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反磁剪切托卡马克等离子体的阿尔芬波电流驱动

ALFVEN WAVE CURRENT DRIVE IN TOKAMAK REVERSED SHEAR PLASMAS

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反磁剪切托卡马克等离子体的阿尔芬波电流驱动

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摘　要

研究了反磁剪切位形托卡马克中旋转等离子体阿尔芬波电流驱动，采用单流体 MHD 模型，考虑磁剪切修正和等离子体旋转修正，在等离子体圆柱模型下，导出阿尔芬波驱动电流密度的表达式，并给出扰动场满足的波方程近似解。仅对极向模数 $m = 0$ 的情况，分别讨论了压缩阿尔芬波、剪切阿尔芬波和圆偏振波电流驱动。结果表明：不考虑磁剪切时，剪切阿尔芬波和圆偏振波电流驱动效率与旋转等离子体密度无关；负磁剪切和正剪切对阿尔芬波电流驱动的作用相反，负磁剪切效应使驱动电流密度提高；旋转效应可以提高阿尔芬波电流驱动效率。结果证明阿尔芬波有利于高约束先进托卡马克等离子体电流驱动。

Alfven Wave Current Drive in Tokamak Reversed Shear Plasmas

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ABSTRACT

The current drive due to Alfven wave in tokamak reversed shear plasmas is studied. In cylindrical geometry, an expression for driving current density J_z is given by means of the single-fluid magnetohydrodynamic (MHD) model taking plasma rotation and magnetic shear into account. The current drive due to the compressional Alfven wave and the shear Alfven wave is considered, respectively. It is found that the efficiency of the Alfven wave current drive without the magnetic shear is independent of rotating plasma density. Moreover, it is shown that a higher efficiency can be obtained in the presence of rotation. For the shear Alfven wave, the magnetic shear has a more distinct effect on the current drive than one for the compressional Alfven wave. The effect of the negative magnetic shear on the Alfven wave current drive is opposite to the effect of the positive, and the negative shear enhances the driven current density J_z. These results show that the Alfven waves may be an excellent current drive candidate for tokamak fusion reactors.

INTRODUCTION

At the beginning of the eighties, one began to experimentally study the lower frequency current drive, a large amount of the experimental current drive is efficient, and the effect of trapped electrons is not so serious as one thinks. Up to the late eighties Ohkawa[1] and others[2~4] pointed out that the efficiency IR/P (I and P are plasma current and absorbed power, respectively, while R is the major radius) of the current drive produced by Alfven wave helicity injection is independent of plasma density. They also pointed out the Alfven wave current drive is more efficient than the other conventional methods of radio frequency current drive, where the efficiency degrades with increasing plasma density. Afterwards, Chan et al.[5] carried out a detailed analysis of current drive within the framework of single-fluid and two-fluid MHD model. They showed that for $\omega \ll \Omega_i$ (Ω_i is the ion cyclotron frequency) circularly polarized waves can drive current far exceeding the current resulting from linearly polarized. Further, the efficiency can be independent of plasma density. The above stated author's results[1~5] motivates us to study the Alfven wave current drive in advanced high confinement tokamak plasma.

An advanced tokamak means that it operates under the circumstances of high confinement, high normalized beta ($\beta_N=\beta/(I/aB)$) and a large bootstrap current fraction. For future tokamak fusion reactors it is very significant and important to improve the plasma confinement, make β_N-value high, increase the fusion products and decrease the scale and cost of the reactor. The safety factor q, one of the important plasma parameters, has directly and accurately been measured in TFTR[6], DIII-D[7] and JT-60U[8], and new reversed shear configurations were formed in experiments. The reversed shear configuration, which has negative magnetic shear in the inner region and positive magnetic shear in the outer region, has been proposed as an advanced tokamak operation[9]. Confinement improvement and formation of internal transport barrier in the negative shear region were observed. On the other hand, the other factor making confinement improve, which is present in all present-day tokamaks, is plasma rotation. It has shown that, in almost all the high confinement regimes of tokamak, e.g., H (high confinement) mode, CH (core high confinement) mode and recently observed enhanced reversed shear (ERS) mode[6, 7, 10~12], plasma rotation is invariably present that is mainly responsible for the suppression of turbulence and associated anomalous losses. The plasma rotation may play an important role in creating or maintaining a thermal transport barrier[13, 14].

However, not only an H-mode edge transport barrier but also the internal transport barrier, formed in the negative magnetic region leads to good confinement, which is favorable for future tokamak reactor. Therefore, we think of advanced high confinement tokamak plasma as being a tokamak rotating plasma in a reversed shear configuration. After the internal transport barrier has been observed in JT-60U reversed magnetic shear lower current discharges, a dramatic improvement in confinement and performance of tokamak plasma within the reversed shear configuration has been proposed in JT-60U. Under sustaining the MHD stability, enhanced plasma current by using radio frequency current drive is expected in order to gain a good fusion performance. Recently, Avinash[15] for the first time studied current drive due to plane polarized compressional Alfven wave in rotating plasma. His results show that current drive efficiency that is as good as that due to circularly polarized waves can be obtained, with a bulk current drive. He showed that the plane-polarized wave could not drive plasma current in the absence of rotation. In summary, Ohkawa et al.[1], Chan et al.[5], and Avinash[15] had pioneered the studies of Alfven wave current drive (AWCD) in plasmas.

