels [14] together with the shell-model [14] and the IBM-4 results. Both shell model and IBM-4 predict a $0^+(T=1)$ ground state and a $1^+(T=0)$ first-excited state. Note that this represents an inversion with respect to the order in ${}^{58}_{29}Cu_{29}$ (Table 1) which agrees with the data. Given that no free parameter is introduced in the IBM-4 calculation, the agreement for the isoscalar levels between shell-model and IBM-4 can be called remarkable and a near one-to-one correspondence between levels can be established, the exceptions being higher-spin (5^+ to 7^+) shell-model states which are absent from the IBM-4 because it does not include high-spin $T=0$ bosons. Note also a low-lying 0^+ state in the IBM-4 calculation which, since the shell-model counterpart is much higher in energy, might have an important spurious component. Experimentally, excited states in ${}^{62}Ga$ were located for the first time very recently [14] in an experiment which populated the nucleus through a fusion-evaporation reaction. However, this type of study of the $N=Z$ nuclei in this region is difficult, requiring high experimental sensitivity, and yields information only on the yrast structure. The vast majority of $T=0$ states predicted by the shell-model or IBM-4 calculations thus remain to be verified.

A similar situation applies to ${}^{66}As$ although in this case, the experimental population was via isomeric states [18]. Only a few states have been identified and without unique spin assigments. Excited states have also been populated in ${}^{70}Br$ [19], which are thought to be built on the known 2.2 sec. isomer, rather than the ground state. However, the excitation energy of the isomeric state is unknown. Thus, a meaningful comparison with the IBM-4 results of Figure 2 is not currently possible for these two nuclei.

Turning now to the $T=1$ states, in ${}^{62}_{31}Ga_{31}$, one notes more levels in experiment and the shell-model as compared to the IBM-4. This deviation grows in the heavier nuclei ${}^{66}As$ and ${}^{70}Br$ where the $T=1$ energies can be taken from the experimental level schemes of the isobaric analogues of ^{66}Ge and ^{70}Se. In particular, one notes the absence from IBM-4 of a second 2^+ level at the observed experimental energy ($E_x \approx 2$ MeV). In a corresponding IBM-3 analysis (which is feasible for the $T=1$ subspace of $N=Z$ nuclei) this state is correctly reproduced but only after allowing a microscopically dictated boson-number dependence of the Hamiltonian. This deficiency of the current calculation for isovector states can thus presumably be traced back to the constancy of the boson hamiltonian for all nuclei shown and indicates the need to derive a boson-number dependence in IBM-4 also.

The present results illustrate the predictive power of the IBM-4. In particular, the $0^+(T=1)$ - $1^+(T=0)$ splitting is correctly reproduced in the known cases, $^{58}_{29}Cu_{29}$ and $^{62}_{31}Ga_{31}$, and is predicted to be about 1 MeV in ^{66}As and 1.25 MeV in ^{70}Br where it is not well established experimentally and where a shell-model calculation is currently not possible. Another result from the present formalism is that an idea of the pair structure of nuclear states can be readily obtained by computing boson-number expectation values in the IBM-4 eigenstates. This is illustrated in Figure 3 where the proportion of isoscalar bosons in the total number of bosons is plotted for various states in the three odd-odd $N=Z$ nuclei discussed above. The most noteworthy feature is the decrease of this proportion in the $T=0$ states as the number of bosons increases. Qualitatively, this decrease can be understood from the corresponding fraction in a simple IBM-4 model with only s bosons where it is given by $(5N+3)/8N$ in the $SU(4)$ limit [20].

A much higher density of $T=0$ states is predicted than observed experimentally in the cases where data exists. As pointed out earlier, this does not represent a deficiency in the calculation but rather a limitation of the experimental techniques currently used to access nuclei on the $N=Z$ line. Note that this limitation is two-fold. It stems not only from the high sensitivity required but also from the nature of the reactions used to populate the nuclei of interest, which yield information only on the yrast structure. To really probe the validity of the predicted $T=0$ structure of these odd-odd nuclei will require the use of a variety of less selective techniques which will only become feasible with the advent of radioactive beams which permit reactions to be studied in inverse kinematics. The results presented here thus pose a challenge for the coming generation of new radioactive beam facilities.

