



FR0106240

I.P.N. - 91406 ORSAY CEDEX

institut de physique nucléaire
CNRS - INP3 UNIVERSITÉ PARIS - SUD

Gestion INIS
Doc. Enreg. le 5/4/2001
N° TRN FR 0106240



IPNO DR 00-32

Modeling a neutron rich nuclei source

M. Mirea, O. Bajeat, F. Clapier, F. Ibrahim.
A.C. Mueller, N. Pauwels and J. Proust

Modeling a neutron rich nuclei source

M. Mirea^{1,2}, O. Bajeat¹, F. Clapier¹, F. Ibrahim¹, A.C. Mueller¹, N. Pauwels¹, and J. Proust¹

¹ Institut de Physique Nucléaire, 91406, Orsay Cedex, France

² Institute of Physics and Nuclear Engineering, Tandem Lab., P.O.Box MG-6, Bucarest, Romania

Abstract. The deuteron break-up process in a suitable converter gives rise to intense neutron beams. A source of neutron rich nuclei based on the neutron induced fission can be realised using these beams. A theoretical optimization of such a facility as a function of the incident deuteron energy is reported. The model used to determine the fission products takes into account the excitation energy of the target nucleus and the evaporation of prompt neutrons. Results are presented in connection with a converter-target specific geometry.

PACS. 25.85.Ca Spontaneous fission - 25.85.Ec Neutron induced fission - 24.75.+i General properties of fission

1 Introduction

The fission process of heavy nuclei at low energy gives rise to neutron rich fragments. Exploiting this property, a source of nuclei towards the stability limit can be conceived [1]. A high flux of neutrons able to produce the fission events can be obtained through the deuteron break-up reaction in some suitable converter placed in the vicinity of the fissioning target. The feasibility study of a facility based on the above concept is the main purpose of the PARRNe [2,3] (Production d'Atomes Radioactifs Riches en Neutrons) ongoing project. The fissioning tar-

get is made from ^{238}U . This work is mainly guided by the NuPECC recommendations established in order to investigate the main options for second-generation radioactive beam facilities in Europe [4]. Such studies were carried also in Ref. [5] using the LAHET code in order to estimate fission rates.

In the following, a theoretical optimization of such a combination is intended by taking into account the influence of the incident deuteron energy on the variation of the fission isotopic product yields. The β -decay of unstable isotopes is not considered here. Throughout the rest of this

paper, the term fission fragment refers to fragments before prompt neutron emission, and the term fission products refers to the products after prompt neutron emission [6]. The fission model, initially reported in Ref. [7], is presented here in an extended version suitable for symmetric decays. The isotopic distribution for all the fission products can now be determined.

The incident energy of the deuteron causes changes in the angular and energetic neutron distributions. So, the converter–target geometry of the facility must be investigated at each incident deuteron energy. The energy of the neutron impinging on the ^{238}U -target rules the excitation of the compound nucleus ^{239}U or the residual ^{238}U obtained in inelastic collisions. It is known that the excitation energy affects the isotopic products yields because the fission barrier is modified for each fragment's binary partition ($A_1 Z_1, A_2 Z_2$) and the number of evaporated neutrons varies.

The paper begins with a theoretical investigation of the relative variations of the isotopic yields of fission products as a function of the excitation energy of the compound nucleus ^{239}U and the excited ^{238}U . The fission model will be presented in the following section. Furthermore, for a given experimental geometry, the yields of the neutron rich isotopes obtained in the fissioning target are determined with respect to the incident energy of the deuteron. The number of fission events produced in the target for a given converter–target geometry as a function of the incident deuteron energy will be deduced. Results are commented in connection with experimental data in the last section.

2 Fission model

This neutron induced fission model is based essentially on the hypothesis of a sudden dissipation process of the excitation energy carried in by the incident neutron.

2.1 Nuclear shape parametrization

Due to the leptodermous character of the nuclear systems, it is convenient to consider that during the fission, the shape degrees of freedom vary and rule the evolution of the system. In general many degrees of freedom are needed to specify accurately the shape of a decaying nucleus. It is however considered that three degrees of freedom is the barest minimum to display the essential features of fission: the elongation, the necking and the mass–asymmetry [8].

The nuclear parametrization is defined by joining smoothly two intersecting spheres of radius R_1 and R_2 with a neck surface generated by the rotation of a circle of radius R_3 around the symmetry axis [9], as presented in Fig. 1. The distance between the center of this circle and the axis of symmetry is given by ρ_3 . By imposing the condition of volume conservation, the surface is perfectly determined by the values of the parameters R (distance between the centers of the spheres), R_3 (the radius of the neck) or $C = S/R_3$ (where $S = +1$ when $\rho_3 - R_3 \geq 0$ and $S = -1$ when $\rho_3 - R_3 < 0$) and R_2 (the radius of the emitted fragment) or $\eta = \frac{V_1 - V_2}{V_0}$. These three parameters characterize the elongation, the necking and the mass–asymmetry, respectively. Due to the axial-symmetry of this system, the

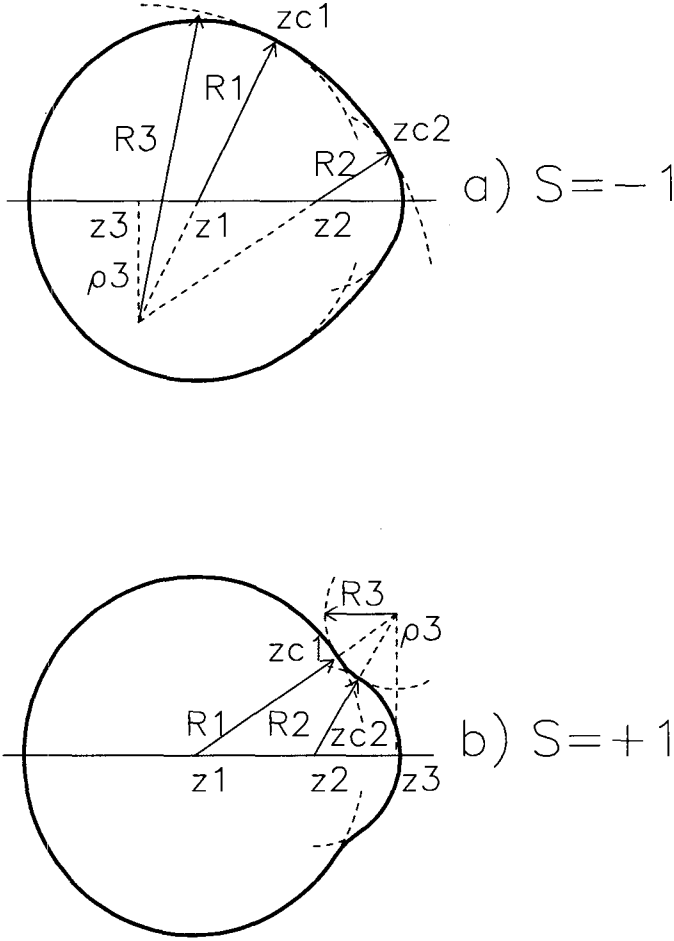


Fig. 1. The nuclear shape parametrization. The upper part corresponds to a negative neck parameter $C = S/R_3$ while the lower one corresponds to positive C values.

surface equation is given in cylindrical coordinates:

$$\rho_s(z) = \begin{cases} [R_1^2 - (z - z_1)^2]^{1/2} & z \leq z_{c1} \\ \rho_3 - S [R_3^2 - (z - z_3)^2]^{1/2} & z_{c1} < z < z_{c2} \\ [R_2^2 - (z - z_2)^2]^{1/2} & z \geq z_{c2} \end{cases} \quad (1)$$

For extremely large values of R_3 , that means for $C = S/R_3 \approx 0 \text{ fm}^{-1}$ the parametrization in the interval $z_{c1} < z < z_{c2}$ is described by relation:

$$\rho_s(z) = [a(z - z_{c1}) + b]^{1/2}$$

with

$$a = \{[R_2^2 - (z_{c2} - z_2)^2]^{1/2} - [R_1^2 - (z_{c1} - z_1)^2]^{1/2}\} / (z_{c2} - z_{c1})$$

$$b = [R_1^2 - (z_{c1} - z_1)^2]^{1/2}$$

The significance of all the geometrical symbols are presented by means of Fig. 1. Throughout this paper, the subscripts 0, 1 and 2 help to assign the parent, daughter (heavy fragment) and the emitted (light fragment) nuclei, respectively. The initial radius of the parent is $R_0 = r_0 A_0^{1/3}$, the final radii of the two fragments are $R_{if} = r_0 A_i^{1/3}$, with $i = 1, 2$ and the constant radius $r_0 = 1.16 \text{ fm}$. In Eq. (1), ρ_s denotes the value of the coordinate ρ on the nuclear surface. "Diamond"-like shapes are obtained for $S = -1$ and necked-in shapes for $S = +1$.

When $S = -1$, the volume of the emitted fragment V_2 cannot be determined unambiguously without introducing a supplementary hypothesis. In this special case, the shape does not evidence a separation plane between the nascent fragments. So, it is considered that V_2 must be computed from half the distance between the centers $z_2 - R/2$ up to the side $z_2 + R_2$. In the case $S = +1$ we compute this volume between $[z_3, z_2 + R_2]$. When $R_3 = 0 \text{ fm}$, the simple parametrization of two intersecting spheres is obtained. This nucleus shape parametrization was widely used in nuclear dynamic calculations [10–12] in a large range of mass asymmetries because it accounts for the most important degrees of freedom encountered in fission processes: elongation, necking and mass-asymmetry. In the following, sometimes, in place of R we use R/Δ which defines the normalized coordinate of elongation with $\Delta = R_1 + R_2 + 2R_3$ being the point of scission. Also, $\Delta_t = R_{1f} + R_{2f}$ denotes the touching point configuration when the two fragments have spherical shapes.

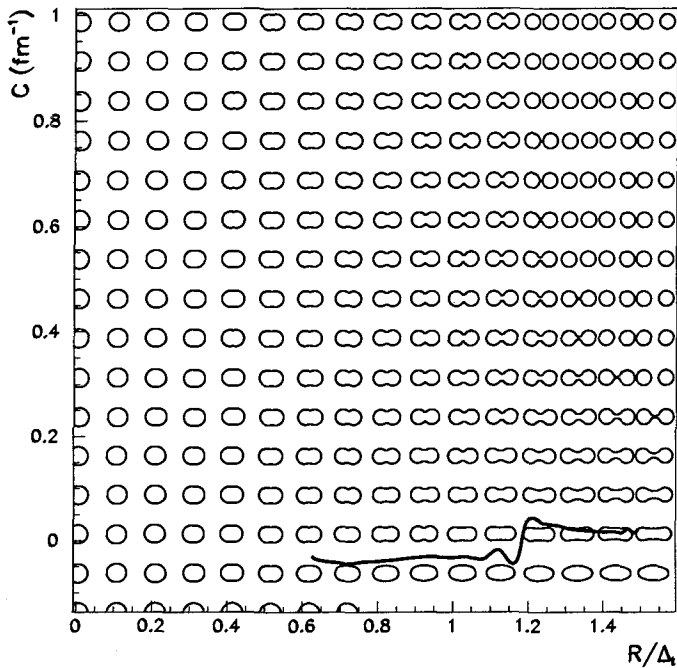


Fig. 2. Family of nuclear shapes in the $(C, R/\Delta_t)$ plane for the symmetric fission of ^{238}U . $\Delta_t = r_0(A_1^{1/3} + A_2^{1/3})$ is the elongation which characterizes the two tangent spheres configuration. The path obtained by minimizing the integral action is plotted with a thick line.

