

On Runaway Transport
under Magnetic Turbulence
in Tokamaks

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22 pp. 6 figs. 15 refs.

Abstract:

The influence of magnetic turbulence on runaway transport has been studied. The evolution of runaway distribution function has been calculated using Electra, a 2D code in momentum space and 1D in radius coordinate. The code considers the effect of averaging the turbulence by runaway orbits. Then Hard X-Ray emission spectrum is estimated and compared with experimental results of TJ-I tokamak, obtaining a remarkable agreement.

Transporte de Electrones Runaways bajo Turbulencia Magnética en Tokamaks

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Resumen:

En este trabajo hemos estudiado la influencia de la turbulencia magnética en el transporte de los electrones 'runaway'. Para el cálculo de la evolución de la función de distribución se ha desarrollado un código denominado 'Electra' que calcula la evolución en dos dimensiones en el espacio de momentos y la difusión en la dirección radial en el espacio real. En el cálculo de la difusión radial el código considera que los electrones de alta energía pueden a lo largo de su trayectoria promediar las turbulencias magnéticas debido a la deriva que sufren sus órbitas respecto a las superficies de flujo magnético. A partir de la función de distribución obtenida se calcula el espectro de emisión de Rayos X y se compara con los resultados experimentales del Tokamak TJ-I, obteniéndose un buen acuerdo.

I - INTRODUCTION

It is well known that the topology of magnetic configuration of confinement devices has a strong influence on their transport properties. Usual transport studies assume that the magnetic configuration is a set of nested magnetic surfaces isomorphic to a set of nested torii in tokamaks as well as in stellarators, due to the Hamiltonian structure of magnetic field equations.

Nowadays more and more evidences show that this idea is too simple. The present plasma diagnostics have now a high spatial resolution. These allow us to detect a big amount of irregularities in the electron density and temperature profiles. This experimental data shows that the magnetic structure is more complicated than the one described before: Filaments¹, structures in the density and temperature profiles², and stochastic layers³ modify the confinement properties of the magnetic traps.

Runaway electrons are a good probe of the devices, magnetic topology as they suffer very rare collisions and are hardly sensitive to electrostatic turbulence. An electron is considered a runaway if its velocity is higher than the Dreicer one, i. e., if the effect of accelerating toroidal electric field is able to overcome the collision drag. In particular, runaway electrons will be sensitive to the modifications of magnetic surface topology induced by magnetic turbulence.

So, runaway transport is mainly governed by the magnetic structure, provided that runaway electrons have enough energy to disregard the effect of radial electric fields. Then the modifications of magnetic topology will have influenced in the runaway electron confinement. Specially, small scale topology changes induced by magnetic turbulence will modify the runaway confinement properties. So the diffusion properties of runaway electrons will depend on velocity in a non-trivial way⁴.

It has been observed that some experimental hard X ray (HXR) intensity spectra present anomalous features: the logarithm of intensity plotted versus photon energy shows two different slopes, join by a flat area, at different energy regions. These spectra are produced by Bremsstrahlung emission of runaway electrons diffused onto the device wall. An appropriate treatment of the evolution of runaway distribution function can explain the experimental results, taking into account that runaway electrons transport is mainly governed by the magnetic topology.

In previous works this spectrum has been used to obtain information of the characteristics of magnetic turbulence, namely amplitude and radial correlation length⁵. The

amplitude was obtained from the first slope of the spectrum, while the correlation length was obtained taking advantage of the fact that high energy runaways are able to average the turbulence if their typical drifts are larger than the typical width of stochastic layers. When the runaway reaches enough energy its drift is wider than the stochastic area; therefore it is able to average its effect and to improve its confinement. Nevertheless, only average or typical quantities could be obtained from this model and all the calculations could be only performed in steady state.

Electra, a 3D code, has been developed to calculate the evolution of the electron distribution function in momentum (2D) and real (1D) space. The code allows to introduce a model for the turbulence that takes into account both radial and time evolutions. Although the HXR measurements are still an average, it is possible to think in methods to examine the influence of these two dependencies on diagnostics. Using a HXR detector array can help to extract the radial dependence of diffusion characteristics⁶.

This paper is organized as follows: In section II the model for the developed code is described. In section III the role of the shape of the critical trajectory in momentum space and its influence on runaway generation is shown. The influence of magnetic turbulence characteristics on runaway generation and evolution is discussed in section IV. Some comparisons with experimental results are shown in section V. Finally the conclusions are in section VI

II - THE MODEL

For the values of the plasma parameters relevant for fusion, two time scales appear in the model: The shortest one is associated to the evolution in the momentum space and the longest one to the diffusion in real space. This fact allows the detachment of evolution of distribution function in both spaces.