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Table 1

Shell-model analysis of the proton-neutron system in the $pf_{5/2}g_{9/2}$ shell

Energy level	Labels $\|\alpha, J, T\rangle_F$	Pseudo-labels $\overline{(\lambda_\alpha \mu_\alpha \nu_\alpha)}\,\bar{L}_\alpha\,\bar{S}_\alpha$ associated	Occupation of the $g_{9/2}$ orbit (%)
0	$\alpha = 1, J = 1, T = 0$	$(\overline{010})\bar{L} = 2, \bar{S} = 1$	0.846
0.52971	$\alpha = 1, J = 0, T = 1$	$(\overline{010})\bar{L} = 0, \bar{S} = 1$	8.55
0.60002	$\alpha = 1, J = 3, T = 0$	$(\overline{010})\bar{L} = 4, \bar{S} = 1$	0.259
1.28587	$\alpha = 2, J = 1, T = 0$	$(\overline{010})\bar{L} = 1, \bar{S} = 0$	1.015
1.44767	$\alpha = 1, J = 2, T = 0$	$(\overline{010})\bar{L} = 2, \bar{S} = 1$	~ 0
2.0335	$\alpha = 1, J = 2, T = 1$	$(\overline{010})\bar{L} = 2, \bar{S} = 0$	1.232
2.20414	$\alpha = 2, J = 3, T = 0$	$(\overline{010})\bar{L} = 2, \bar{S} = 1$	0.161
2.3487	$\alpha = 1, J = 4, T = 0$	$(\overline{010})\bar{L} = 4, \bar{S} = 1$	~ 0
2.47888	$\alpha = 1, J = 5, T = 0$	$(\overline{010})\bar{L} = 4, \bar{S} = 1$	0.072
2.6003	$\alpha = 2, J = 2, T = 0$	$(\overline{010})\bar{L} = 2, \bar{S} = 1$	~ 0
2.77479	$\alpha = 1, J = 4, T = 1$	$(\overline{010})\bar{L} = 4, \bar{S} = 0$	0.5015

