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NUMERICAL ANALYSIS OF BRANCHED CRACKS IN BI-AXIAL STRESS FIELDS

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ABSTRACT

The stress corrosion cracks as seen for example in PWR steam generator tubing made of Inconel 600 are usually found to be of highly irregular kinked and branched shapes.

Numerical analysis of kinked and branched cracks in bi-axial plane stress fields using methods such as finite or boundary element method may provide useful and cost effective solutions. However, accurate analysis of complex shaped cracks requires very fine meshes and, consequently, excessively high computational efforts.

This paper discusses some possible strategies of numerical modeling of kinked and branched cracks in general bi-axial stress field using the general-purpose finite element code ABAQUS. The strategies discussed include *J*-integral and stress intensity factor solutions with different mesh densities. The accuracy of the numerical results obtained is compared with reference solutions from the literature.

The main result of the paper is an optimal numerical strategy, which maximizes the accuracy of the results at as low computational efforts as feasible. The selected optimal strategy is expected to be used in the future simulations of large networks of inter-granular stress corrosion cracks at the grain-size scale using incomplete random tessellation.

1. INTRODUCTION

The inter-granular stress corrosion cracks are one of the main causes for early retirement of tubes in steam generators of PWR nuclear power plants [1.]. Combined influence of mechanical loads and aggressive environment causes the development of random crack networks. Typical pattern obtained by numerical simulation of inter-granular crack propagation, which is in agreement with metalographical analyses of pulled out tubes from the steam generators [2.], is presented in Figure 1. .

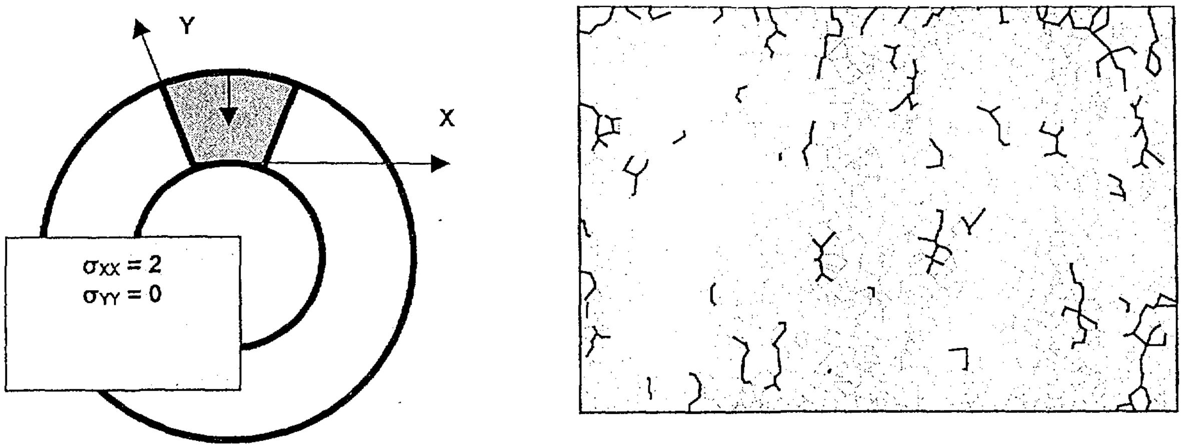

Figure 1: Simulated crack pattern within a radial slice of a steam generator tube [2.]

The most important part of the simulated crack growth (Figure 1) is – besides of realization of the random grain structure – a model of crack initialization and propagation. Adequate crack initialization is obtained by a random process, which considers for example the contact of cracks with the aggressive medium and orientation of stress field. The crack propagation is also a random process assuming more frequent propagation of cracks with larger stress intensity factors. Large degree of branching and relatively important interactions between neighboring cracks make the use of available methods of linear elastic fracture mechanics [4.] hardly possible.

At the moment, the simulations of crack patterns are based upon estimation of stress intensity factors by appropriate empirical models, which correlate the actual crack shape with a simple replacement crack [3.]. This approach is – like any interpolation – unpredictable, when unknown crack forms are considered.