Of late years, both the experiments and the theories have shown that the synergistic effects of the reversed (negative in core region) magnetic shear and the rotation can remarkably enhance the core confinement of tokamak plasma. Therefore, it is natural that we call the tokamak plasma with rotation and reversed magnetic shear a high confinement advanced tokamak plasma. We think studying the AWCD in this system to be significant to further enhancing the performance of high confinement advanced tokamaks. In the present paper, we examine the effects of the magnetic shear and rotation on the AWCD in cylindrical geometry.

1 MODEL

1. 1 Basic equations

Our study is based on the single-fluid MHD model. A single-fluid equation of motion is given by

$$nm_{\mathrm{i}}\frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t}+\nabla p=\frac{1}{c}\boldsymbol{j}\times\boldsymbol{B}-\nabla\cdot(\vec{\pi}^{\mathrm{e}}+\vec{\pi}^{\mathrm{i}})+\boldsymbol{F} \tag{1}$$

and Ohm's law is

$$\boldsymbol{j}=\sigma\left[(\boldsymbol{E}+\frac{1}{c}\boldsymbol{V}\times\boldsymbol{B})-\frac{1}{enc}\boldsymbol{j}\times\boldsymbol{B}+\frac{1}{en}\nabla p_{\mathrm{e}}+\frac{1}{en}\nabla\cdot\vec{\pi}^{\mathrm{e}}\right] \tag{2}$$

Where n, $\boldsymbol{V}$, $\boldsymbol{j}$, $\boldsymbol{B}$, and $\boldsymbol{E}$ are plasma density, velocity, current density, magnetic field and electric field, respectively. p and σ are plasma pressure and conductivity, respectively. The second term on the right-hand side of Eq.(1), $\nabla\cdot(\vec{\pi}^{\mathrm{e}}+\vec{\pi}^{\mathrm{i}})$, is defined as the viscosity force. $\boldsymbol{F}$ is related to electric-field force and other force except for Lorentz one.

We consider current drive due to low-frequency waves—Alfven waves, $\omega<<\Omega_{\mathrm{i}}$. In this paper the viscosity force related to plasma fluid expansion and deformation is expressed in terms of $\nu\nabla^2\boldsymbol{V}$, where ν is kinematic viscosity. In an advanced high confinement tokamak plasma, plasma rotation in the poloidal and/or toroidal direction is present owing to high-power neutral beam injection, biased electrode, radio frequency wave, and so on. Because in all non-inductive current drive experiments the plasma current is driven surely in the toroidal direction, we only consider the poloidal rotation that has a constant angular frequency $\boldsymbol{\Omega}$ (i.e., Ω is independent of the minor radius of plasma) as a first approximation. Therefore, for the rotating plasma Eq. (1) can be written as following form with the viscosity force and Coriolis force produced by the poloidal rotation

$$\rho\frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t}+2\rho\boldsymbol{\Omega}\times\boldsymbol{V}=\frac{1}{c}\boldsymbol{j}\times\boldsymbol{B}+\rho\nu\nabla^2\boldsymbol{V}-\nabla p \tag{3}$$

Where $\rho = nm_{\mathrm{i}}$. In this paper, we can neglect the second, third and fourth terms on the right-hand side of Eq. (2) because of well-known reason[15]. Then, Eq. (2) is simplified as

$$\eta\boldsymbol{j}=\boldsymbol{E}+\frac{1}{c}\boldsymbol{V}\times\boldsymbol{B} \tag{4}$$

Where $\eta = 1/\sigma$, the plasma resistivity. Eqs. (3) and (4) are the basic equations used to study the current drive due to Alfven waves in an advanced high confinement tokamak plasma.

1. 2 An expression for J_z in cylindrical geometry

The expression of driven current density due to Alfven waves, J_z, is obtained by linearizing the Eqs. (3) and (4), and assuming that the variation of all linear perturbation quantities with time is of the form $\exp(-\mathrm{i}\omega t)$. We have assumed that the plasma rotation, $\boldsymbol{\Omega}$, is a first approximation, the plasma fluid velocity is only the perturbation quantity. The equilibrium magnetic field $\boldsymbol{B}_0$ is non-uniform. For the sake of simplicity, the perturbation of plasma density and temperature are neglected in this paper. The linearized version of Eq. (3) is

$$-\mathrm{i}\rho\omega\delta\boldsymbol{V}+2\rho\boldsymbol{\Omega}\times\delta\boldsymbol{V}=\frac{1}{c}\boldsymbol{j}_0\times\delta\boldsymbol{B}+\frac{1}{c}\delta\boldsymbol{j}\times\boldsymbol{B}_0+\rho\nu\nabla^2\delta\boldsymbol{V} \tag{5}$$