Instead the mass-asymmetry generalized coordinate R_{2f} used in Ref. [7], that is, the final radius of the light fragment, in the following, the quantity $\eta = \frac{A_1 - A_2}{A_0} = \frac{V_1 - V_2}{V_0}$ will be taken as the mass-asymmetry. The equation of volume conservation is solved in order to obtain R_1 and R_2 , the radii of the heavy and the light fragment, such that the volumes V_1 and V_2 correspond to the masses A_1 and A_2 .

In Fig. 2, the possible shapes for the symmetric fission of ^{238}U are plotted with respect the elongation and the necking of the nuclear system. For positive values of the parameter $C = S/R_3$, we obtain necked shapes, while for negative values of C the parametrization gives swollen

shapes in the central region. Swollen shapes are geometrically possible only up to a fixed value of the elongation, depending on C . In the positive region, the elongation for the scission configuration decreases with increasing C , reaching the lower limit $\Delta_t = r_0(A_1^{1/3} + A_2^{1/3})$ for $R_3 = 0$ fm.

2.2 Penetrabilities

The probability to obtain a binary partition $(A_H Z_H, A_L Z_L)$ is ruled by the barrier penetrability. Using the last action trajectory principle it is possible to obtain the path in the configuration space followed by the fissioning system [13].

The quantum penetrability

$$P = \exp \left\{ -\frac{2}{\hbar} \int_{R_i}^{R_f} \sqrt{2B \left(q_i, R, \frac{\partial q_i}{\partial R} \right) [E_d(q_i, R) - E_v]} dR \right\} \quad (2)$$

is calculated by using the semi-classical Wentzel-Kramers-Brillouin (WKB) approximation. The region of interest is classically forbidden. However the action trajectory is determined along a classical trajectory between the initial R_i and final R_f turning points corresponding to the entrance into the sub-barrier region and the exit on the other side of the potential. The two turning points are fixed by the same value of the deformation energy, that is, $E_d(q_i, R_i) = E_d(q_i, R_f) = E_v$. Here, $\{q_i\}$ denotes the set of generalized deformation parameters, excepting the elongation R . In the functional (2), B gives the inertial mass, E_d is the deformation energy determined so that its value in the ground state is zero and $E_v \approx 0.5$ MeV is the zero point vibration energy. The condition of consis-

tence must be fulfilled: the same shape for the inertia, the microscopic and macroscopic energies.

2.3 Deformation energy

The main key steps to deduce the deformation energy E_d are presented in the following.

2.3.1 Macroscopic deformation energy

The macroscopic deformation energy was computed in the framework of the Yukawa-plus-exponential [14] model extended for binary systems with different charge densities [15] where another degree of freedom is introduced, namely the charge asymmetry. The charge density of the system is kept initially unchanged (up to 0.7Δ), and in the final stages of the process where very necked forms are reached, the charge densities of the two fragments are linearly varying function of R up to their final values in the output channel. The values of the numerical parameters are from Ref. [16]. Both nuclear, E_n ,

$$E_n = -\frac{a_2}{8\pi^2 r_0^2 a^4} \int_V \int_V \left(\frac{r_{12}}{a} - 2\right) \frac{\exp\left(-\frac{r_{12}}{a}\right)}{\frac{r_{12}}{a}} d^3 r_1 d^3 r_2 \quad (3)$$

and Coulomb, E_c ,

$$E_c = \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\rho_e(\mathbf{r}_1)\rho_e(\mathbf{r}_2)}{r_{12}} d^3 r_1 d^3 r_2 \quad (4)$$

(with $r_{12} = |\mathbf{r}_1 - \mathbf{r}_2|$) energies are expressed as a sum of three shape dependent terms: the self-energies of the fragments (B_{ni} , B_{ci} , $i=1,2$), plus the interaction energy (B_{n12} , B_{c12}):

$$E_n/E_n^0 = (c_{s1}/c_s)B_{n1} + [(c_{s1}c_{s2})^{1/2}/c_s]B_{n12} + (c_{s2}/c_s)B_{n2} \quad (5)$$

$$E_c/E_c^0 = (\rho_{1e}/\rho_{0e})^2 B_{c1} + (\rho_{1e}\rho_{2e}/\rho_{0e}^2)B_{c12} + (\rho_{2e}/\rho_{0e})^2 B_{c2} \quad (6)$$

where E_n^0 and E_c^0 correspond to the spherical shapes, ρ_{ie} are charge densities and

$$c_{si} = a_s[1 - \kappa_s(N_i - Z_i)^2/A_i^2] \quad (7)$$

$$\rho_{ie} = 3eZ_i/(4\pi r_0^3 A_i) \quad (8)$$

with the parameters: $a_s = 21.13$ MeV, $\kappa_s = 2.3$, $a = 0.68$ fm, $a_V = 15.9937$ MeV, $\kappa_V = 1.927$. The quantities B_{ni} and B_{ci} in the above equations are dependent on the nuclear shape; their expressions containing two- and three-fold integrals are evaluated by numerical quadrature.

Further more, the contribution of the volume energy E_V is added proportionally to the light fragment radius so that $E_V = 0$ when $R_2 = R_0$ and

$$E_V = E_{V1} + E_{V2} - E_{V0} \quad (9)$$

when $R_2 = R_{2f}$. The volume self-energies of the fragments are

$$E_{Vi} = -a_V[1 - \kappa_V(N_i - Z_i)^2/A_i^2] \quad (10)$$

A linear interpolation also proportional to the light volume fragment is used to simulate the existence of the Wigner energy E_W :

$$E_{Wi} = -7 \exp(-6 |I_i|) \text{ MeV}, \quad (11)$$

where $I_i = (N_i - Z_i)/A_i$ is the relative neutron excess of each nucleus.

So, the total deformation macroscopic energy becomes:

$$E_{LDM} = E_n + E_c + (E_V + E_W) \frac{R_2 - R_0}{R_{2f} - R_0} - E_0 \quad (12)$$

where R_2 is the radius of the light fragment during the fission process (begins from R_0 at for the initial parent considered spherically and reaches its final radius R_{2f} after the scission). E_0 is the liquid drop energy of the spherically parent.

In the barrier region, the modulation of the potential energy surface by shell effects leads to the famous double-humped barrier for actinides. These effects are based on the following microscopic treatment.

2.3.2 Level scheme

The single-particle energetic levels are obtained with an improved version of the Supersymmetric Two-Center Shell Model (STCSM) inspired from the work of Maruhn and Greiner [17]. For a rigorous analysis of the microscopic model, the reader can refer to Ref. [18] and Refs. [19,20] where the model was improved. The STCSM was proved a useful tool in the microscopic treatment of supersymmetric fission, including cluster [19–21] and alpha [22] decay.

For the nuclear shape parametrization presented above, the microscopic potential (in cylindrical coordinates) is split into several parts which are treated separately:

$$V(\rho, z, \varphi) = V_0(\rho, z) + V_{as}(\rho, z) + V_n(\rho, z) + V_{Ls}(\rho, z, \varphi) + V_{L^2}(\rho, z, \varphi) - V_c \quad (13)$$

where $V_0(\rho, z)$ represents the two-center harmonic potential whose eigenvectors can be analytically obtained by solving the Schrödinger's equation. It is given by the relation:

$$V_0(\rho, z) = \begin{cases} \frac{1}{2}m_0\omega_{z_1}^2(z+z_1)^2 + \frac{1}{2}m_0\omega_\rho^2\rho^2, & z < 0 \\ \frac{1}{2}m_0\omega_{z_2}^2(z-z_2)^2 + \frac{1}{2}m_0\omega_\rho^2\rho^2, & z \geq 0 \end{cases} \quad (14)$$

where m_0 is the nucleon mass, z_1, z_2 (reals, positives) represent the distances (conditioned by the relation $z_1 + z_2 = R$) between the centers of the two spherical fragments and the intersection plane between the spherical shape of the daughter and the spheroid shape which overlaps the emitted nucleus (see Fig. 2 in Ref. [18]), and ω_i are the oscillator stiffnesses. In the case of symmetric fragmentation, $\omega_{z_1} = \omega_{z_2} = \omega_\rho$ and the parity becomes a good quantum number. The eigenvalues and the eigenvectors for reflection symmetric systems must be deduced using the mathematical formalism presented in Refs. [23,24].

The role played by the other terms in the total potential, i.e., $V_{as}, V_n, V_{Ls}, V_{L^2}, V_c$ is related to the mass-asymmetry, to the necking, to the spin orbit coupling, to the l^2 correction and to the depth of the potential, respectively and are defined as in Ref. [18].

Due to the cylindric symmetry of the system, the quantum numbers along the ρ -axis (n_ρ) and φ -coordinates (m) are constants of the motion. This property leads us to know which adiabatic levels are occupied by a unpaired nucleon during the process as function of the final splitting of the system. In these circumstances, we are able to apply the blocking effect to the unpaired nucleons during the fission process for each studied channel.

The spin-orbit and l^2 coefficients are obtained from Ref. [25] by interpolating the published values as a function of their mass. V_c is considered 0. These coefficients are mainly determined for nuclei close to the stability line. Therefore, we do not expect very accurate results in the mass determination of neutron rich isotopes.

2.3.3 Shell effects

The shell effects are computed using the well known Strutinsky procedure [13, 26]. The total energy is the sum of the liquid drop energy and the shell and pairing effects due to the proton and neutron level schemes.

The shell correction is:

$$\delta V_{p(n)} = \left[\sum_{\nu=1}^{Z(N)} \epsilon_{\nu} - \int_{-\infty}^{\tilde{\lambda}} \epsilon \tilde{g}(\epsilon) d\epsilon \right] \quad (15)$$

where ϵ_{ν} are the single-particle energies, $\tilde{\lambda}$ is the Fermi energy for the smoothed level density,

$$\tilde{g}(\epsilon) = \frac{1}{\tilde{\gamma}} \int_{-\infty}^{\infty} \mathcal{F}\left(\frac{\epsilon - \epsilon'}{\tilde{\gamma}}\right) \sum_{\nu} \delta(\epsilon - \epsilon_{\nu}) d\epsilon' \quad (16)$$

is the mean density of single particle levels, $\tilde{\gamma}$ is an interval around the Fermi energy which is taken 1.15 (in $\hbar\omega$ units) times the value of the mean distance between the major shells of the light fragment, while

$$\mathcal{F}(x) = \frac{1}{\pi^{1/2}} \exp(-x^2) \sum_{k=0,2}^{2m} a_k H_k(x^2)$$

is the well known Strutinsky smoothing function with Hermite polynomials. The single-particle levels are normalized with $\hbar\omega$. As remarked in Ref. [27] for asymmetric fission processes, the shell effects are determined more accurately if $\hbar\omega$ has the value associated with the mass of the light fragment. So, it is considered that $\hbar\omega$ varies linearly from $\hbar\omega_0 = 41/A_0^{1/3}$ to $\hbar\omega_2 = 41/A_2^{1/3}$ at scission, in a way similar to that mentioned for the charge asymmetry. The sum over ν means for all the levels with energies below the last occupied level. The smooth value of the Fermi energy $\tilde{\lambda}$ is obtained as usual from the condition of number conservation. If the parent possess an odd number of protons (Z) or neutrons (N), the procedure is the

same, taking care that in the sum (15) and in the particle number conservation equation, to consider the last level non-degenerate. If the nucleus is excited, the pair initially located in the last occupied level ϵ_0 is broken and two unpaired nucleons reach the levels ϵ_1 and ϵ_2 . Consequently, a correction $C = |\epsilon_1 - \epsilon_0| + |\epsilon_2 - \epsilon_0|$ is added to the shell effects.