Another possibility is direct numerical analysis of stress intensity factors of all the cracks of the crack pattern (Figure 1). Regardless of the numerical method used for the analysis, an automatic mesh generation is appropriate. The aim of this paper is to optimize the analysis of the stress intensity factors of branched cracks in the bi-axial stress field with finite element method [5.]. The goal of optimization is as accurate stress intensity factors as possible with as reasonable computational effort as feasible. The development of the automatic mesh generation and simulations of the entire pattern of cracks are closely linked future tasks, based on the results discussed in this paper.

2. MATHEMATICAL MODEL

2.1 CRACK TIP LOADING

Stress intensity factor K is the most frequently used measure of the crack tip loading. It determines the stress tensor σ_{ij} around the crack tip with known geometry [4.]:

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} \cdot f_{ij}(\vartheta) \tag{1}$$

where r and ϑ are polar coordinates of a point of interest ($r = 0$ at the crack tip) and f_{ij} is weight function, which basically depends on the geometry of the crack. According to the type of the crack loading, two independent stress intensity factors K_I (opening) and K_{II} (shear) [4., 6.] are used in planar problems.

Stresses or deformations in at least one point should be known to determine stress intensity factors K_I and K_{II}. However, the singularity of the stress tensor around crack tip (Eq. 1) makes numerical determination of the stress intensity factor with finite element method rather difficult. The estimates of stress intensity factors K_I and K_{II} must be extrapolated towards the crack tip. Therefore models with large numbers of elements are needed to support the extrapolation.

2.2 *J*- INTEGRAL

Better numerical stability of the results could be obtained using energy quantities, such as J-integral [4., 6.]:

$$J = \int_{\Gamma} \left(W dy - T_i \frac{\partial u}{\partial x} ds \right) \tag{2}$$

where Γ is an arbitrary closed integration contour around crack tip, T_i is stress vector perpendicular to curve Γ:

$$T_i = \sigma_{ij} n_j \tag{3}$$

u is displacement within direction x and W is deformation energy given by:

$$W = W(x,y) = W(\varepsilon) = \int_0^{\varepsilon} \sigma_{ij} d\varepsilon_{ij} \tag{4}$$

ε_{ij} is the deformation tensor.
State-of-the-art techniques for numerical evaluation of J-integral value are based upon integration over area (or volume for three-dimensional cases) instead of integration along the curve (Green's theorem). Additional dimension of integration contributes to the stability of results obtained using coarse meshes and therefore less computational efforts.

2.3 DECOMPOSITION OF *J*-INTEGRAL TO STRESS INTENSITY FACTORS K_I AND K_{II}

In elastic fracture mechanics the J-integral and stress intensity factors are related as:

$$J = \frac{K^2}{E}\left(1 - \nu^2\right); \qquad K^2 = K_I^2 + K_{II}^2 \tag{5}$$

Exact procedures for decomposition of J-integral into the stress intensity factors K_I and K_{II} are given in the literature [7.]. In general, the stress field should be divided into symmetrical and anti-symmetrical part, resulting in symmetrical and anti-symmetrical part of J-integral. In our case (Figure 1), such procedure should be repeated for every crack tip. This would exceed available computational capacity. Therefore, we developed and tested the method, which is described below.

Finite element method enables calculations of displacements around the crack tip. Displacements u at point T (r, ϑ) are given as:

$$u_x = \frac{1+\nu}{2E}\sqrt{\frac{r}{2\pi}}\left(f_{xI} \cdot K_I + f_{xII} \cdot K_{II}\right); \qquad u_y = \frac{1+\nu}{2E}\sqrt{\frac{r}{2\pi}}\left(f_{yI} \cdot K_I + f_{yII} \cdot K_{II}\right) \tag{6}$$

where ν is Poisson number, E is Young's module and f is weight factor with index denoting the coordinate and the loading type of the crack. Stress intensity factors K_{I} and K_{II} depend upon the displacement field, the size and the geometry of the crack. Values of stress intensity factors K_{I} and K_{II} can be calculated using:

$$K_{I} = \frac{f_{yII} \cdot u_x - f_{xII} \cdot u_y}{f_{xI} \cdot f_{yII} - f_{xII} \cdot f_{yI}} \cdot \frac{2E}{1+\nu}\sqrt{\frac{2\pi}{r}}; \quad K_{II} = \frac{f_{xI} \cdot u_y - f_{yI} \cdot u_x}{f_{xI} \cdot f_{yII} - f_{xII} \cdot f_{yI}} \cdot \frac{2E}{1+\nu}\sqrt{\frac{2\pi}{r}} \quad (7)$$