The evolution of the distribution function in momentum space, after an interval time Δt , can be obtained as follows:

$$f_i(p_{\parallel}, p_{\perp}, r) = f_{i-\Delta t}(p_{\parallel}(t - \Delta t), p_{\perp}(t - \Delta t), r) \quad (1)$$

Where the evolution of a single particle is obtained from Langevin equations for a particle test ensemble, averaging the stochastic terms:

$$\begin{aligned} \frac{dp_{\parallel}}{dt} &= -eE_{\parallel} - v_{\parallel} p_{\parallel} - F_S p_{\parallel} \\ \frac{dp_{\perp}}{dt} &= -v_{\perp} p_{\perp} - F_S p_{\perp} \end{aligned} \quad (2)$$

The toroidal electric field, Coulomb collisions⁷ and synchrotron radiation losses⁸ are included to estimate particle evolution. The several quantities that appear in the equations are:

$$v_{\parallel} = \frac{n_e e^4 \ln \Lambda}{4\pi\epsilon_0^2} \gamma (Z_{eff} + 1 + \gamma) \frac{1}{m_e^2 c^3 p^3} \quad (3)$$

$$v_{\perp} = \frac{(vp^2 - v_{\parallel} p_{\parallel}^2)}{p_{\perp}^2} \quad (4)$$

$$v = \frac{n_e e^4 \ln \Lambda}{4\pi\epsilon_0^2} \frac{\gamma^2}{m_e^2 c^3 p^3} \quad (5)$$

$$F_S = \frac{e^2}{6\pi\epsilon_0 m_e c} \gamma p^2 \left(\frac{1}{R_0^2} + \frac{e^2 B^2}{m_e^2 c^2} \frac{p_{\perp}^2}{p^4} \right) \quad (6)$$

Where $p = v\gamma/c$, is the electron momentum normalized to $m_e c$, p_{\parallel} is the component of electron momentum parallel to the magnetic field, p_{\perp} is the perpendicular one, γ is the relativistic factor, $\ln \Lambda$ is the Coulomb logarithm, R_0 is the tokamak major radius, B and E_{\parallel} are the toroidal magnetic and electric fields. Collisionality is approximated by the slowing-down equations in the high speed limit and the proportional terms to F_S accounts for synchrotron radiation losses that are only important for high energy.

As the model is only valid for high-speed particles, it only allows us to estimate the evolution of the part of the distribution function corresponding to the runaway electrons. The non runaway part of the distribution function is assumed to keep Maxwellian, therefore the electrons that become runaway must be provided at the same rate that they are lost. We have also considered the Maxwellian value as the lower limit to the runaway distribution function.

The calculation starts with a Maxwellian distribution function, that is allowed to evolve in momentum space according to the equations (2). After a single step evolution in momentum space, the distribution function suffers radial diffusion. A 1D differential equation of diffusion in cylindrical coordinates is introduced to evaluate the distribution function evolution in radial space.

$$\frac{\partial f(p_{\parallel}, p_{\perp}, r)}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r D(\bar{v}, r) \frac{\partial f(p_{\parallel}, p_{\perp}, r)}{\partial r} \right) \quad (7)$$

This equation is solved numerically after each evolution step in momentum space.

The diffusion coefficient, $D(\mathbf{v}, r)$, must include the processes that are important for runaway evolution. In this work the only process considered is magnetic turbulence (see section IV), because the magnetic topology the most important factor that modifies runaway confinement.

III - PARTICLE TRAJECTORIES IN MOMENTUM SPACE.

The particle trajectories in momentum space, plotted in figure 1, has been obtained solving the equations (2). The deterministic part of Langevin equations define the evolution in the momentum space and they divide the space in two isolated regions. The particle behaviors in both regions are very different, in one region the electrons remain as thermal while they become runaway electrons in the other region.

From these trajectories it is possible to obtain the critical curve that separates the runaway and non-runaway areas in 2D momentum space. The critical curves are shown for different radial positions in figure 2. These curves have the important property that are not circular, so it is not possible from here on to speak about a critical velocity, but about a separatrix in momentum space. In the test particle Langevin equations (2) only the deterministic part has been considered, it causes that both regions in momentum space are isolated. In fact the particles can jump from the non-runaway to the runaway zone, and vice versa, due to the random movement of the particles caused by collisions. This behavior could be introduced in the equations through stochastic terms. Then the shape of critical curves and their position in momentum space would be very important as they would have strong influence on runaway generation. These terms are not considered in this work, because we are only interested in the evolution of runaway electrons.