The pairing correction is for [protons – label p and for neutrons – label n]:

$$\delta P_{p(n)} = P - \frac{1}{2} g(\tilde{\lambda}) \tilde{\Delta}^2 \quad (17)$$

where P is the pairing value obtained directly from the level scheme while $\tilde{\Delta} = 12/A^{1/2}$ is the smoothed distribution gap parameter. Δ and λ are deduced from the BCS equations on the form:

$$\sum_{\nu=n-n_c}^{n+n_c} \frac{1}{[(\epsilon_{\nu} - \lambda)^2 + \Delta^2]^{1/2}} = 2\tilde{g}(\tilde{\lambda}) \ln\left(\frac{2\Omega}{\tilde{\Delta}}\right) \quad (18)$$

$$Z(N) = \sum_{\nu} 1 - \frac{\epsilon_{\nu} - \lambda}{[(\epsilon_{\nu} - \lambda)^2 + \Delta^2]^{1/2}} \quad (19)$$

and Ω is an energetic interval of the order of $\tilde{\gamma}$, $2\Omega = 2n_c/\tilde{g}(\tilde{\lambda})$ is the number of states taken into account. The phenomenological gap parameter $\tilde{\Delta}$ was forced to vary linearly from $12/A_0^{1/2}$ up to $12/(A_1^{1/4} A_2^{1/4})$ as considered for the charge asymmetry. For odd filled level we appealed to the blocking effect in computing this correction.

The deformation energy

$$E_d = E_{\text{LDM}} + \delta V_p + \delta V_n + \delta P_p + \delta P_n \quad (20)$$

in MeV is represented in Fig. 3 for the symmetric fission of ^{238}U . In this representation, the energy reference was modified such that $E_d=0$ in the ground-state position.

This plot evidences a very pronounced minimum which belongs to the ground state, marked with a point. A second minimum, at 4 MeV, is located at a more elongated shape and smaller values of C .

2.4 Excitation energy

The excitation energy brought by the incident neutron modifies the potential barrier and the ground state energy. It is considered that a preformation of the fission channel occurs when the target nucleus receives energy. For example, in the case of protons, the excitation energy can broke a pair of protons and the atomic numbers of the nascent fragments Z_1 and Z_2 become odds. In this early stage, two single particle levels, ϵ_H and ϵ_L belonging to the heavy and light fragments will be occupied by unpaired protons. As briefly mentioned above, the deformation energy for odd Z_1 and Z_2 must be corrected and must be now determined using the relation $\delta V_p^{\text{odd}} = \delta V_p + |\epsilon_H - \epsilon_0| + |\epsilon_L - \epsilon_0|$. It must be pointed out that the pairing correction takes into account the blocking of the levels ϵ_H and ϵ_L . The excitation energy will be $E^* = E_c^* - |\epsilon_H - \epsilon_0| - |\epsilon_L - \epsilon_0|$ where E_c^* is the initial energy brought by the neutron. In this particular case, if $E^* < 0$, the barrier is increased with $|E^*|$.

We use the Franck–Condon principle in a sense similar to that presented in Ref. [28] where it is evidenced that in an ideal collective model, the excitation process takes place in such a way that the positions and velocities coordinates of the heavy part of the system remains unchanged. For a fixed point in the configuration space

of the deformation coordinates, associated to the ground-state of the system, we assume the system shifts from one potential to another. In our interpretation, this principle is equivalent to say that the "damping", as one way to speak of the exchange between vibration and nucleonic excitation, is produced in a very short time at the beginning of the process. So, this model is based on a sudden dissipation process.

The excitation energy will damp the shell effects. For simplicity, we assume an exponential decay [29]

$$U_{\text{tot}} = E_{\text{LDM}} + \delta U(T=0) \exp(-T^2/T_0^2) \quad (21)$$

where E_{LDM} is the liquid drop potential along the trajectory path and $\delta U = \delta V_p + \delta V_n + \delta P_p + \delta P_n$ is the sum of shell and pairing corrections (for both neutron and proton scheme). The nuclear temperature T is connected with the excitation energy [30]

$$E^* = \frac{1}{9} A_0 T^2 - T \quad (22)$$

where the value $T_0=1.5$ MeV allows the shell corrections to vanish if $E^* > 60$ MeV. So, when the nucleus is excited, according to the Franck–Condon effect, the system will skip from ground state potential energy to another, characterized by a fixed nuclear temperature. The deformation energy at a temperature T will be now $E_d(T) = U_{\text{tot}}(T) - U_{\text{tot}}^{\text{gs}}(T)$ where $U_{\text{tot}}^{\text{gs}}(T)$ is the deformation energy at the same temperature T in the ground-state point of the configuration space at $T=0$.

It must be mentioned that the STCSM only simulates the variation of the single particle levels towards swollen nuclear shapes in the central region, that is, for small val-

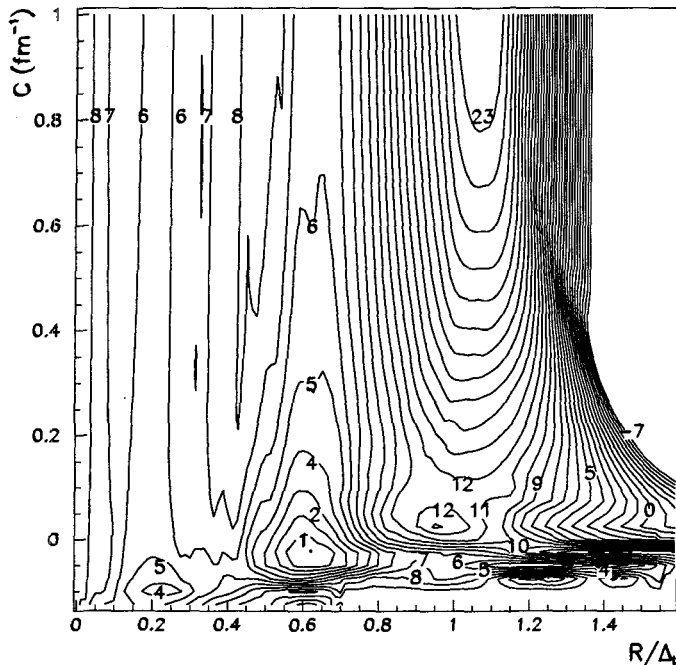


Fig. 3. The ^{238}U deformation energy E_d in MeV for symmetric nuclear shapes. The step value between the equipotential lines is 1 MeV. Some values of the energies are marked on the plot. The ground state is located by a point of coordinates $R/\Delta_t=0.62$ and $C=-0.022 \text{ fm}^{-1}$.

ues of C . In other words, the model is not adequate to obtain good positions of the single-particle energies in the region of small values of C because the spin-orbit and L^2 terms are constructed for a two intersected spheres parametrization. For example, the definition of the angular momentum \mathbf{L} operator is $\mathbf{L} = \nabla V \times \mathbf{p}$ where V is the deformed single particle potential. The STCSM uses the two center potential of two intersected spheres to simulate V in the definition of \mathbf{L} . So, the experimental values of the heights of the humped barrier cannot be reproduced exactly.

2.5 Tensor of inertia

In a n -dimensional deformation space, where the nuclear shape is described by a set of independent coordinates $\{q_i\}$, the inertia tensor $\{B_{ij}\}$ is defined by the equation of kinetic energy E_k [31,32]:

$$E_k = \frac{1}{2} \sum_{i,j=1}^n B_{ij}(q_1, q_2, \dots, q_n) \frac{\partial q_i}{\partial t} \frac{\partial q_j}{\partial t} \quad (23)$$

There are different methods of calculation for these quantities. One of the most used method to obtain the effective mass is the Werner-Wheeler approximation. It is assumed

that this tensor is a measure of the collective motion of the nuclear matter. The flow of the fluid is idealized as non rotational, nonviscous and hydrodynamical. To obtain the inertia $B(s)$ along the trajectory, the multidimensional problem can be reduced to a single-dimensional one:

$$B(s) = \sum_{i,j=1}^n B_{ij}(q_1, q_2, \dots, q_n) \frac{\partial q_i}{\partial s} \frac{\partial q_j}{\partial s} \quad (24)$$

for a path given parametrically by the equations $q_i = q_i(s)$. s is generally an independent, arbitrary parameter, but could be chosen as one of the independent generalized coordinates, in particular R , like in our case.

In the case of symmetric decays, the shapes are ruled by two degrees of freedom: the elongation R and the necking C . The variations of the diagonal elements B_{RR} and B_{CC} , together with those of the off-diagonal $B_{RC} = B_{CR}$ term are plotted in Figs. 4,5 and 6. It is interesting to note that, while the lower values of the deformation energy E_d can be found in the region characterized by small values of C , the effective masses increase in this region of the configuration space. Very large values can be obtained in the

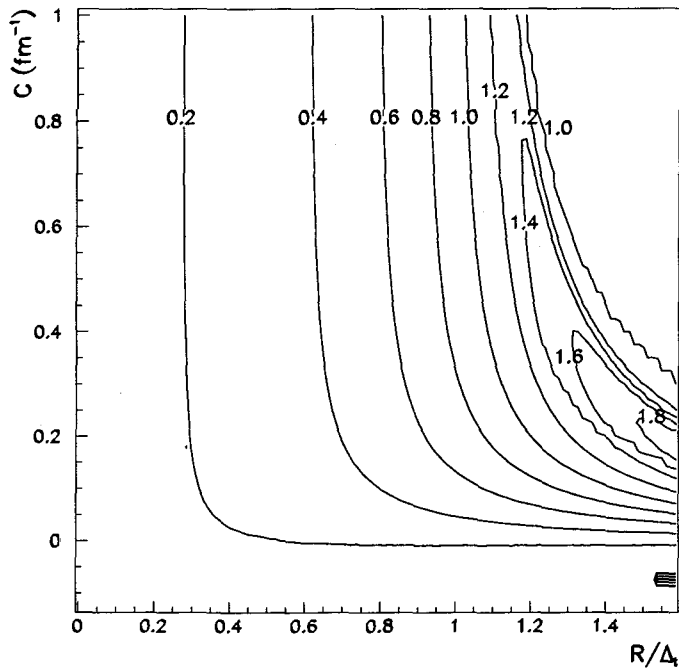


Fig. 4. The ^{238}U B_{RR}/μ diagonal element of the inertia tensor for symmetric nuclear shapes. $\mu = A_1 A_2 / A_0$ is the reduced mass of the system. The difference between two contour level values is 0.2.

scission region. After the rupture, $B_{RR} = \mu = A_1 A_2 / A_0$, i.e., the reduced mass of the system and $B_{CC} = B_{RC} = 0$.