Numerical values of stress intensity factors K_{I} and K_{II} depend strongly on position of the chosen point with respect to the crack tip. Therefore the ratio of both stress intensity factors was introduced:

$$k = \frac{K_{\mathrm{II}}}{K_{\mathrm{I}}} = \frac{f_{xI} \cdot u_y - f_{yI} \cdot u_x}{f_{yII} \cdot u_x - f_{xII} \cdot u_y} \quad (8)$$

Ratio k was found to be reasonably stable, especially at nodes, which are collinear with observed crack tips. Hence there is no need for extrapolation of stress intensity factors K_{I} and K_{II} towards the crack tip. Apart from k ratio, the value of J-integral, which is a sum of contributions of K_{I} and K_{II} (Eq. 5), is needed for decomposition. Stress intensity factors K_{I} and K_{II} are:

$$K_{\mathrm{I}} = \sqrt{\frac{J_{num} E}{\left(1+k^2\right) \cdot \left(1-\nu^2\right)}} \quad (9)$$

$$K_{\mathrm{II}} = k \cdot K_{\mathrm{I}} \quad (10)$$

at which J_{num} is a numerically obtained value of J-integral.

3. NUMERICAL EXAMPLES

Two numerical examples will be presented here. The first is kinked crack shown in Figure 2, and the second is branched crack shown in Figure 3. An analytical result of the first example is known from literature [9.] within 3%. This result is used as a reference value. Uni-axial stress is – due to the assumed elastic behavior of material (principle of superposition) – enough to verify the results in a bi-axial stress field. Reference results for branched crack in an equi-bi-axial stress field were obtained by calculation with boundary element method [3.].