The trajectories of the electrons in momentum space are plotted in figure 1 show that even electrons with velocity in the same direction as electric field can become runaways. These electrons in its movement can first lose energy, which cause a depletion in the runaway distribution function near to the critical trajectory in such region. Though it is not considered in our model, the structure of trajectories in momentum space shows that perpendicular diffusion in momentum space can generate runaway electrons, when they go across the critical trajectory, or can make them to increase their parallel velocity. This effect can be understood comparing electron perpendicular diffusion in momentum space, due to the stochastic wave-particle interaction, with displacements of the test particle in the perpendicular direction, which allows those particles to cross the critical trajectory or to reach trajectories that correspond, in the limit, to higher parallel velocities. An effect similar to this one could explain the generation of fast particles in the parallel direction due to Electron Cyclotron Resonance Heating⁹.

IV - INFLUENCE OF MAGNETIC TURBULENCE

As we have seen in section II, an 1D differential equation of diffusion is introduced to evaluate the distribution function evolution in real space. The effect of magnetic turbulence is taken into account through the diffusion coefficient, $D(\mathbf{v},r)$. This coefficient, $D(\mathbf{v},r)$, will be given by:

$$D(\bar{v},r) = D_M(r) |v_{||}| Y(\Delta) \quad (8)$$

I. e., it is proportional to the magnetic line diffusion coefficient, D_M , to the absolute value of parallel speed and considers the runaway orbit averaging of the turbulence through the factor $Y(\Delta)^{10}$. The argument of the function Y is:

$$\Delta = d_r(v,r) k_r(r) = k_r c q \frac{\gamma}{\omega_c} (1 - \gamma^2)^{1/2} \quad (9)$$

Where d_r is the typical runaway drift, c is the speed of light, q is the safety factor, ω_c is the cyclotron frequency, γ is the relativistic Lorentz factor, and k_r^{-1} is the radial correlation length of the turbulence. Y is a monotonic function and takes the value 1 when Δ is 0 and tends to 0 when Δ tends to infinite. It takes account of the orbit averaging of the turbulence: when runaway drifts are larger than the radial correlation length of the turbulence, the effect of this one is weaker and the runaway confinement is improved¹¹. The runaway diffusion coefficient is shown in figure 3 versus energy (with the condition $p_{||} = p$) and, as it is shown, it is non-monotonic. The diffusion coefficient is increasing with runaway energy up to a certain value, due to the dependence on $Y(\Delta) v_{||}$, which provokes the degradation of confinement for increasing energies. For parallel velocities above this value, the confinement is improved. Runaway electrons can reach or not this critical energy, depending on the parameters of the tokamak discharge and on the characteristics of the magnetic turbulence. This fact and the different energy range measured by the HXR detectors explain the discrepancies that have been observed in different runaway confinement times in tokamaks¹². As it can be seen in figure 3, the critical velocity cannot be defined with high precision at inner radial positions.

V - RESULTS

As key result we are obtaining the influence of magnetic turbulence on the evolution of runaway distribution function. Electra allows to obtain runaway distribution function in the momentum space for different radial positions at any time. Here we show the results for diverse cross sections, using the typical parameters of the TJ-I tokamak¹³: magnetic field $B=1$ T, parallel electric field $E_{\parallel}=-2.1$ V/m, electron density $n_e=0.7 \cdot 10^{19}$ m⁻³, electron temperature $T_e=1$ keV, major radius $R=0.3$ m, minor radius $a=0.1$ m, effective charge $Z_{\text{eff}}=2$ and safety factor $q=1-4.3$. Distribution function in velocity spaces is shown in figure 4 for four pitch angles and four radial positions in TJ-I, for the following magnetic turbulence characteristics: magnetic diffusion coefficient $D_M=5 \cdot 10^{-8}$ m and inverse of correlation length $k_r=100$ m⁻¹.

External magnetic surfaces receive runaway electrons coming from the inner ones, and the rate of diffusion depends on the velocity in a non-trivial way. The shape of the distribution function depends basically on the form of the diffusion coefficient in velocity space in the inner magnetic surfaces. It is even possible that radial diffusion causes that distribution function presents positive slope in some areas of momentum space. These positive slopes were predicted theoretically before in reference ¹⁴, but they were attributed to kinetic instabilities in that work.

Once the distribution function is calculated by the code, HXR emission can be estimated using a relativistic cross section that includes Elwert's factor¹⁵. The emission calculated in this way can be compared with experimental HXR spectra. In figure 5 we show a comparison between the calculated and the measured HXR spectra. The experimental values of density, temperature, and loop voltage are introduced in the model and the characteristics of the turbulence are varied to adjust the spectra. We obtain two different slopes and a flat zone between them. The second slope starts at an energy value that coincides approximately with the critical value, ϵ_k ⁴. The typical runaway drifts d_r from ϵ_k are larger than the radial correlation length of the turbulence k_r^{-1} (see figure 3)

The former approximated models could not explain the flat area that appears in the spectra. They did not take into account any diffusive process and all the quantities were averaged. Now this area can be explained considering the fact that several critical energies appear at different radial positions in the plasma and the final spectra will have a footprint of all of them.