2.6 Action integral minimization

It is not practical possible to compute the least action trajectories for all fragment partitions ($A_1 Z_1, A_2 Z_2$) and for all excitation energies E^* . Therefore, the least action trajectory was determined for the symmetric cold fission process of the parent ^{238}U and it was considered that the parametrical form obtained for this path represent a good approximation for the other channels.

In the case of symmetric decays, the shapes are ruled by two independent degrees of freedom: the elongation and the necking. The action integral is minimized in a bidimen-

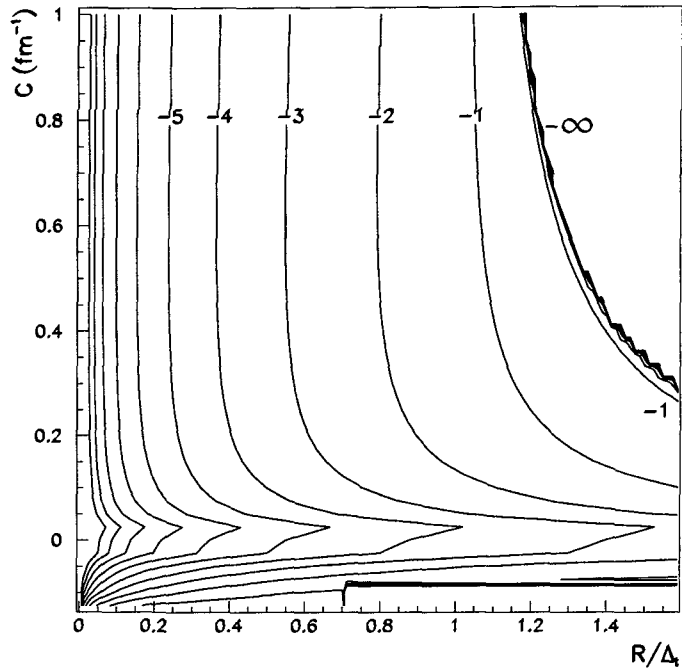


Fig. 5. Same as Fig. 4 for the $\log_{10}(B_{CC}/\mu)$ diagonal element of the inertia tensor. The dimension of B_{CC}/μ is fm^{-4} . $\mu = A_1 A_2 / A_0$ is the reduced mass of the system. The difference between two contour level values is 1.

sional configuration space. The first turning point R_i of the action integral denotes the ground state of the parent. The second turning point R_f must be on the equipotential line of the exit barrier deformation energy surface fulfilling the condition $E_d(C, R_f) = E_d(C, R_i) = \text{constant}$. This condition determines a dependence $R_f = f(C)$. For each R_f a local minimum of the action integral can be found. Using a simple numerical method [33,34] the path, associated to the local minimum, followed by the nuclear system can be parametrized. The action integral functional is approximated with Gauss-Legendre quadrature, that is, a function which depends on a set variables $\{C_j\}$ giving the values of the necking-in for different values of the elonga-

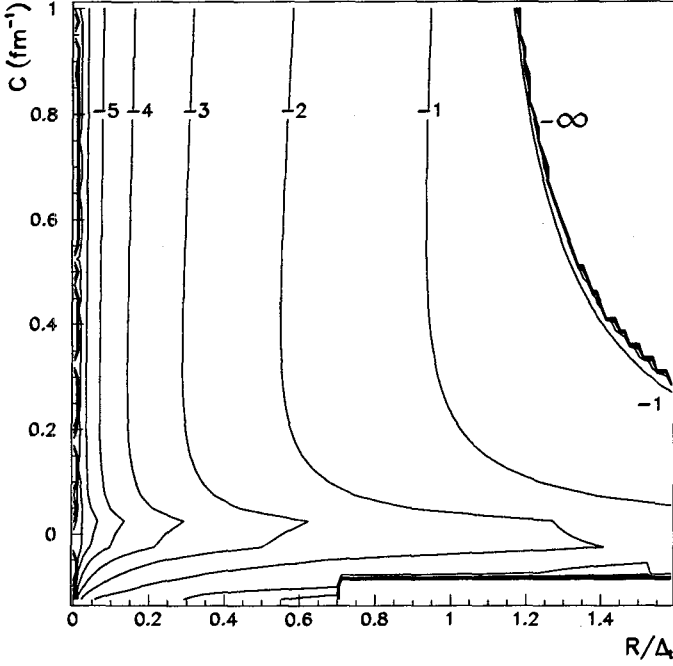


Fig. 6. Same as Fig. 4 for the $\log_{10}(B_{RC}/\mu)$ off-diagonal element of the inertia tensor. The dimension of B_{RC}/μ is fm^{-2} . $\mu = A_1 A_2 / A_0$ is the reduced mass of the system. The difference between two contour level values is 1.

tions $\{R_j\}$, $R_j \subset (R_i, R_f)$:

$$\int_{R_i}^{R_f} \sqrt{2B\left(R, C, \frac{\partial C}{\partial R}\right) E(R, C)} dR \approx \sum_{j=1}^{N_g} w_j \sqrt{2B\left(R, C_j, \frac{\partial C}{\partial R}\bigg|_{R=R_j}\right) E(R_j, C_j)} (R_f - R_i) \quad (25)$$

where

$$B\left(R, C, \frac{\partial C}{\partial R}\right) = B_{RR} + B_{CC} \left(\frac{\partial C}{\partial R}\right)^2 + 2B_{RC} \frac{\partial C}{\partial R} \quad (26)$$

is the inertia along the trajectory. Here, w_j represent the Gauss-Legendre weights, R_j are the mesh points of the quadrature and, in the present calculations, $N_g = 32$. So, the action integral which was a functional of the path $C(R)$ in the configuration space, depends now on an even number of variables C_j fixed in the mesh points. The derivatives $\frac{\partial C}{\partial R}\bigg|_{R=R_j}$ were obtained using spline interpolations. The global minimum can be found by evaluating

the local minimum for all the possible exit point of the barrier R_f . The global minimum path evaluated numerically in this manner is presented in Fig. 2. As mentioned, because the evaluation of the trajectories for all possible partitions and all the excitation energies is not reasonable in practice, this parametrization of the path will be extrapolated for all mass-asymmetries and any excitations. For asymmetric fission processes, a linear variation of the asymmetry parameter η from 0 (in the ground state of the parent) and the final $\frac{A_1 - A_2}{A_0}$ (when the scission is reached) is taken into account. This kind of treatment is partially supported by the affirmation found in Refs. [35,36] that the trajectories for spontaneous and induced fission are identical or at least similar.

2.7 Fragment yields

The fission path being known, it is possible to compute the penetrabilities for each fission fragment partition $(A_1 Z_1, A_2 Z_2)$.

The following simple formula is used for the penetrabilities

$$P(A_1, Z_1, A_2, Z_2) = \exp\left\{-0.43921\sqrt{\mu} \int_{R_i}^{R_f} \sqrt{E_d - E_v} dR\right\} \quad (27)$$

where, to avoid cumbersome calculations, the effective mass was considered approximatively equal to the reduced mass.

Having in mind that the probability to obtain a partition is proportional to its penetrability, it is possible to obtain the yields y normalized to 200% of the fission fragments:

$$y(A_1, Z_1, A_2, Z_2) = \frac{P(A_1, Z_1, A_2, Z_2)}{\sum_{N_Z=N_{Z1}}^{N_{Z2}} \sum_{N_A=N_{A1}(N_Z)}^{N_{A2}(N_Z)} P(A_0 - N_A, Z_0 - N_Z, N_A, N_Z)} \times 100 \quad (28)$$

where $Z_0=92$ and $A_0=238$ or 239 . In this application, the penetrabilities were calculated only for the fission products ranging between $N_{Z1}=34$, $N_{Z2}=48$, $N_{A1} \approx N_Z$, $N_{A2} \approx 2N_Z$ which gives reasonable values for the penetrabilities.

Up to now, the isotopic distribution of the fragments was obtained as a function of the excitation energy E^* of the parent. The energy of the compound nucleus modifies the potential barrier, that is, the penetrabilities, and the number of prompt neutrons emitted. These two effects modify the distributions of the final products essentially.

2.8 Neutron evaporation

The fission is accompanied by an almost instantaneous emission of neutrons, which are of decisive importance in calculating the final mass yields. The average number of neutrons emitted per fission is called $\bar{\nu}$. The dependence between the number of emitted neutrons and the excitation energy of the compound nucleus was taken as [37]:

$$\Delta(\bar{\nu}(E^*)) = \frac{1}{8.5} E^* \quad (29)$$

It is also considered that the energy supplied by the incident neutron is shared between the fragments proportionally to their mass. The fragment mass dependence of $\bar{\nu}$ is obtained from the universal neutron yield curves [38]

$$\bar{\nu} \approx 0.08(A_L - 82) + 0.10(A_H - 126) \quad (30)$$

that is

$$\bar{\nu}_H \approx 0.10(A_H - 126) \quad (31)$$

$$\bar{\nu}_L \approx 0.08(A_L - 82) \quad (32)$$

where $\bar{\nu}_H$ and $\bar{\nu}_L$ refer to the evaporated neutrons from the heavy and light fragments, respectively. Here, $A_1 = A_H$ and $A_2 = A_L$. The distributions of $\bar{\nu}_H$ and $\bar{\nu}_L$ are Gaussian with $\sigma^2 \approx 0.6$. Such a curve rises linearly with mass above $A=82$ and above 128. In the central range $118 \leq A \leq 128$, the values of ν were interpolated linearly. The fission product yields $Y(A'_1, Z'_1)$ and $Y(A'_2, Z'_2)$ normalized to 200% can be obtained by a folding procedure:

$$Y(A'_H, Z_1) = \frac{\sum_{A_1=A'_H}^{A'_H+2\bar{\nu}_H(A_1)} y(A_1 Z_1, A_2 Z_2) \times \exp\left[-\left(\frac{\bar{\nu}_H - |A_1 - A'_H|}{\sigma}\right)^2\right]}{\sum_{A''_H=A_1-2\bar{\nu}_H(A_1)}^{A_1} \exp\left[-\left(\frac{\bar{\nu}_H - |A_1 - A''_H|}{\sigma}\right)^2\right]} \quad (33)$$

$$Y(A'_L, Z_2) = \frac{\sum_{A_2=A'_L}^{A'_L+2\bar{\nu}_L(A_2)} y(A_1 Z_1, A_2 Z_2) \times \exp\left[-\left(\frac{\bar{\nu}_L - |A_2 - A'_L|}{\sigma}\right)^2\right]}{\sum_{A''_L=A_2-2\bar{\nu}_L(A_2)}^{A_2} \exp\left[-\left(\frac{\bar{\nu}_L - |A_2 - A''_L|}{\sigma}\right)^2\right]} \quad (34)$$

The fission product yields obtained in this way depend strongly on the parent excitation energy deposited by the incident neutron.