Assuming that the last term with kinematic viscosity ν is proportional to $(-\rho\nu k^2\delta\boldsymbol{V})$

and letting $\varpi = \omega + \mathrm{i}k^2 v$, Eq. (5) became

$$-\mathrm{i}\rho\varpi\delta\boldsymbol{V} + 2\rho\boldsymbol{\Omega}\times\delta\boldsymbol{V} = \frac{1}{c}\boldsymbol{j}_0\times\delta\boldsymbol{B} + \frac{1}{c}\delta\boldsymbol{j}\times\boldsymbol{B}_0 \tag{6}$$

Through cross- and dot-product Eq. (6) by $\boldsymbol{\Omega}$, we can get a relation of the perturbation velocity $\delta\boldsymbol{V}$ and the perturbation current $\delta\boldsymbol{j}$ and magnetic field $\delta\boldsymbol{B}$

$$\begin{aligned}\delta\boldsymbol{V} = \frac{\mathrm{i}}{c\rho\varpi(1-4\Omega^2/\varpi^2)}\Big\{&[(\boldsymbol{j}_0\times\delta\boldsymbol{B}) + (\delta\boldsymbol{j}\times\boldsymbol{B}_0)] + \frac{2\mathrm{i}}{\varpi}[(\boldsymbol{\Omega}\cdot\boldsymbol{j}_0)\delta\boldsymbol{B} - (\boldsymbol{\Omega}\cdot\delta\boldsymbol{B})\boldsymbol{j}_0 + \\ &(\boldsymbol{\Omega}\cdot\delta\boldsymbol{j})\boldsymbol{B}_0 - (\boldsymbol{\Omega}\cdot\boldsymbol{B}_0)\delta\boldsymbol{j}] - \frac{4}{\varpi^2}[(\boldsymbol{j}_0\times\delta\boldsymbol{B})\cdot\boldsymbol{\Omega} + (\delta\boldsymbol{j}\times\boldsymbol{B}_0)\cdot\boldsymbol{\Omega}]\boldsymbol{\Omega}\Big\}\end{aligned} \tag{7}$$

Substituting Eq. (5) into the Ohm's law, Eq. (4), and linearizing it, we have

$$\eta\,\boldsymbol{j}_0 + \eta\,\delta\bar{\boldsymbol{j}}^{(2)}(\boldsymbol{r},t) = \boldsymbol{E}_0 + \frac{1}{2}\mathrm{Re}\left(\frac{1}{c}\delta\boldsymbol{V}(\boldsymbol{r})\times\delta\boldsymbol{B}^*(\boldsymbol{r})\right) \tag{8}$$

Where $\delta\boldsymbol{B}^*$ is the conjugate plural. Obviously, the equilibrium electric field $\boldsymbol{E}_0$ produces the equilibrium current $\boldsymbol{j}_0$, while the second-order current $\delta\bar{\boldsymbol{j}}^{(2)}$ is driven by the second term on the right-hand side of Eq. (8),

$$\eta\,\delta\bar{\boldsymbol{j}}^{(2)} = \frac{1}{2}\mathrm{Re}\left(\frac{1}{c}\delta\boldsymbol{V}(\boldsymbol{r})\times\delta\boldsymbol{B}^*(\boldsymbol{r})\right) \tag{9}$$

In this paper, we employ a cylindrical geometry with orthogonal unit vectors $\boldsymbol{e}_r$, $\boldsymbol{e}_\theta$ and $\boldsymbol{e}_z$. The equilibrium magnetic field is given by

$$\boldsymbol{B}_0 = B_0\left(\boldsymbol{e}_z + \bar{\delta}(r)\boldsymbol{e}_\theta\right) \tag{10}$$

Where B_0=constant, $\bar{\delta} = r/qR$, and $q(r)$ is the safety factor. In this coordinate system, the z-component of Eq. (9) becomes

$$\eta\,\bar{j}_z^{(2)} = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{c}(\delta V_r\delta B_\theta^* - \delta V_\theta\delta B_r^*)\right\} \tag{11}$$

According to Eq. (7) in the cylindrical geometry, we have

$$\delta V_r = \frac{\mathrm{i}}{c\rho\varpi(1-4\Omega^2/\varpi^2)}\left\{[(\boldsymbol{j}_0\times\delta\boldsymbol{B})_r + (\delta\boldsymbol{j}\times\boldsymbol{B}_0)_r] + \frac{2\mathrm{i}\Omega}{\varpi}[(\boldsymbol{j}_0\times\delta\boldsymbol{B})_\theta + (\delta\boldsymbol{j}\times\boldsymbol{B}_0)_\theta]\right\} \tag{12}$$

$$\delta V_\theta = \frac{\mathrm{i}}{c\rho\varpi(1-4\Omega^2/\varpi^2)}\left\{[(\boldsymbol{j}_0\times\delta\boldsymbol{B})_\theta + (\delta\boldsymbol{j}\times\boldsymbol{B}_0)_\theta] - \frac{2\mathrm{i}\Omega}{\varpi}[(\boldsymbol{j}_0\times\delta\boldsymbol{B})_r + (\delta\boldsymbol{j}\times\boldsymbol{B}_0)_r]\right\} \tag{13}$$