VI - CONCLUSIONS

Electra is a code that has been developed for studying runaway diffusive transport under the effect of magnetic turbulence. The model that has been introduced in the code allows to reproduce the experimental hard X ray intensity spectra that present two different slopes at different energy regions. The energy of these two different regions is related to the ability of high energy runaway electrons to average the effect of the turbulence. This fact causes an improvement of their confinement when their typical drifts, d_r , are larger than the radial correlation length of the turbulence, k_r^{-1} . This characteristic is included in the model through the orbit averaging factor, $Y(\Delta)$, in the runaway diffusion coefficient.

The model allows to include radial as well as time dependence of the relevant magnitudes that cause runaway electron transport, namely parallel electric field and magnetic turbulence. Nevertheless, the distribution function evolution is estimated in this work considering only radial dependence and assuming that all the magnitudes are constant in the time.

The HXR spectrum is evaluated using a relativistic cross section that takes into account Coulomb screening through Elwert's factor. A remarkable agreement is obtained between the results of the model and previous experimental data obtained in the small TJ-I tokamak. These spectra can be used as diagnostic not only to investigate runaway properties but also to study the characteristics of magnetic turbulence.

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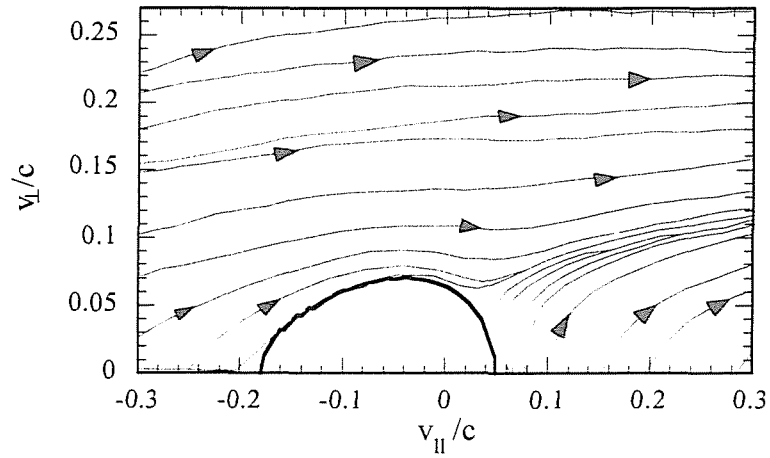


Figure 1. Trajectories in momentum space obtained solving equations (2) for the following plasma parameters: magnetic field $B=1$ T, parallel electric field $E_{\parallel}=-2.1$ V/m, central electron density $n_e=0.7 \cdot 10^{19} \text{ m}^{-3}$, central electron temperature $T_e=1$ keV, and effective charge $Z_{\text{eff}}=2$.