2.9 Distributions of fission products

In Fig. 7 are displayed the isotopic and the mass distributions of the ^{238}U fission for $E^*=3$ MeV (top), $E^*=50$ MeV (middle) and $E^*=200$ MeV (bottom). The gross features of the distributions are reproduced. Mass yields observed in low-energy particle induce fission of actinides are generally characterized by double humped curves showing the preference for asymmetric mass division. The peak to valley ratio reaches several order in magnitude. In this energy region, the distribution evidences that products with mass 100–110 and 130–140 are widely produced while the symmetric fission process has a lower probability. Increasing

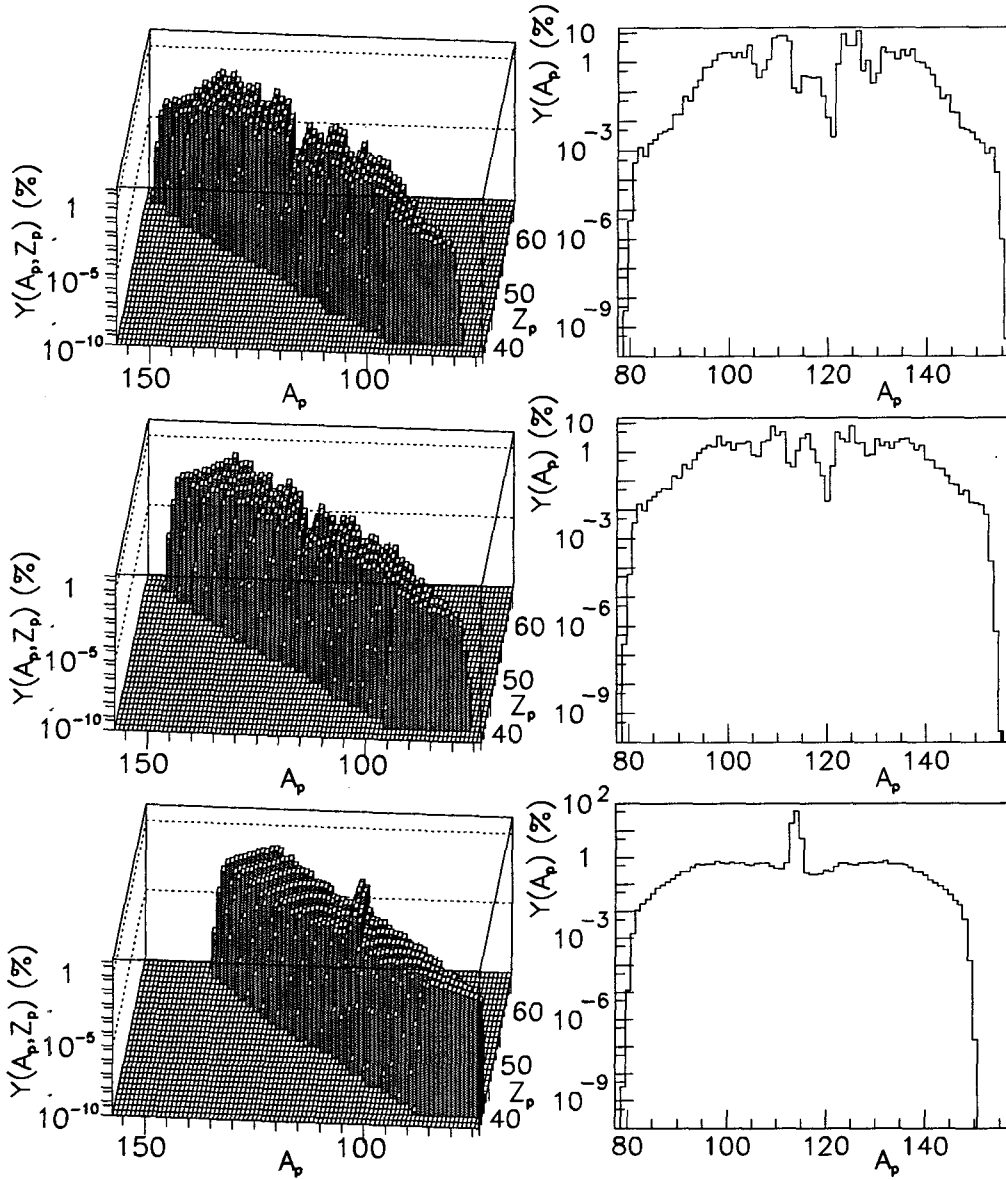


Fig. 7. Isotopic and mass products distributions for three values of the excitation energy (3 MeV top, 50 MeV middle and 200 MeV bottom) for the ^{238}U induced fission normalized to 200%.

the excitation energy, the isotopic distribution becomes broader and the yields towards symmetric fission are enhanced. Also, the maximal values of the yields are shifted towards the stability line, mainly due to the neutron evaporation. The distributions of the ^{239}U fission are displayed in Fig. 8.

3 Fission products yields

In the following, fission product yields obtained in the target are determined as a function of the initial deuteron energy for a specific converter–target geometry. In this section, E_d means the incident deuteron energy and E_n the neutron energy.

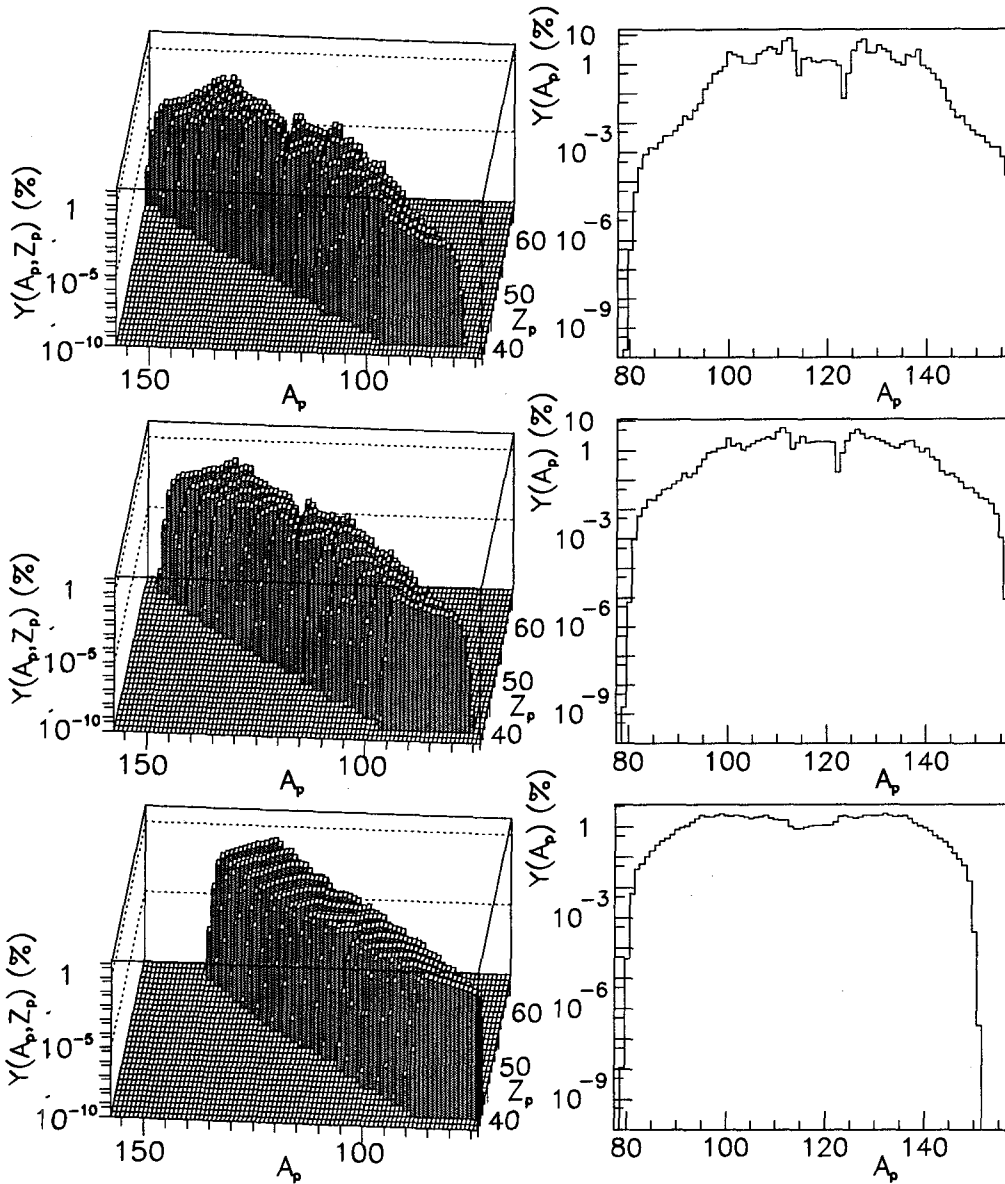


Fig. 8. Isotopic and mass products distributions for three values of the excitation energy (3 MeV top, 50 MeV middle and 200 MeV bottom) for the ^{239}U induced fission normalized to 200%.

3.1 Neutron distributions

The neutron angular $P(\theta, E_d)$ and energy distributions $P(\theta, E_n, E_d)$ produced in the converter depend strongly on the initial deuteron energy E_d . Increasing the deuteron energy, the neutron flux in the forward direction increases rapidly up to $E_d \approx 100$ MeV. From $E_d = 100$ MeV, a saturation effect is visible. This effect rules an increase of the

neutron yields at almost all energies, including the range [0-20] MeV where the neutron-rich isotopes produced by fission can survive.

Angular and energy distributions of neutrons obtained by bombarding thick targets with incident deuterons up to 200 MeV are reported [39–41], scanning a large energy interval where quantitative data were not available. A sim-