Letting the driven current density $J_z \equiv \bar{j}_z^{(2)}$ and substituting Eqs. (12) and (13) into Eq. (11), we have

$$\begin{aligned}B_0\eta J_z = \frac{1}{2}\mathrm{Re}\Big\{\frac{4\pi\mathrm{i}V_{\mathrm{A}}^2}{c^2\varpi(1-4\Omega^2/\varpi^2)}\Big\{&[\delta j_r\delta B_r^* + (\delta j_\theta - \bar{\delta}\delta j_z)\delta B_\theta^*] - \frac{2\mathrm{i}\Omega}{\varpi}[\delta j_r\delta B_\theta^* - (\delta j_\theta - \\ &\bar{\delta}\delta j_z)\delta B_r^*] + \frac{1}{B_0}\left[j_{0z}\left(-\delta\boldsymbol{B}_\perp\cdot\delta\boldsymbol{B}_\perp^* + \frac{2\mathrm{i}\Omega}{\varpi}(\delta\boldsymbol{B}\times\delta\boldsymbol{B}^*)_z\right) + (\tilde{\boldsymbol{j}}_{0\perp}\cdot\delta\boldsymbol{B}_\perp^*)\delta B_z\right]\Big\}\Big\}\end{aligned} \tag{14}$$

Where $V_{\mathrm{A}}^2 = (B_0^2/4\pi m_{\mathrm{i}}n)$ is the Alfven velocity, and

$$\widetilde{\boldsymbol{j}}_{0\perp} = (j_{0r} + \frac{2\mathrm{i}\Omega}{\overline{\omega}} j_{0\theta})\boldsymbol{e}_r + (j_{0\theta} - \frac{2\mathrm{i}\Omega}{\overline{\omega}} j_{0r})\boldsymbol{e}_\theta = \boldsymbol{j}_{0\perp} + \frac{2\mathrm{i}}{\overline{\omega}}(\boldsymbol{j}_0 \times \boldsymbol{\Omega})_\perp \tag{15}$$

For the magnetic field configuration shown as Eq. (10), from Maxwell field equation, the equilibrium current density $\boldsymbol{j}_0$ is given by

$$j_{0r} = j_{0\theta} = 0 \tag{16.1}$$

$$j_{0z} = \frac{cB_0}{4\pi}\frac{(r\overline{\delta})'}{r} \tag{16.2}$$

Where $(r\overline{\delta})' \equiv \frac{\mathrm{d}}{\mathrm{d}r}(r\overline{\delta}) = \frac{r}{Rq}(2 - \frac{r}{q}\frac{\mathrm{d}q}{\mathrm{d}r}) = \overline{\delta}(2-\hat{s})$, which is related to the magnetic shear that is defined as $\hat{s} = \frac{r}{q}\frac{\mathrm{d}q}{\mathrm{d}r}$. In a tokamak plasma with reversed magnetic shear, we notice that q has a maximum near the magnetic axis and a minimum at a certain magnetic surface. We can see that $\hat{s}<0$ appears in the region with the negative magnetic shear and $\hat{s}>0$ in the positive shear region. Therefore, Eq. (14) becomes

$$\begin{aligned} B_0\eta J_z = \frac{1}{2}\mathrm{Re}\Bigg\{ \frac{4\pi \mathrm{i} V_{\mathrm{A}}^2}{c^2\overline{\omega}(1-4\Omega^2/\overline{\omega}^2)} \Big[(\delta j_r \delta B_r^* + \delta\widetilde{j}_\theta \delta B_\theta^*) - \frac{2\mathrm{i}\Omega}{\overline{\omega}}(\delta j_r \delta B_\theta^* - \\ \delta\widetilde{j}_\theta \delta B_r^*) + \frac{c}{4\pi}\frac{\overline{\delta}}{r}(2-\hat{s})\Big(-\delta\boldsymbol{B}_\perp \cdot \delta\boldsymbol{B}_\perp^* + \frac{2\mathrm{i}\Omega}{\overline{\omega}}(\delta\boldsymbol{B}\times\delta\boldsymbol{B}^*)_z \Big) \Big] \Bigg\} \end{aligned} \tag{17}$$

where $\delta\widetilde{j}_\theta = \delta j_\theta - \overline{\delta}\delta j_z$. If the effects of the rotation and the magnetic shear are not considered, Eq. (17) reduces to