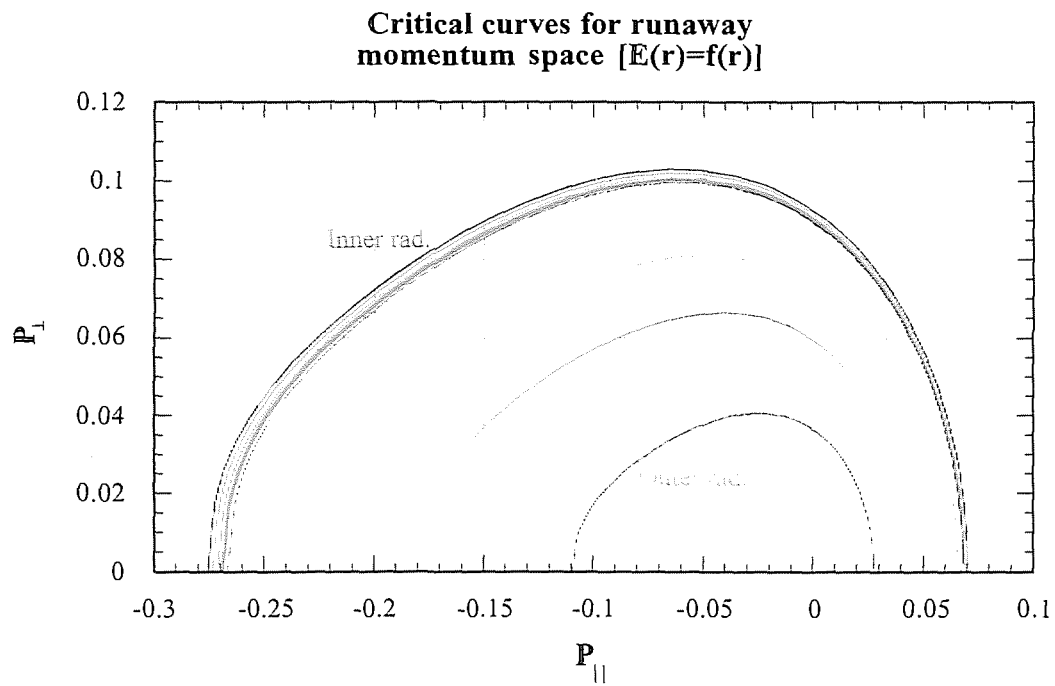


Figure 2. Critical trajectories in momentum space for several radial positions and for the same parameters as in figure 1.