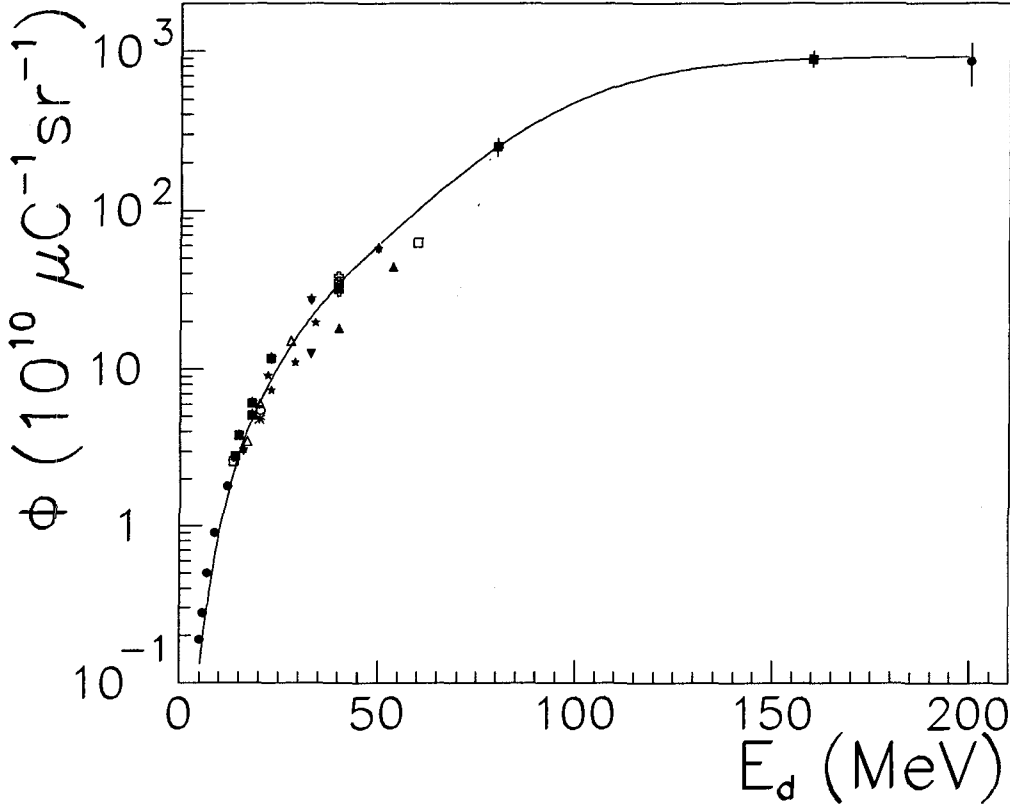


Fig. 9. A systematic of total yields Φ at 0° versus the incident energy of the deuteron E_d for the bombardment of Be is obtained by plotting experimental results given in different references: filled circles from Ref. [44] (5, 6, 7, 9, 12, 18 and 23 MeV), filled squares from Ref. [45] (14, 14.8, 18, 23 and 40 MeV), up-point filled triangles deduced from Ref. [46] (40 and 53.8 MeV), down-point filled triangle from Ref. [47] (12.5 MeV), empty circle from Ref. [48] (20.2 MeV), empty square from Ref. [43] (13.54 MeV), up-point empty triangles from [40] (17, 20 and 28 MeV), empty rhombs from Ref. [49] (16, 33 and 50 MeV), empty crosses from Ref. [50] (4 points at 40 MeV), filled stars from the compilation of Ref. [51] (14, 16, 18, 22, 23, 29, 33, 34, 40 and 50 MeV), empty stars from Ref. [52] (14.8, 18 and 23 MeV), asterisk from Ref. [53] (20 MeV), filled circle at $E_d=200$ MeV from Ref. [41], empty square at 60 MeV deduced from Ref. [54], filled squares at 80 and 160 MeV from Ref. [39]. In the low energy deuteron energy the plot looks like with the curve drawn in Ref. [55]. The curve is obtained with the semi-empirical parametrization given by Rel. (35).

ple phenomenological formalism intended to simulate the distributions in the forward direction was developed and presented in detail in Ref. [40] by extending the formalism based on Serber's theory [42]. The experimental results compared with phenomenologic simulations proved to be in a good agreement. Following the Serber's prescriptions,

two distributions can be modeled semi-classically in the case of thin targets: one distribution is due to the stripping reaction and another is due to direct collisions between nucleons, resulting in the escape of some particles from the target nucleus. For an initial deuteron energy, these distributions determine the neutron probability $P(\theta, E_n, E_d)$

to be emitted at an angle θ at an energy E_n . In the case of thick targets, this probability must be integrated over the deuteron range weighted with the cross section (d, n). Theoretical angular and energy neutron yields followed by renormalization of the integrated probability with the experimental value of the total yield at 0° . This formalism needs, first of all, an estimation of the neutron yields at 0° for thick Be targets as a function of the deuterons incident energies. This dependence gives the approximate productivities for other kinds of targets by using simple geometric relations involving atomic and mass numbers of the concerned elements. Semi-empirical formulae for this quantity are given in the Ref. [43] (for incident energies smaller than 15 MeV) and Ref. [44] (over 15 MeV), but their validity becomes uncertain for deuteron energies greater than 50 MeV due to the lack of experimental data over this value. Using the experimental yields obtained in Ref. [39–41], together with a systematic of experimental data, a fit of the data has been made and the following formula is proposed for the yields at 0° up to 200 MeV deuteron incident energy on thick Be targets:

$$\Phi(\theta = 0^\circ) = \begin{cases} 10.1 \times 10^{12} E_d^{2.95} & \text{for } E_d < 15 \text{ MeV,} \\ 3.4 \times 10^{13} E_d^{2.5} & \text{for } 15 \text{ MeV} \leq E_d \leq 50 \text{ MeV,} \\ \left\{ \frac{928.3}{1 + \exp[0.053(98 - E_d)]} - 6.89 \right\} \times 10^{16} & \text{for } 50 \text{ MeV} < E_d, \end{cases} \quad (35)$$

where Φ is the number of neutrons over the incident charge unit in $\text{sr}^{-1}\text{C}^{-1}$ and the incident energy of the deuteron E_d is given in MeV. The experimental systematic and the empirical parametrization are displayed in Fig. 9.

Our estimations require the cross section for neutron production in order to weight along the target thickness. One choice for the dependence of the forward direction cross section in the Be target is given by the formula [45]

$$\left. \frac{d\sigma(E)}{d\Omega} \right|_{\theta = 0^\circ} = 0.18 \ln(E) + 0.007E \quad (36)$$

and another proposed dependence [40] as

$$\left. \frac{d\sigma(E)}{d\Omega} \right|_{\theta = 0^\circ} = 2. \times 10^{-4} E^{1.5} \left| \frac{dE}{dt} \right| \quad (37)$$

where dE/dt is the stopping power in $\text{MeV}/(\text{g}/\text{cm}^2)$, E is the deuteron energy in MeV for the range t in the target, and the cross section is expressed in b/sr. Both approximations were used in calculations up to 40–50 MeV deuteron energies. In the following, the dependences exhibited by these formulae were extrapolated for all the energy domain concerned with the imposed restriction that when σ reaches the value 1.35 b, this value being regarded as an upper limit [54], σ remains constant. In this context, it was considered that at the highest energies, the major part of the cross section is concentrated in the forward direction. The theoretical simulations for incident deuteron at energies of 80 and 160 MeV obtained by using these two parametrizations [Rels. (36) and (37)] were almost identical. Also, as usually done for thick targets, an attenuation of the initial beam was taken into account by multiplying the cross section with a factor directed by a law of the type $\exp(-\int N \sigma dt)$, where $\int N dt$ is the number of nuclei on the surface unit. Another modification of the formalism of Ref. [40] is that the straggling angle is computed by extrapolating the table of Ref. [56] as a function of the

deuteron position in the target while in our previous work [40] the mean value over the range of the deuteron was considered.

3.2 Number of fission events

The dimensions of the ^{238}U fissionable target are small, so that it is possible to neglect the attenuation of the neutrons in the uranium. The geometry illustrated in Fig. 10 will be considered in the following because it is the most often geometry used in practice. The set-up consists of two cylindrical pieces: the converter and the fissionable target positioned horizontally along the deuteron beam direction. Usually, the target is made from uranium carbide UC_x . The converter delivers the neutrons obtained mainly by break-up processes as mentioned previously. Using the above theoretical distributions $P(\theta, E_n, E_d)$ and $P(\theta, E_d)$, a mean range of the neutrons emitted from a slice of the converter in the fissionable target can be calculated:

$$R_m = \frac{4 \int_0^{R_c} \int_0^{\sqrt{R_c^2 - y^2}} d_m(x, y) dx dy}{\pi R_c^2} \quad (38)$$

where

$$d_m(x, y) = \frac{\int_0^{2\pi} [\int_0^{\pi/2} P(\theta, E_d) d(\theta, \varphi) \sin(\theta) d\theta] d\varphi}{2\pi \int_0^{\pi/2} \sin(\theta) P(\theta, E_d) d\theta} \quad (39)$$

is the mean range of the neutron in ^{238}U emitted in a point (x, y) in the converter and $d(\theta, \varphi)$ is the range of the neutron emitted at the angles θ and φ . R_c represents the radius of the converter, considered equal to that of the incident deuteron beam. The values of $d(\theta, \varphi)$ are computed using geometrical relations. It was considered easier to replace the variables θ and φ with θ_x and θ_y fulfilling the

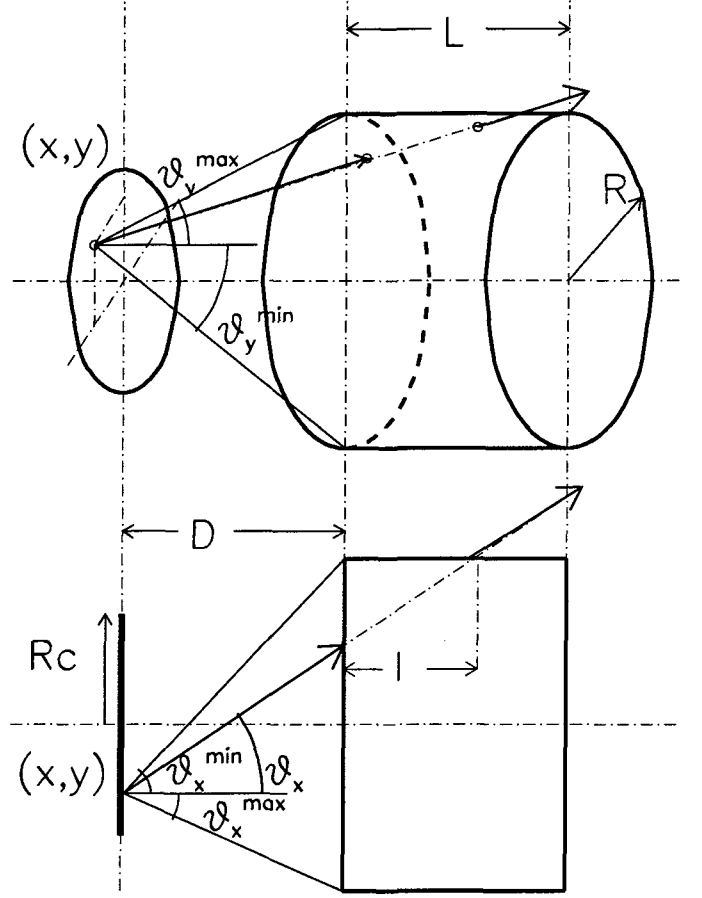


Fig. 10. The experimental schematic set-up. Top: perspective view and bottom: section along the symmetry axis. (x, y) represents a point in one slice of the converter where the deuteron is broken and a neutron emerges at angles θ_x and θ_y . The maximal values of these angles allowing that the neutron hits the target are also represented. D is the distance between the slice of the converter and the entry edge of the target, L is the thickness of the cylindrical target, R is the radius of the target, R_c is the radius of the converter (which equals the radius of the beam).

condition of the same angular domain:

$$\int_0^{2\pi} [\int_0^{\pi/2} P(\theta, E_d) d(\theta, \varphi) \sin(\theta) d\theta] d\varphi = \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} P(\theta[\theta_x, \theta_y], E_d) d(\theta_x, \theta_y) \sin(\theta[\theta_x, \theta_y]) \times \mathcal{D} \left(\frac{\theta, \varphi}{\theta_x, \theta_y} \right) d\theta_x d\theta_y \quad (40)$$

where

$$\theta(\theta_x, \theta_y) = \arctan \left\{ [\tan^2(\theta_x) + \tan^2(\theta_y)]^{1/2} \right\} \quad (41)$$

$$\varphi(\theta_x, \theta_y) = \arctan \left(\frac{\tan(\theta_x)}{\tan(\theta_y)} \right) \quad (42)$$

and the Jacobi determinant is

$$\mathcal{D} \left(\begin{matrix} \theta, \varphi \\ \theta_x, \theta_y \end{matrix} \right) = \left| \frac{d\theta}{d\theta_x} \frac{d\varphi}{d\theta_y} - \frac{d\theta}{d\theta_y} \frac{d\varphi}{d\theta_x} \right| \quad (43)$$

The significance of θ_x and θ_y can be found inspecting the Fig. 10. In these calculations, it is considered that the neutrons are emitted in a weighted position in the converter

$$R_p = \frac{\int_0^T \sigma[E(t)] t dt}{\int_0^T \sigma[E(t)] dt} \quad (44)$$

where $\sigma[E(t)]$ represents the cross section for neutron production at the depth t in the converter and T is the thickness of the converter.