$$J_z = \frac{1}{2B_0\eta}\mathrm{Re}\left(\frac{4\pi \mathrm{i} V_{\mathrm{A}}^2}{c^2\overline{\omega}}(\delta\boldsymbol{j}_\perp \cdot \delta\boldsymbol{B}_\perp^*) \right) \tag{18}$$

which is the same as the expression given by Chan et al[5]. If the rotation effect is considered and the magnetic shear effect does not, from Eq. (17) we can obtain

$$J_z = \frac{1}{2B_0\eta}\mathrm{Re}\left\{ \frac{4\pi \mathrm{i} V_{\mathrm{A}}^2}{c^2\overline{\omega}(1-4\Omega^2/\overline{\omega}^2)} \left[(\delta\boldsymbol{j}_\perp \cdot \delta\boldsymbol{B}_\perp^*) - (\frac{2\mathrm{i}\Omega}{\overline{\omega}})(\delta\boldsymbol{j}\times\delta\boldsymbol{B}^*)_z \right] \right\} \tag{19}$$

This equation is the same as Eq .(5) in Ref.15. If we only consider the effect of magnetic shear on the Alfven wave current drive, from Eq. (17) we have the following expression of driven current density

$$J_z = \frac{1}{2B_0\eta}\mathrm{Re}\left\{ \frac{4\pi \mathrm{i} V_{\mathrm{A}}^2}{c^2\overline{\omega}} \left[(\delta\boldsymbol{j}_\perp \cdot \delta\boldsymbol{B}_\perp^*) - \overline{\delta}\left(\delta j_z \delta B_\theta^* + \frac{c}{4\pi r}(2-\hat{s})\delta\boldsymbol{B}_\perp \cdot \delta\boldsymbol{B}_\perp^* \right) \right] \right\} \tag{20}$$

We conclude that the effect of the negative magnetic shear on the Alfven wave current drive is opposite to the effect of the positive magnetic shear on it. For compressional Alfven wave and shear Alfven wave, the driven current density with the negative magnetic shear ($\hat{s}<0$) is larger than that with the positive shear ($\hat{s}>0$).

1. 3 The wave equations

For the sake of simplicity, we only consider an approximate case: we assume that both the rotation factor $2\Omega/\varpi$ and the shear factor δ are the first-order corrections and neglect the effects of these two factors on the perturbation quantities $\delta\boldsymbol{j}$ and $\delta\boldsymbol{B}$ in Eq. (17). Then, we have

$$\delta\boldsymbol{V} = \frac{\mathrm{i}B_0}{c\rho\varpi}(\delta j_\theta \boldsymbol{e}_r - \delta j_r \boldsymbol{e}_\theta) \tag{21}$$

and

$$\frac{1}{c}\delta\boldsymbol{V}\times\boldsymbol{B}_0 = -\frac{\mathrm{i}B_0^2}{c^2\rho\varpi}(\delta j_r \boldsymbol{e}_r + \delta j_\theta \boldsymbol{e}_\theta) \tag{22}$$

According to Ohm's law and Maxwell equations, the wave equation for $\delta\boldsymbol{B}$ is obtained

$$(\frac{\omega\varpi}{V_\mathrm{A}^2})\delta\boldsymbol{B} - \mathrm{i}\frac{c^2\eta}{4\pi\omega}(\frac{\omega\varpi}{V_\mathrm{A}^2})\nabla^2\delta\boldsymbol{B} + \left(\frac{\partial^2}{\partial z^2}\delta\boldsymbol{B} - \nabla(\frac{\partial}{\partial z}\delta B_z) + \nabla^2\delta B_z\boldsymbol{e}_z\right) = 0 \tag{23}$$

We express the $\boldsymbol{e}_z$-component of Eq. (23) as

$$\nabla^2\delta B_z = -(\frac{\omega\varpi}{V_\mathrm{A}^2})\left(1 - \mathrm{i}\frac{c^2\eta}{4\pi\omega}(\frac{\omega\varpi}{V_\mathrm{A}^2})\right)^{-1}\delta B_z \tag{24}$$

Assuming an $\mathrm{e}^{im\theta+\mathrm{i}k_z z}$ dependence, the component δB_z satisfy Bessel's equation and is given by

$$\delta B_z(r,\theta,z) = b_\mathrm{c}\mathrm{J}_m(l_z r)\mathrm{e}^{im\theta+\mathrm{i}k_z z} \tag{25}$$

with

$$l_z^2 = (\frac{\omega\varpi}{V_\mathrm{A}^2})\left(1 - \mathrm{i}\frac{c^2\eta}{4\pi\omega}(\frac{\omega\varpi}{V_\mathrm{A}^2})\right)^{-1} - k_z^2 \tag{26}$$

Where b_c is a constant.