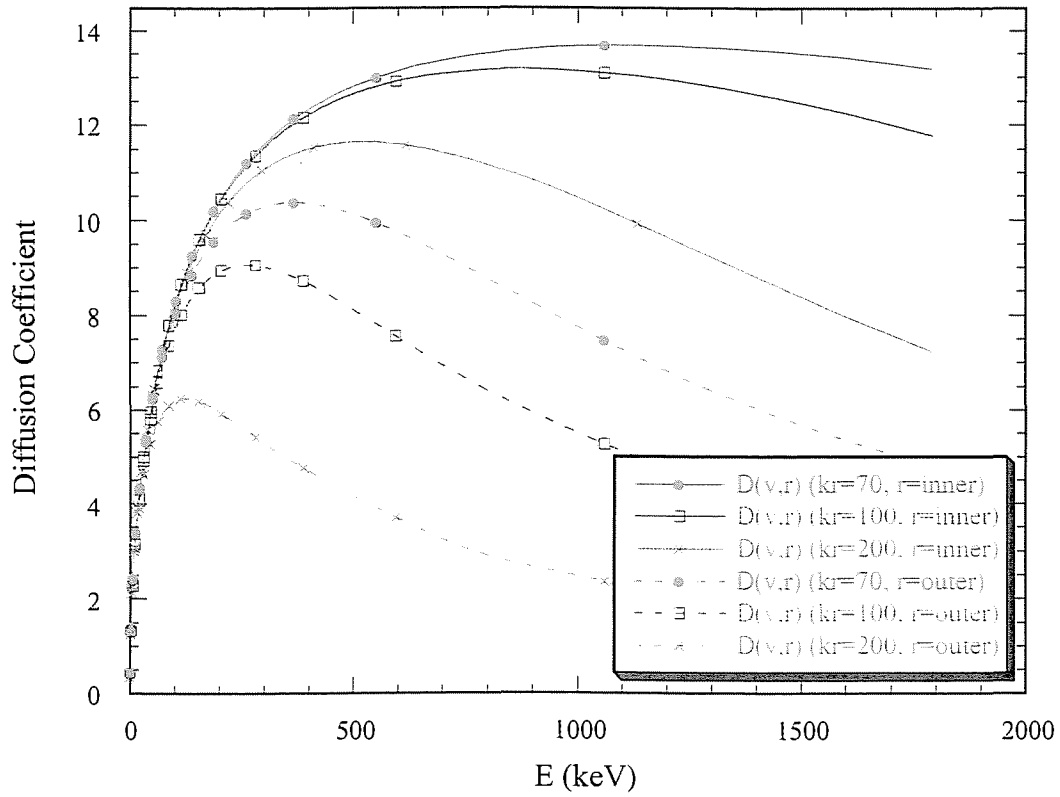
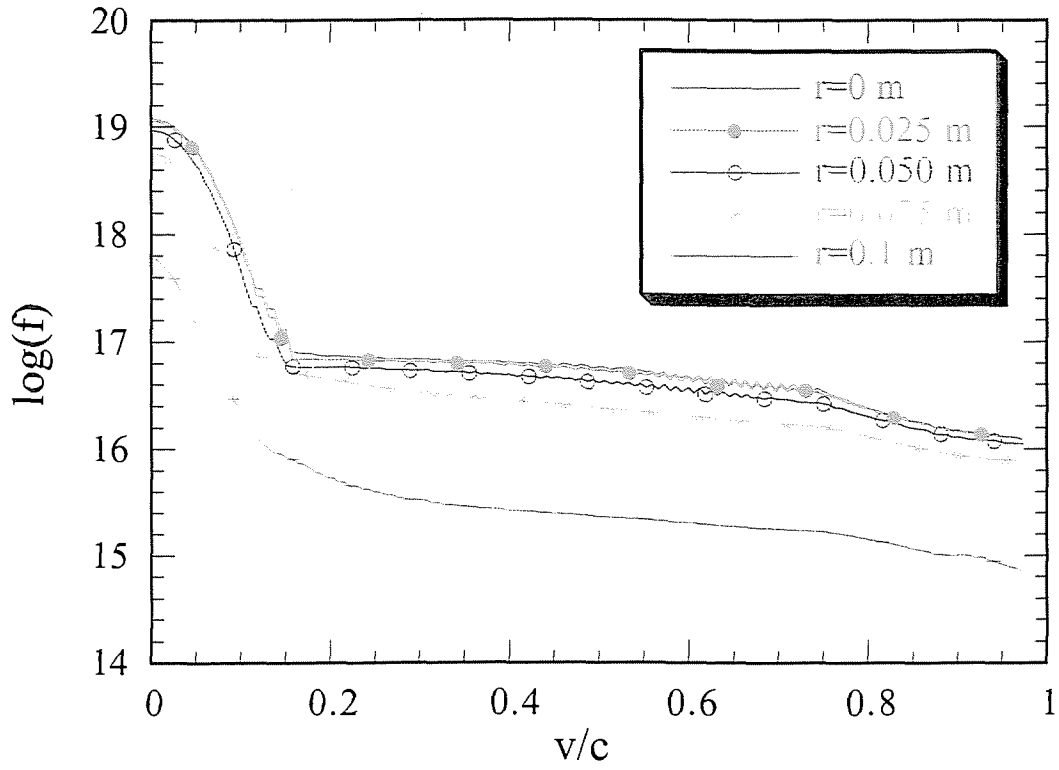
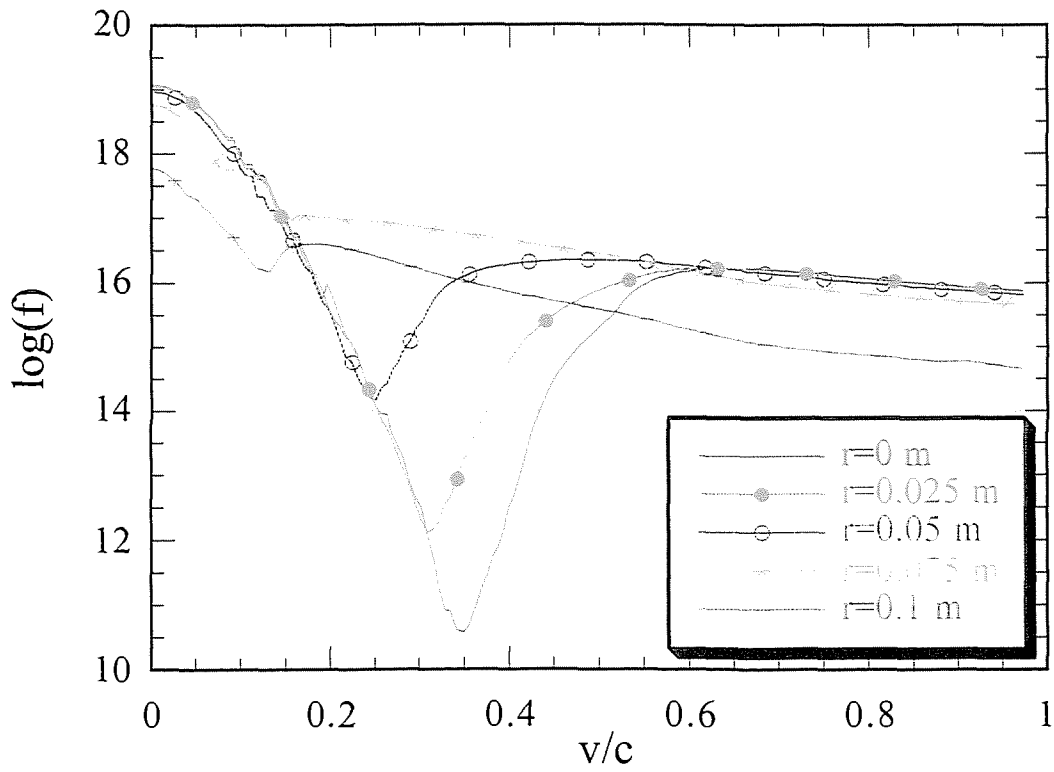


Figure. 3: Diffusion coefficient vs. energy for $p_{\parallel} = p$. Plasma parameters: magnetic field $B=1$ T, parallel electric field $E_{\parallel}=-2.1$ V/m, electron density $n_e=0.7 \cdot 10^{19} \text{ m}^{-3}$, electron temperature $T_e=1$ keV, magnetic diffusion coefficient $D_M=5 \cdot 10^{-8}$ m, inverse of correlation length $k_r=70, 100, 200 \text{ m}^{-1}$, effective charge $Z_{\text{eff}}=2$ and safety factor at the edge $q_a=4.3$.