Now, the total number of fission events can be approximately estimated by charge unit of incident deuteron.

$$N_f(E_d) = \frac{R_m N_t \rho_U}{M_U} \frac{\int_0^{E_d} \sigma_f(E) D(E, E_d) dE}{\int_0^{E_d} D(E, E_d) dE} \quad (45)$$

where

$$D(E_n, E_d) = 2\pi \int_0^{\pi/2} P(\theta, E_n, E_d) \sin \theta d\theta, \quad (46)$$

$\sigma_f(E)$ is the fission cross section, ρ_U is the U density, M_U is the U mass, $D(E, E_d)$ is the neutron energetic distribution. The total number N_t of neutrons emitted by the converter is

$$N_t = 2\pi \int_0^{\pi/2} P(\theta, E_d) \sin(\theta) d\theta \quad (47)$$

For a given geometry, these formulae allow us to estimate the number of fission events in the target. If the cross section is multiplied with the products distributions normalized to 200% obtained at each neutron energy, we are able

to determine the isotopic number of elements produced for an incident deuteron energy E_d .

4 Results and discussion

First of all, a simple estimation of the deuteron energy required to produce the largest number of unstable products can be easily realised. If the energy of the neutron is too high, the fragment emits neutrons, so that it will be shifted towards the stability line. It was shown in Refs. [1, 7], that for neutron energies larger than 20 MeV, the formed neutron rich fragments are too excited to survive. Including the following approximation in the numeric code

$$\sigma(E) = \left[1 - \mathcal{H} \left(\frac{E - E_0}{E_0} \right) \right] \sigma(E) \quad (48)$$

the number of fragments produced the fission induced by neutrons with incident kinetic energies lower than 20 MeV can be obtained. Here, $\mathcal{H}(E_0)$ is the Heaviside function with the properties that $\mathcal{H}(x) = 0$ when $x < 0$ and $\mathcal{H}(x) = 1$ when $x \geq 0$, and $E_0 = 20$ MeV. That means, only fission fragments obtained at energies smaller than 20 MeV neutrons are retained. This procedure enable us to extract an approximate value of the best deuteron energy.

In the following estimations, the thickness of the converter was always choosed to be equal with the range of the ^2H at each energy. The radius of the ^{238}U cylinder was 7 mm, its thickness is 60 mm and it was considered to be built from UC_x . The distance between the exit edge of the converter and the entrance face of the ^{238}U target

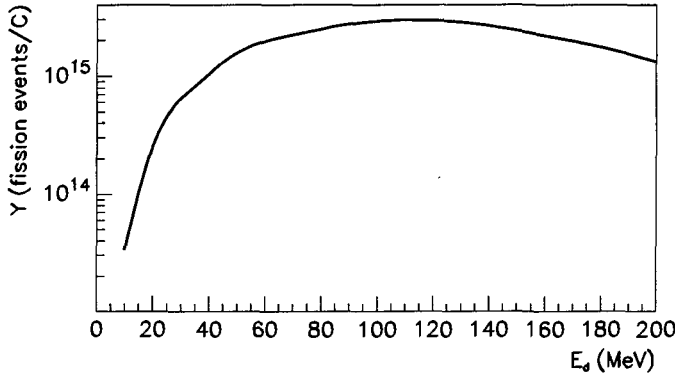


Fig. 11. The number of fission events Y able to produce neutron rich nuclear fragments per incident charge of deuteron as a function of the deuteron energies E_d .

corresponds to 10 mm. The converter was considered to be built from C. The deuteron beam radius equals 1 mm.

In Fig. 11, the number of fission events able to produce neutron rich fragments per charge of incident deuterons is plotted as a function of the deuteron energy. It can be observed that the maximum number of interesting fission events is produced when the ^2H have an energy close to 100 MeV. In the same time, the yield does not increase very much in the interval 60 MeV to 100 MeV, and it will be more convenient to construct an accelerator for ^2H at 60 MeV, due to the costs of such a facility.

In the following, in connection with the fixed converter-target geometry, the number of fission products produced in the fission target per deuteron incident charge will be determined for neutron rich isotopes of Se up to Ce. First, an estimation of the excitation energy of the uranium must be realised.

The excitation energy is estimated crudely in connection with the incident neutron energy. If the compound ^{239}U nucleus is formed, the excitation energy deposited by

the neutron is considered equal to its incident energy. This estimation does not take into account the variation of the mass defect of the ^{239}U nucleus versus that of the $^{238}\text{U}+n$ system. Having in mind that this variation is about 2 MeV, and that we are concerned by the influence of the excitation energy in a wide range of neutron energies, up to hundreds of MeV, it can be assessed that for our purpose this variation of the mass defect has little influence. In the case of ^{238}U residual nucleus, the energy of the recoil neutron was considered as approximatively 3 MeV and the excitation energy deposited in the ^{238}U of about $E_n - 3$ MeV.

The cross section curve exhibits a threshold behavior characterized by a second step that occurs at a bombarding energy of about 6–7 MeV. This is the threshold for a "second chance fission" where the ^{239}U compound nucleus can emit a neutron and the residual ^{238}U nucleus undergoes fission. Therefore, the cross fission is divided in two components: a cross section for ^{239}U with the threshold at approximatively 1 MeV and a maximum value of ≈ 0.5 b and a cross section for the ^{238}U with the threshold a 6 MeV and a maximum value of ≈ 0.5 b. The total fragment distribution is obtained by overlapping these two distributions.

The number of products $Y(A_p, Z_p)$ produced in the ^{238}U fission target by charge of incident deuterons are displayed in Fig. 12 as a function of the incident energy E_d . Only one value of the mass number A_p is selected for isotopes with half-lives near to one or several seconds. A very pronounced odd-even effect in the atomic number Z_p

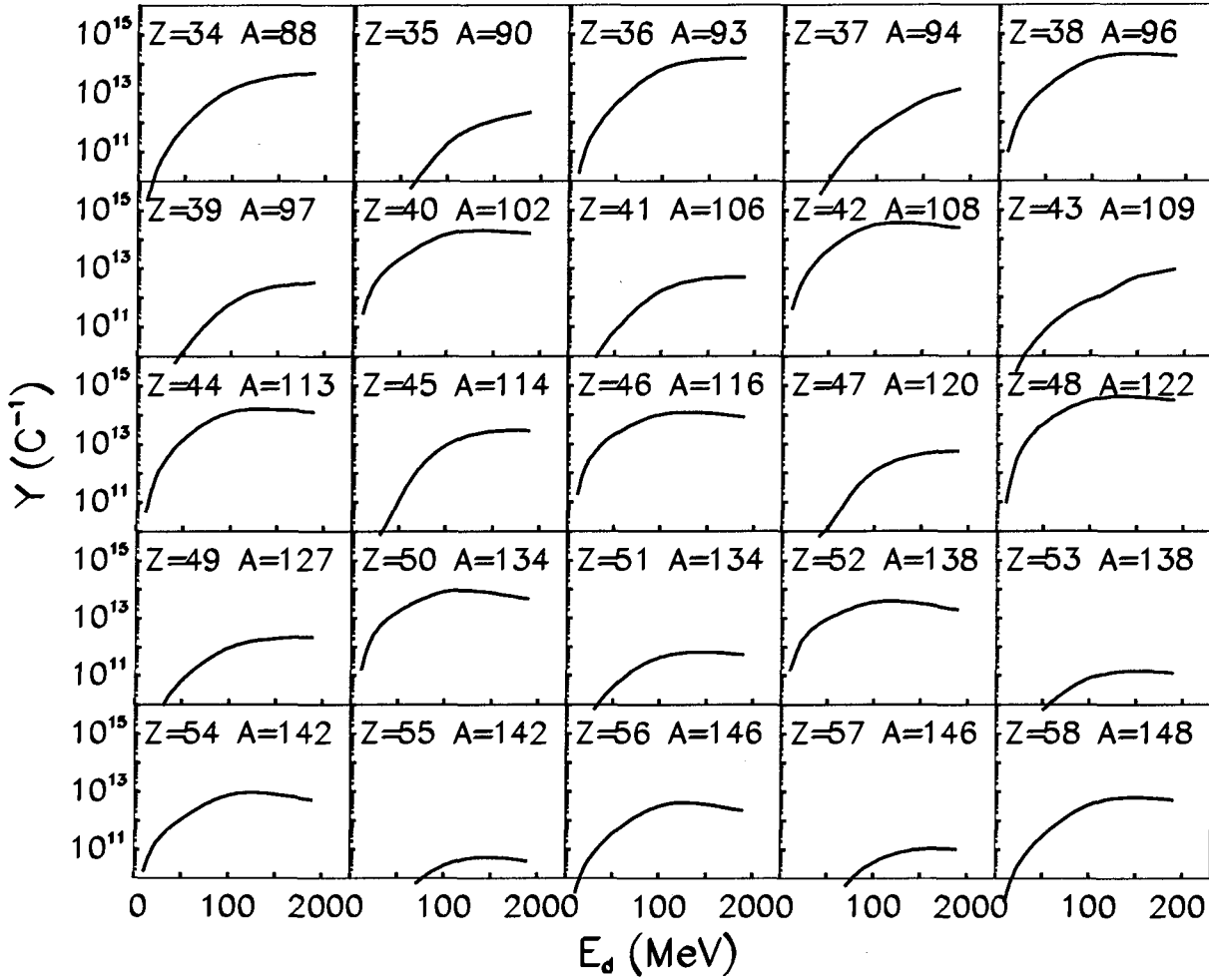


Fig. 12. The number of fission products $Y(A_p, Z_p)$ per incident charge of deuteron as a function of the deuteron energies E_d for several neutron rich isotopes.

can be remarked. First, the even fragments will be discussed. It can be evidenced that, as previously remarked, for $E_d \approx 100$ MeV, the number of neutron rich nuclei supplied by the facility reaches values close to the maximal ones. In the case of light elements, in the vicinity of $E_d=100$ MeV, the slope of the curves decreases dramatically and a saturation behavior begins to be evidenced for larger values of the incident deuteron energy. Concerning the heavy products, it is interesting to note that even their yields begin to decrease slowly for E_d energies above 100 MeV. In almost all cases, the increase of the neutron rich

product yields from 60 to 100 MeV is lower than one order of magnitude. In the case of odd Z_p , the maximal values of the yields are attained for higher E_d than for even Z_p . In this case, the increase of the potential barrier due to single-particle effects is greater and these microscopical effects vanish at larger excitation energy of the parent.