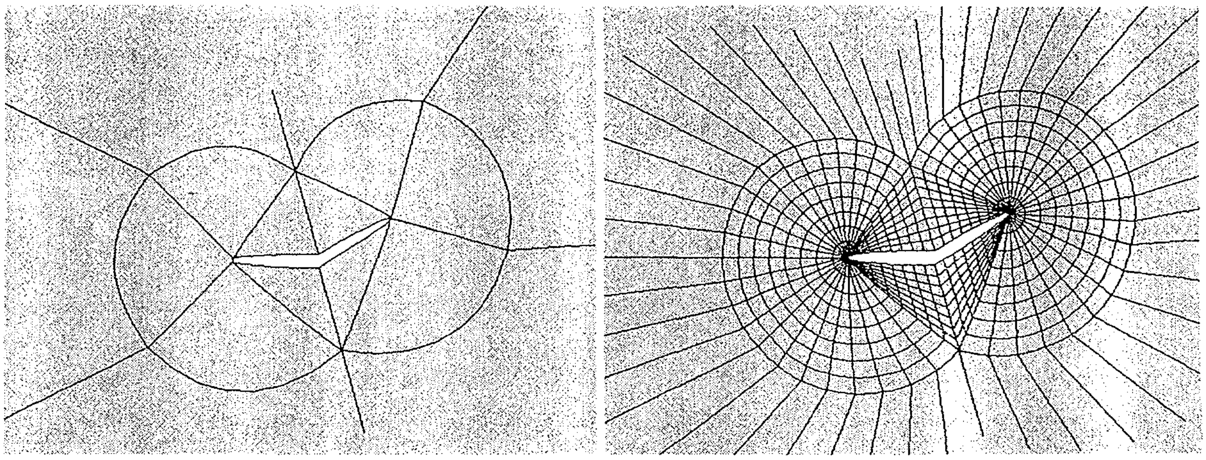

Figure 2: Coarse (left) and fine (right) finite element mesh of a kinked crack

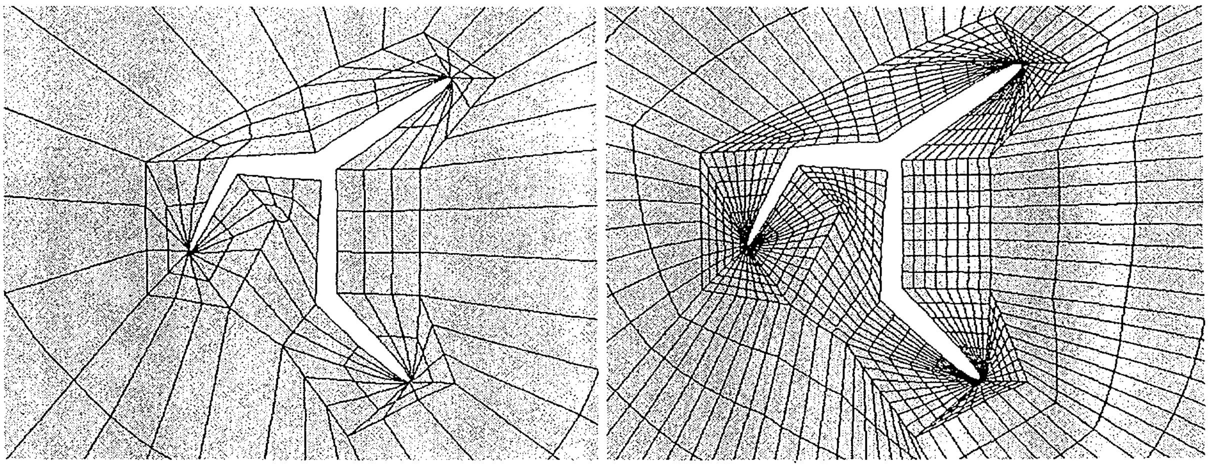

Figure 3: Coarse (left) and fine (right) finite element mesh of a branched crack.

The cracks were modeled with meshes of different densities. The finest and the coarsest meshes used in calculations are shown in Figures 2 and 3. As a measure of the mesh density typical length of element at the crack tip was used as shown in Figure 4. Therefore the finest mesh has a relative size of a typical element equal to 12.5% and the coarsest mesh has a relative size of a typical element equal to 100% of the crack length.

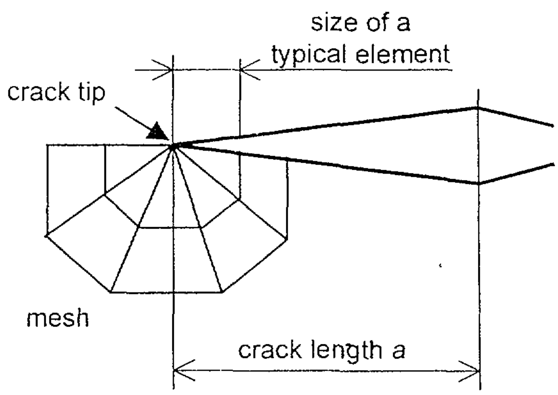

Figure 4: Size of a typical element

The calculation was performed using finite element code ABAQUS/Standard. Meshes around crack tips were modeled using collapsed singular elements. Singular elements describe the singularity of the displacements at the crack tip [5.]. Figure 5 presents the value of k ratio as a $1/\sqrt{r}$ function of the different mesh density (different relative size of typical element) and compare it to the reference solutions (100%) [9.]. Accuracy of the numerical results is higher, if finer meshes are used. Nevertheless, coarse meshes with typical element length of 50% give reasonably accurate results (within 10%).