Angle $(p, p_{||}) = 25^\circ$



Angle $(p, p_{||}) = 65^\circ$



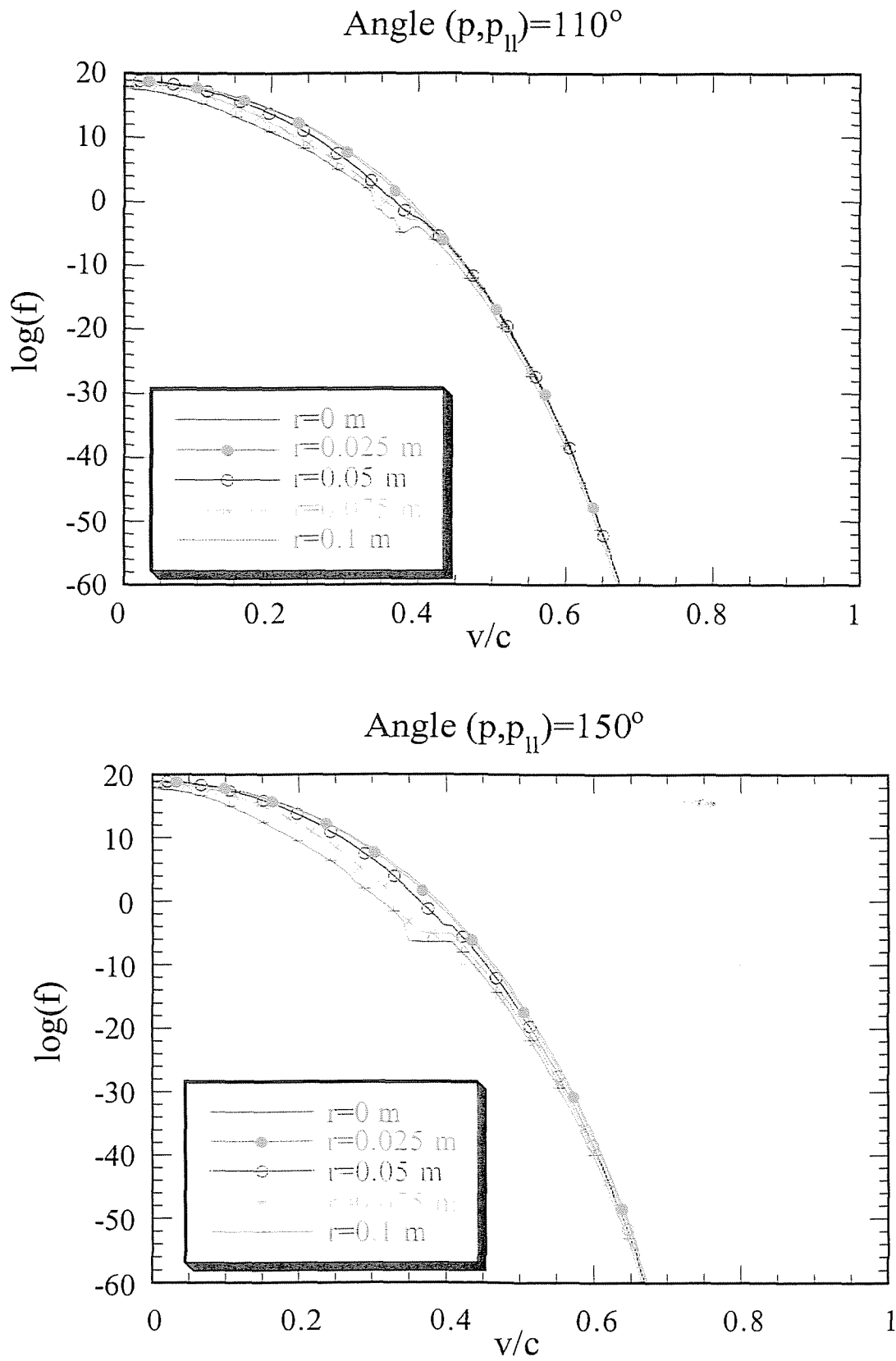


Figure 4a: Electron distribution function for five radial positions and for four pitch angles. The plasma parameters are the same as in the former figures.