The r component of Eq. (23) is as follows

$$-(\nabla^2\delta\boldsymbol{B})_r = -\left(\frac{\omega\varpi}{V_\mathrm{A}^2} - k_z^2\right)\left(\frac{\mathrm{i}c^2\eta}{4\pi\omega}\frac{\omega\varpi}{V_\mathrm{A}^2}\right)^{-1}\delta B_r + \mathrm{i}k_z\left(\frac{\mathrm{i}c^2\eta}{4\pi\omega}\frac{\omega\varpi}{V_\mathrm{A}^2}\right)^{-1}\frac{\mathrm{d}}{\mathrm{d}r}\delta B_z \tag{27}$$

Where

$$-(\nabla^2\delta\boldsymbol{B})_r = -\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\delta B_r - \frac{\partial^2}{\partial z^2}\delta B_r + \frac{\partial^2}{\partial r\partial z}\delta B_z + \frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}\delta B_\theta + \frac{1}{r^2}\frac{\partial}{\partial\theta}\delta B_\theta \tag{28}$$

Letting

$$l_\mathrm{b}^2 = \left((\frac{\omega\varpi}{V_\mathrm{A}^2}) - k_z^2\right)\left(-\mathrm{i}\frac{c^2\eta}{4\pi\omega}(\frac{\omega\varpi}{V_\mathrm{A}^2})\right)^{-1} - k_z^2 \tag{29}$$

And using Eq. (25), we obtain

$$(l_b^2-\frac{m^2}{r^2})\delta B_r-(\frac{\mathrm{i}m}{r})\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}r\delta B_\theta=\mathrm{i}k_z\frac{l_b^2}{l_z^2}\frac{\mathrm{d}}{\mathrm{d}r}\delta B_z \tag{30}$$

The perturbation magnetic field $\delta\boldsymbol{B}$ obeys

$$\nabla\cdot\delta\boldsymbol{B}=0 \tag{31}$$

If we define

$$\psi(x)=\frac{\mathrm{i}x}{m}\delta B_r-\frac{\mathrm{i}x}{m}\frac{\mathrm{i}k_z}{l_z}\frac{\mathrm{d}}{\mathrm{d}r}\delta B_z \tag{32}$$

Where $x=l_\mathrm{b}r$. Eq. (30) is just as the Bessel's equation for $\psi(x)$, so

$$\psi(x)=b_\mathrm{s}\mathrm{J}_m(x) \tag{33}$$

Where b_s is a constant. It is evident from Eqs. (25), (32) and (33) that

$$\delta B_r(r,\theta,z)=\left(-\frac{\mathrm{i}m}{l_\mathrm{b}r}b_\mathrm{s}\mathrm{J}_m(l_\mathrm{b}r)+\frac{\mathrm{i}k_z}{l_z}b_\mathrm{c}\mathrm{J}_m'(l_zr)\right)\mathrm{e}^{\mathrm{i}m\theta+\mathrm{i}k_zz} \tag{34}$$

From Eqs. (25), (34) and (31), we easily obtain the poloidal component of $\delta\boldsymbol{B}$.

$$\delta B_\theta(r,\theta,z)=\left(b_\mathrm{s}\mathrm{J}_m'(l_\mathrm{b}r)-\frac{mk_z}{l_z^2r}b_\mathrm{c}\mathrm{J}_m(l_zr)\right)\mathrm{e}^{\mathrm{i}m\theta+\mathrm{i}k_zz} \tag{35}$$

Finally, we can use Faraday's law and Ohm's law to solve the corresponding electric fields, yielding

$$\delta E_r=\left[\frac{k_zV_\mathrm{A}^2}{c\varpi}\left(1-\frac{\mathrm{i}c^2\eta}{4\pi\omega}\frac{\omega\varpi}{V_\mathrm{A}^2}\right)b_\mathrm{s}\mathrm{J}_m'(l_\mathrm{b}r)-\frac{\omega}{c}\frac{k_\theta}{l_z^2}b_\mathrm{c}\mathrm{J}_m(l_zr)\right]\mathrm{e}^{\mathrm{i}m\theta+\mathrm{i}k_zz} \tag{36}$$

$$\delta E_\theta=\left[\frac{k_zV_\mathrm{A}^2}{c\varpi}\left(1-\frac{\mathrm{i}c^2\eta}{4\pi\omega}\frac{\omega\varpi}{V_\mathrm{A}^2}\right)\frac{\mathrm{i}k_\theta}{l_\mathrm{b}}b_\mathrm{s}\mathrm{J}_m(l_\mathrm{b}r)-\mathrm{i}\frac{\omega}{c}\frac{1}{l_z}b_\mathrm{c}\mathrm{J}_m'(l_zr)\right]\mathrm{e}^{\mathrm{i}m\theta+\mathrm{i}k_zz} \tag{37}$$

$$\delta E_z=\left(-\frac{c\eta}{4\pi}l_\mathrm{b}b_\mathrm{s}\mathrm{J}_m(l_\mathrm{b}r)\right)\mathrm{e}^{\mathrm{i}m\theta+\mathrm{i}k_zz} \tag{38}$$

There are a number of ways to characterize each branch of Alfven wave. The shear Alfven wave is incompressible, so it has δB_z=0 and $\nabla\cdot\delta\boldsymbol{E}\neq0$, while the compressional wave has δE_z=0 and $\nabla\cdot\delta\boldsymbol{E}=0$. Therefore, $b_\mathrm{s}\neq0$ represents the sheared Alfven wave and $b_\mathrm{c}\neq0$ does the compressional branch.