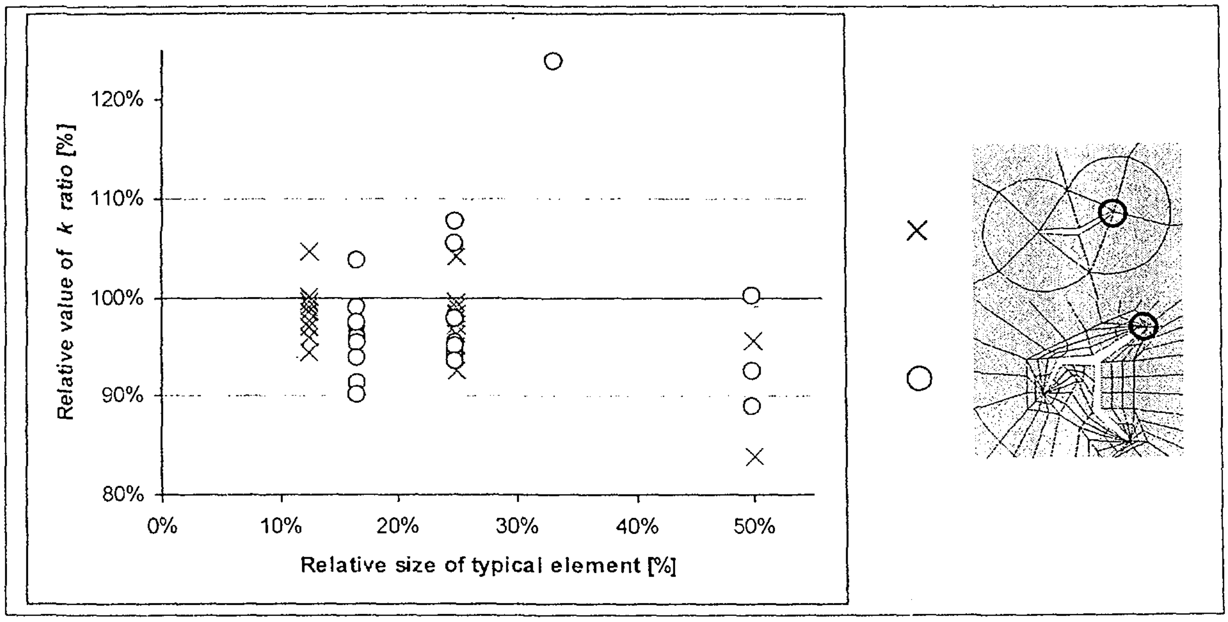

Figure 5: Value of k ratio as a function of relative size of typical element and crack model

Figure 6 and Figure 7 compare the scatter of the k ratio results at the nodes that are nearly collinear with the analyzed crack tip. Comparison between the k ratio results for kinked and branched crack is shown in Figure 6. The majority of the results are within 10% error margins; furthermore average error is in the order of 5%.

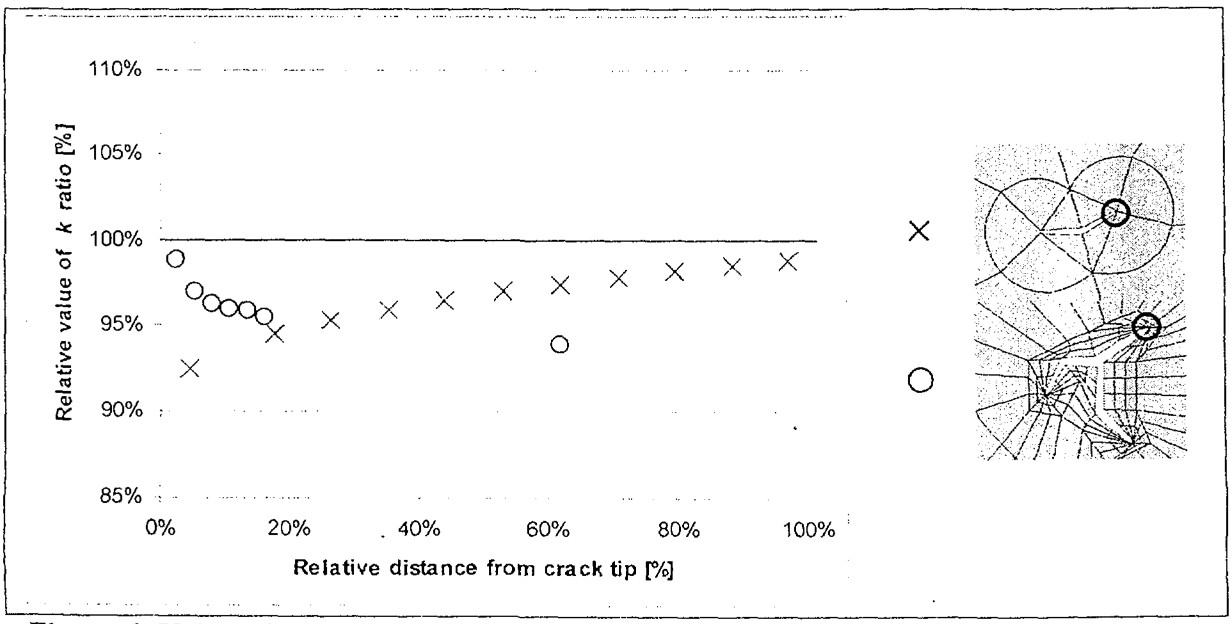

Figure 6: Value of k ratio as a function of relative distance from the crack tip and crack model