Figure 1

Spectroscopy of Gallium 62

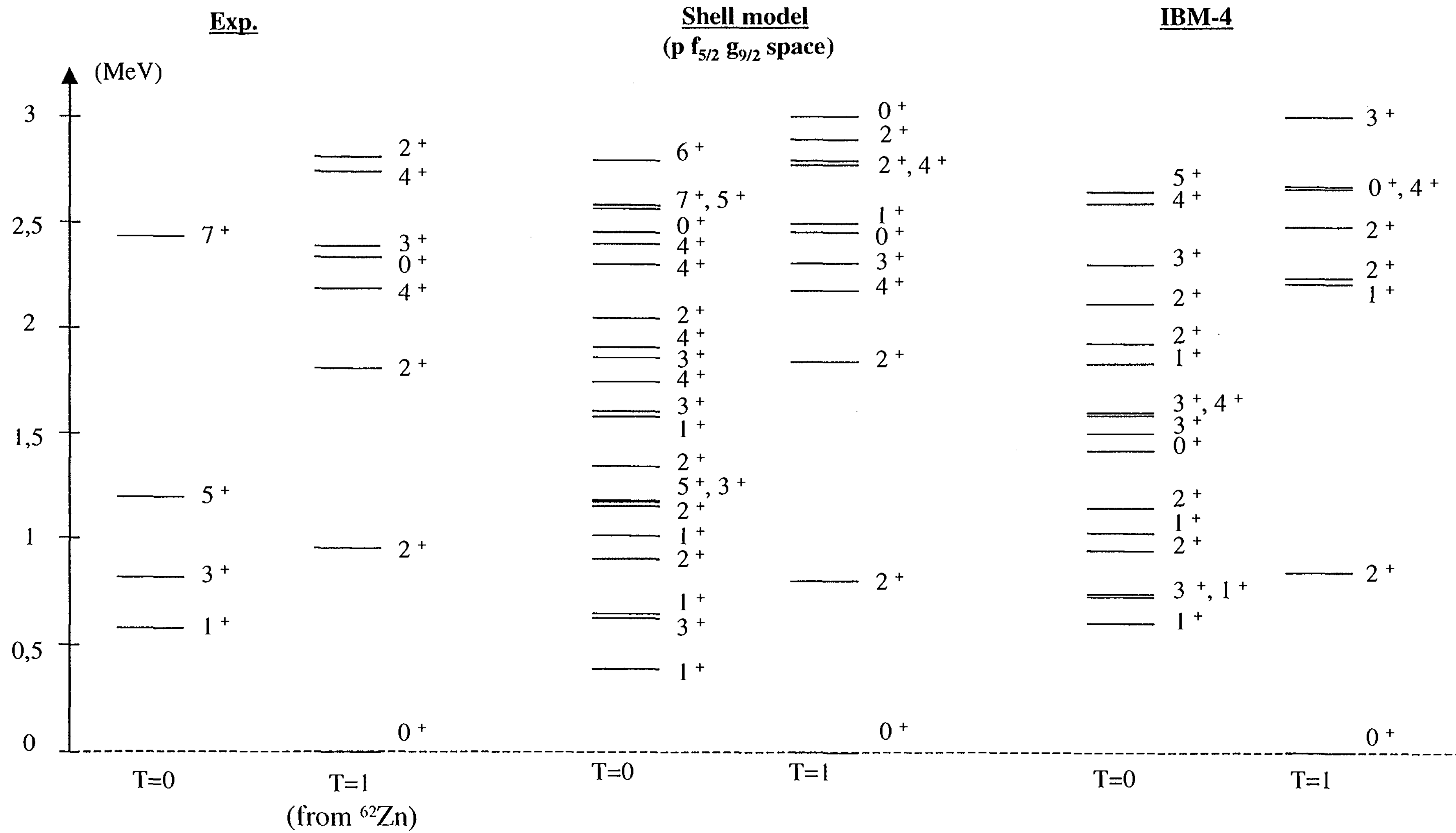

Figure 2

Spectra of ^{66}As and ^{70}Br predicted in IBM-4

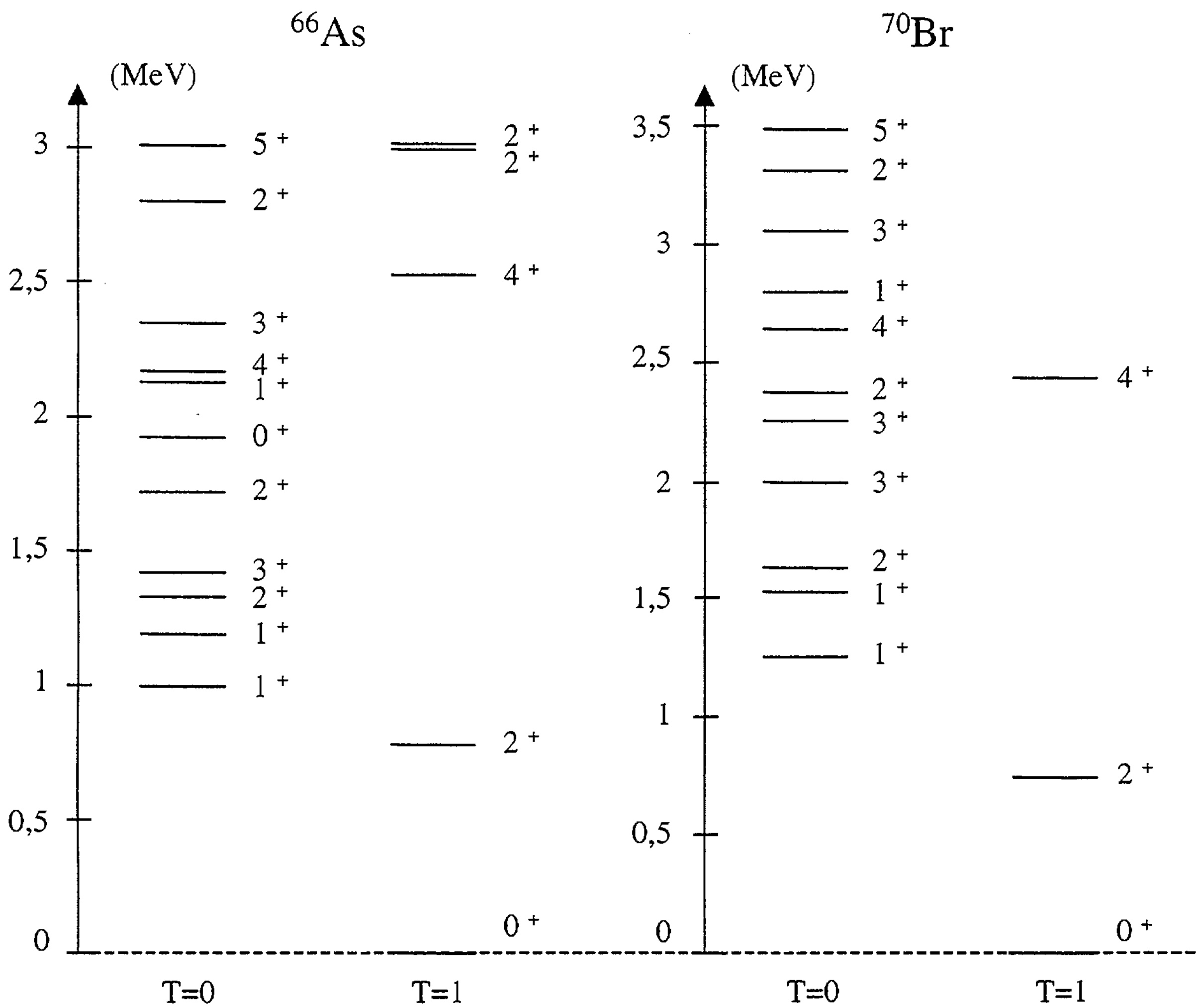

Figure 3

Pair structure of $N = Z$ odd-odd nuclei