2 ALFVEN WAVE DRIVEN CURRENT AND EFFICIENCY

In the cylindrical geometry, the total driven current can be evaluated by

$$I=2\pi\int_0^a J_zr\mathrm{d}r \tag{39}$$

Where a is the minor radius of the tokamak plasma.

From Eq. (4) and neglecting the effects of the plasma rotation and the magnetic shear on the first-order perturbation current density, we can obtain the relation between linear perturbation quantities

$$\delta j_r = \frac{1}{\eta + \frac{4\pi \mathrm{i} V_A^2}{c^2 \varpi}} \delta E_r \tag{40}$$

$$\delta j_\theta = \frac{1}{\eta + \frac{4\pi \mathrm{i} V_A^2}{c^2 \varpi}} \delta E_\theta \tag{41}$$

$$\delta j_z = \frac{1}{\eta} \delta E_z \tag{42}$$

Substituting Eqs. (40)～(42) into Eq. (17), we can obtain an expression of the driven current density

$$J_z = \frac{1}{2B_0\eta} \mathrm{Re}\left\{ (1-\frac{4\Omega^2}{\varpi^2})^{-1} \left(1 - \frac{\mathrm{i}c^2\eta}{4\pi\omega}\frac{\omega\varpi}{V_A^2}\right)^{-1} \left[(\delta E_r \delta B_r^* + \delta E_\theta \delta B_\theta^*) - (\frac{2\mathrm{i}\Omega}{\varpi})(\delta E_r \delta B_\theta^* - \delta E_\theta \delta B_r^*) \right] - (1-\frac{4\Omega^2}{\varpi^2})^{-1} \frac{4\pi \mathrm{i} V_A^2}{c^2\varpi} \bar{\delta} \left[\delta E_z \delta B_\theta^* + \frac{c}{4\pi r}(2-\hat{s})(\delta B_r \delta B_r^* + \delta B_\theta \delta B_\theta^*) \right] \right\} \tag{43}$$

In the following, we only retain the terms up to the first-order of $2\Omega/\varpi$ or $\bar{\delta}$ in Eqs. (17) and (43).

In our system, the total power absorption is given by

$$P = \mathrm{Re}\int \delta \boldsymbol{j} \cdot \delta \boldsymbol{E}^* \mathrm{d}V = 4\pi^2 R\, \mathrm{Re}\left\{ \int_0^a (\delta j_r \delta E_r^* + \delta j_\theta \delta E_\theta^* + \delta j_z \delta E_z^*) r \mathrm{d}r \right\} \tag{44}$$

Then, the current drive efficiency is defined as

$$\bar{\eta} = \frac{IR}{P} \tag{45}$$

Now we can analyze in more detail the current drive due to the comperssional Alfven mode, the shear Alfven mode, and mixing modes in tokamak rotating plasma with magnetic shear, respectively.

In this paper, we only consider m=0 in order to compare the Avinash's[15] and Chan et al.'s[5] results. We discuss three simple cases. Firstly, we consider a plane polarized comperssinal mode given by δE_z=0, propagating in the r-z plane. From the solution of wave equations, Eqs. (25) and (34)～(38), we have

$$\delta B_r = \frac{\mathrm{i}k_z}{l_z} b_c \mathrm{J}_0'(l_z r) \mathrm{e}^{\mathrm{i}k_z z} \tag{46.1}$$

$$\delta B_\theta = 0 \tag{46.2}$$

$$\delta B_z = b_c \mathrm{J}_0(l_z r) \mathrm{e}^{\mathrm{i}k_z z} \tag{46.3}$$

And

$$\delta E_r = \delta E_z = 0 \tag{47.1}$$

$$\delta E_\theta = -\mathrm{i}\frac{\omega}{c l_z} b_\mathrm{c} \mathrm{J}_0'(l_z r)\mathrm{e}^{\mathrm{i}k_z z} \tag{47.2}$$

For this mode, $\delta j_r = \delta j_z = 0$, and

$$\delta j_\theta = -\frac{c}{4\pi} l_z \left(1 + \frac{k_z^2}{l_z^2}\right) b_\mathrm{c} \mathrm{J}_0'(l_z r)\mathrm{e}^{\mathrm{i}k_z z} \tag{48}$$