Distribution Function

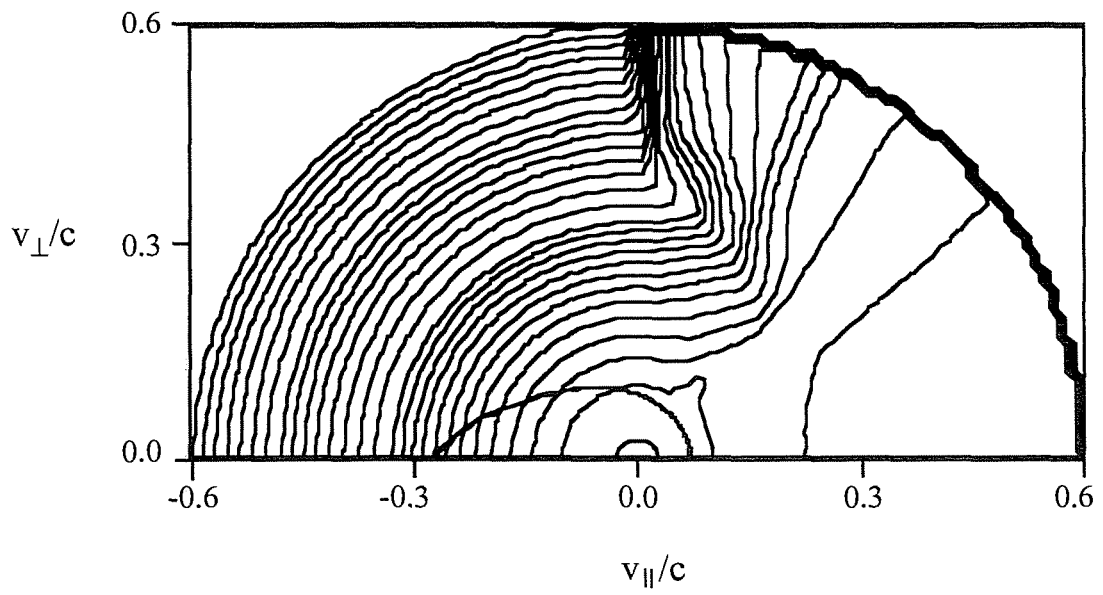


Figure 4b: Level curves of the electron distribution function corresponding to the radial position $r=3$ cm of figure 4a.

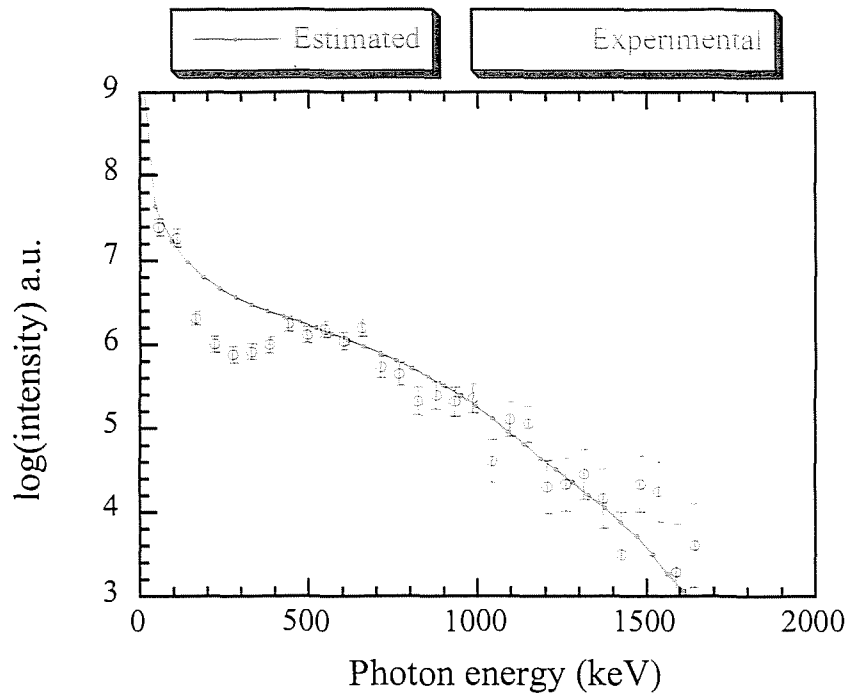


Figure 5: Comparison of the calculated HXR spectrum with the measured one for the following plasma parameters: magnetic field $B=0.9$ T, parallel electric field $E_{\parallel}=-1.7,-0.8$ V/m, electron density $n_e=1.2 \cdot 10^{19} \text{ m}^{-3}$, electron temperature $T_e=1$ keV, major radius $R=0.3$ m, minor radius $a=0.1$ m, magnetic diffusion coefficient $D_M=2 \cdot 10^{-8}$, inverse of correlation length $k_r=100 \text{ m}^{-1}$, effective charge $Z_{\text{eff}}=2$ and safety factor $q_a=4.3$.