In the frame of the PARRNe project, the production of noble gases was investigated experimentally using a UC_x cylindrical fissionable target containing about 30 g uranium. Experiments were realised at 80 and 130 MeV deuteron energies using the AGOR cyclotron at KVI Grö-

ningen [57]. The noble gases were transported through a transfer line in order to be collected on a cold finger. The resulting activity was measured under good background conditions by γ -spectroscopy. Comparisons with the theory will be realised under the assumption that the number of isotopes detected is proportional to the number of fission products (of the same isotope) produced in the fission target. Therefore, the ratios between the experimental yields measured at $E_d=130$ MeV and 80 MeV were computed. We obtained 1.91, 1.5, 1.3 and 1.53 for ^{89}Kr , ^{137}Xe , ^{138}Xe and ^{139}Xe , respectively. The theoretical values of the ratios of the yields produced in the fission target at the same energies were 5.11, 2.06, 1.82 and 2.06, for the same isotopes, respectively. Quantitatively, the calculated and the theoretical values in the case of Xe isotopes show a remarkable agreement. Qualitatively, the theory evidence that the increase of the Kr yield is more pronounced with the deuteron energy than those of the Xe isotopes. Relatively to the even Xe isotope, the increase of the odd Xe yields is more pronounced, both theoretically and experimentally. The theory, in connection with the experimental data, shows that increasing the energy above $E_d=80$ MeV, the increase of the neutron rich isotope yields is not too consistent.

These calculations suggest that without expensive facilities, a large number of isotopes far from the stability line can be obtained at a deuteron energy of 60 MeV. Our estimations take into account the angular and energy distributions of neutrons produced in the converter, the excitation of the fission nucleus which modifies the chan-

nel partition (A_1, Z_1, A_2, Z_2) probabilities and the neutron evaporation.

Acknowledgements

This work was sponsored by the European Contract SPIRAL II No. ERB 4062 PL 975009 and by IN2P3 No. FMGE CT 980100.

References

1. I.C. Gomes and J.A. Nolen, in *Proc. of the 2nd Int. Topical Meeting on Nuclear Applications of Accelerator Technology* (Gatlinburg, TN, USA, September 20–23, 1998); J.A. Nolen, private communication 1998.
2. F. Clapier, A.C. Mueller, C. Obert, O. Bajeat, M. Ducourtieux, A. Ferro, A. Horbowa, L. Kotfila, C. Lau, H. Lefort, S. Kandry-Rody, N. Pauwels, J.C. Poitier, J. Proust, J.C. Putaux, C.F. Liang, P. Paris, A.C.C. Vilari, R. Lichtenhaler, L. Maunoury, and J. Lettry, *Phys. Rev. ST Accel. Beams* **1**, 013501 (1998).
3. S. Kandry-Rody, J. Obert, E. Cottereau, O. Bajeat, M. Ducourtieux, C. Lau, H. Lefort, J.C. Poitier, J.C. Putaux, F. Clapier, J. Lettry, A.C. Mueller, N. Pauwels, J. Proust, C.F. Liang, P. Paris, H.L. Ravn, B. Roussiere, J. Sauvage, J.A. Scarpaci, F. Le Blanc, G. Lahu, I. Lhenry, T. Von Egidy and R. Antoni, *Nucl. Instr. Meth. B* **160**, 1 (2000).
4. R. Bennet, P. Van Duppen, H. Geissel, K. Heyde, B. Jonsson, O. Kester, G.-E. Korner, W. Mittig, A.C. Mueller, G. Munzenberg, H.L. Ravn, K. Riisager, G. Schrieder, A. Shotter, J.S. Vaagen and J. Vervier, NuPECC Report, 2000, (<http://www.nupecc.org>).
5. D. Ridikas, W. Mittig and A.C.C. Villari, in *Int. Conf. Shielding Aspects of Accelerators, Targets and Irradiation*

- Facilities (SATIF 5)* (Paris, 17–21 July 2000, OECD), (to be published).
6. A.C. Wahl, in *Symposium on Physics and Chemistry of Fission*, (Salzburg 22–26 March, IAEA Vienna, Vol. 1, p. 317, 1965).
 7. M. Mirea, F. Clapier, N. Pauwels and J. Proust, *Nuovo Cimento A* **111**, 267 (1998).
 8. W.J. Swiatecki and S. Bjornholm, *Phys. Rep.* **4**, 324 (1972).
 9. K. Depta, R. Hermann, J.A. Maruhn and W. Greiner: in *Dynamic of Collective Phenomena*, edited by P. David (World Scientific, Singapore, p. 29, 1987).
 10. K. Depta, J.A. Maruhn, Hou-Ji Wang, A. Săndulescu and W. Greiner, *Int. J. Mod. Phys. A* **5**, 3901 (1990).
 11. M. Mirea, D.N. Poenaru and W. Greiner, *Z. Phys. A* **349**, 39 (1994).
 12. M. Mirea, D.N. Poenaru and W. Greiner, *Nuovo Cimento A* **105**, 571 (1992).
 13. M. Brack, J. Damgaard, A. Jensen, H. Pauli, V. Strutinsky and W. Wong, *Rev. Mod. Phys.* **44**, 320 (1972).
 14. K.T.R. Davies and J.R. Nix, *Phys. Rev. C* **14**, 1977 (1976).
 15. D.N. Poenaru, M. Ivaşcu and D. Mazilu, *Comp. Phys. Comm.* **19**, 205 (1980).
 16. P. Möller and J.R. Nix, *At. Data. Nucl. Data Tables* **26**, 165 (1981).
 17. J. Maruhn and W. Greiner, *Z. Phys.* **251**, 431 (1972).
 18. M. Mirea, *Phys. Rev. C* **54**, 302 (1996).
 19. M. Mirea and F. Clapier, *Europhys. Lett.* **40**, 509 (1997).
 20. M. Mirea, *Phys. Rev. C* **57**, 2484 (1998).
 21. M. Mirea, *Eur. Phys. J. A* **4**, 335 (1999).
 22. M. Mirea, *Phys. Rev. C*, (to be published).
 23. E. Badraxe, M. Rizea and A. Săndulescu, *Rev. Roum. Phys.* **19**, 63 (1974).
 24. P. Holzer, U. Mosel and W. Greiner, *Nucl. Phys. A* **138**, 241 (1969).
 25. S.G. Nilsson, Chin Fu Tsang, A. Sobiczewski, Z. Szymanski, S. Wycech, C. Gustafson, I.-L. Lamm, P. Möler and B. Nilsson, *Nucl. Phys. A* **131**, 1 (1969).
 26. V.M. Strutinsky, *Nucl. Phys. A* **122**, 1 (1968).
 27. F.A. Ivanyuk, M.S. Ivascu and D. Poenaru, in *Particle Emission from Nuclei*, D.N. Poenaru and M.S. Ivascu Editors, (Boca Raton, Florida, vol. 1, chap. 6, p. 168, 1989).
 28. D.L. Hill and J.A. Wheeler, *Phys. Rev.* **89**, 1102 (1953).
 29. H.J. Lustig, J.A. Maruhn and W. Greiner, *J. Phys. G* **6**, L 25 (1980).
 30. R.K. Gupta and W. Greiner, in *Heavy Elements and Related New Phenomena*, W. Greiner and R.K. Gupta Editors (World Scientific, Singapore, vol. 1, chap. 14, p. 551, 1999).
 31. K.T.R. Davies, A.J. Sierk and J.R. Nix, *Phys. Rev. C* **13**, 2385 (1973).
 32. D.N. Poenaru, M. Ivascu, I. Ivascu, M. Mirea, W. Greiner, K. Depta and W. Renner, in *International Conference 50 Years with Nuclear Fission*, J.W. Behrens and A.D. Carlson Editors (American Nuclear Society, Lagrange Park, p. 617, 1989).
 33. D.N. Poenaru, M. Mirea, W. Greiner, I. Cata and D. Mazilu, *Mod. Phys. Lett. A*, **5** 2101 (1990).
 34. T. Benfoughal and M. Mirea, *J. Phys. III*, **6**, 1403 (1996).
 35. T. Ledergerber and H.C. Pauli, *Nucl. Phys. A* **207**, 1 (1973).
 36. H.C. Pauli, *Nucleonika* **20**, 601 (1975).

37. E. Segré, in *Nuclei and Particle* (W.A. Benjamin, Inc., New York, chap. 11, p. 494, 1964).
38. J. Terrel, *Phys. Rev.* **127**, 880 (1962)
39. N. Pauwels, S. Brandenburg, H. Laurent, J.P.M. Beijers, F. Clapier, L. Lebreton, M. Mirea, M.G. Saint-Laurent and R.G.T. Zegers, Preprint IPNO 00-01, 2000 (unpublished).
40. S. Menard, M. Mirea, F. Clapier, N. Pauwels, J. Proust, C. Donzaud, D. Guillemaud-Mueller, I. Lhenry, A.C. Mueller, J.A. Scarpaci and O. Sorlin, *Phys. Rev. ST Accel. Beams* **2**, 033501 (1999).
41. N. Pauwels, F. Clapier, P. Gara, M. Mirea and J. Proust, *Nucl. Instr. Meth. B* **160**, 315 (2000).
42. R. Serber, *Phys. Rev.* **72**, 1008 (1947).
43. H.J. Brede, G. Dietze, K. Kudo, U.J. Schrewe, F. Tancu and C. Wen, *Nucl. Instr. Meth. A* **274**, 332 (1989).
44. M.A. Lone, A.J. Ferguson and B.C. Robertson, *Nucl. Instr. Meth.* **189**, 515 (1981).
45. D. Christenson, M.W. McNaughton and J.A. Jungerman, *Nucl. Instr. Meth.* **160**, 499 (1979).
46. G.W. Schweimer, *Nucl. Phys. A* **100**, 537 (1967).
47. Kazuo Shin, Kouki Hibi, Masahito Fujii, Yoshitomo Uwamino and Takashi Nakamura, *Phys. Rev. C* **29**, 1307 (1984).
48. A. Belymam, A. Hoummada, J. Collot, P. de Saintignon, G. Mahout and B. Merkel, *Nucl. Instr. Meth. B* **134**, 217 (1998).
49. J.P. Meulders, P. Leleux, P.C. Macq and C. Pirart, *Phys. Med. Biol.* **20**, 235 (1975).
50. M.J. Saltmarsh, C.A. Ludemann, C.B. Fulmer and R.C. Styles, *Nucl. Instr. Meth.* **145**, 81 (1977).
51. P. Grand and A.N. Goland, *Nucl. Instr. Meth.* **145**, 49 (1977).
52. M.A. Lone, C.B. Bigham, J.S. Fraser, H.R. Schneider, T.K. Alexander, A.J. Ferguson and A.B. McDonald, *Nucl. Instr. Meth.* **143**, 331 (1977).
53. J. Collot, P. De Saintignon, P. Gabor, A. Hoummada, G. Mahout, D. Marchand, F. Merchez, E. Leon Florian, C. Leroy, Ph. Jean and B. Merkel, *Nucl. Instr. Meth. A* **350**, 532 (1994).
54. K. Goebel and A.J. Miller, *Nucl. Instr. Meth.* **96**, 581 (1971).
55. J.A. Stokes, L.A. Parks, C.A. Preskitt and J. John, *IEEE Trans. Nucl. Sci.* **NS-30**, 1623 (1983).
56. H. Bichsel, Preprint USC-136-150, 1970.
57. F. Ibrahim et al., (to be published).