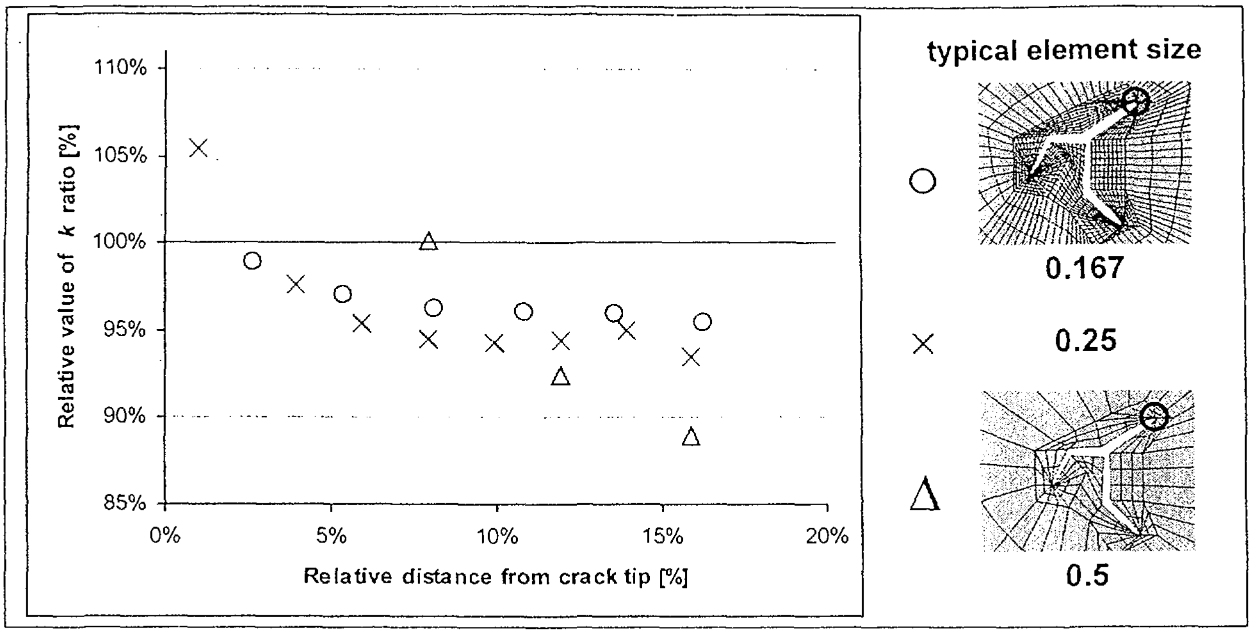

Figure 7: Value of *k* ratio as a function of relative distance to the crack tip and relative size of typical element (branched crack)

Comparison of the *k* ratio results as a function of the relative distance to the crack tip and different mesh densities is shown in Figure 7. The finest mesh provides the best results. However, even the coarsest mesh can provide results within averaged errors in the order of 6%.

4. SUMMARY

The paper discusses optimized analysis of stress intensity factors of kinked and branched cracks in bi-axial stress field using finite element method. Analyses were made by simple decomposition of *J*-integrals, which were obtained by computer code ABAQUS. The following actions are recommended for more accurate results:

- Use of singular elements to model the crack tip.
- Meshes with typical element length of 50% of a crack length should be used for reasonable accurate results (within 10%).
- Decomposition of *J*-integral should be performed at nodes that are nearly collinear with analyzed crack tip.
- Decomposition of *J*-integral should be performed at nodes within 50 % relative distance from the crack tip.

The averaged estimates of stress intensity factors are then expected to be within 6% error margin.

Future work will focus on simulations of large networks of inter-granular stress corrosion cracks at the grain-size scale using incomplete random tessellation. The optimal strategy discussed in this paper will be used as a basis.

5. ACKNOWLEDGMENT

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6. LITERATURE

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