According to Eqs. (17), (39) and (44), the current driven by the plane polarized compressional wave can be obtained

$$I = \frac{\pi |b_\mathrm{c}|^2}{c B_0 \eta} \int_0^a r\mathrm{d}r\, \mathrm{Re}\left\{\left(\frac{\mathrm{i}\omega k_z}{|l_z|^2}\right)\left(\frac{2\Omega}{\overline{\omega}} + \frac{k_z^2 V_\mathrm{A}^2}{\omega\overline{\omega}} \frac{1}{k_z R q}(2-\hat{s})\right) \mathrm{J}_0'(l_z r)\mathrm{J}_0^{*\prime}(l_z r)\right\} \tag{49}$$

And the absorbed power can be obtained

$$P = \pi R |b_\mathrm{c}|^2 \int_0^a r\mathrm{d}r\, \mathrm{Re}\left\{\left(\frac{-\mathrm{i}\omega k_z^2}{|l_z|^2}\right)\left(1 + \frac{l_z^2}{k_z^2}\right) \mathrm{J}_0'(l_z r)\mathrm{J}_0^{*\prime}(l_z r)\right\} \tag{50}$$

In some reasonable approximations, the efficiency $\overline{\eta}$ is given by

$$\overline{\eta} = \frac{1}{B_0 \eta c k_z} \frac{\int_0^a \left\{\left(\frac{2\Omega}{\omega} + \frac{k_z^2 V_\mathrm{A}^2}{\omega^2} \frac{1}{k_z R q}(2-\hat{s})\right) \mathrm{J}_0'(l_z r)\mathrm{J}_0^{*\prime}(l_z r)\right\} r\mathrm{d}r}{\int_0^a \frac{\omega^2}{k_z^2 V_\mathrm{A}^2} \mathrm{J}_0'(l_z r)\mathrm{J}_0^{*\prime}(l_z r) r\mathrm{d}r} \tag{51}$$

This equation is dependent on the plasma density. It is clear that, in the cylindrical geometry, the plane polarized compressional wave can drive current in tokamak rotating plasmas with magnetic shear. If the magnetic shear is omitted in Eq. (51), the efficiency is independent of the plasma density, which agrees with Avinash's result[15]. The effect of the magnetic shear makes the efficiency inversely proportional to the density. However, the efficiency with the negative magnetic shear is higher than that with the positive shear. Secondly, we consider a shear Alfven branch, $b_\mathrm{c} = 0$. For this case, we have $\delta B_r = \delta B_z = 0$, and $\delta B_\theta = b_\mathrm{s} \mathrm{J}_0'(l_\mathrm{b} r)\mathrm{e}^{\mathrm{i}k_z z}$;

$$\delta E_r = \frac{k_z V_\mathrm{A}^2}{c\overline{\omega}} b_\mathrm{s} \mathrm{J}_0'(l_\mathrm{b} r)\mathrm{e}^{\mathrm{i}k_z z} \tag{52.1}$$

$$\delta E_\theta = 0 \tag{52.2}$$

$$\delta E_z = -\frac{c\eta}{4\pi} l_\mathrm{b} b_\mathrm{s} \mathrm{J}_0(l_\mathrm{b} r)\mathrm{e}^{\mathrm{i}k_z z} \tag{52.3}$$

and

$$\delta j_r = -\frac{\mathrm{i}c}{4\pi} k_z b_\mathrm{s} \mathrm{J}_0'(l_\mathrm{b} r)\mathrm{e}^{\mathrm{i}k_z z} \tag{53.1}$$

$$\delta j_\theta = 0 \tag{53.2}$$

$$\delta j_z = \frac{c}{4\pi} l_b b_s J_0(l_b r) e^{ik_z z} \tag{53.3}$$

The driven current due to the shear Alfven wave is obtained

$$\begin{aligned} I = \frac{\pi}{B_0 \eta} |b_s|^2 \int_0^a r\mathrm{d}r \left(\frac{k_z V_A^2}{c\omega} \right) \left(\frac{k^2 \nu}{\omega} \right) \left\{ \frac{4\Omega}{\omega} + \frac{1}{k_z R q} (2 - \hat{s}) + \right. \\ \left. \bar{\delta} (1 - \frac{\omega}{k^2 \nu}) \left(\frac{2\pi V_A^2}{c^2 \eta \omega} \right)^{1/2} \left(\frac{\omega^2}{k_z^2 V_A^2} - 1 \right)^{1/2} \frac{J_0(l_b r)}{J_0'(l_b r)} \right\} J_0'(l_b r) J_0^{*'}(l_b r) \end{aligned} \tag{54}$$

The absorbed power is

$$P = \pi R |b_s|^2 \int_0^a r\mathrm{d}r \left(\frac{k_z^2 V_A^2}{\omega} \right) \left\{ \frac{k^2 \nu}{\omega} J_0'(l_b r) J_0^{*'}(l_b r) + \left(1 - \frac{\omega^2}{k_z^2 V_A^2} \right) J_0(l_b r) J_0^*(l_b r) \right\} \tag{55}$$

then, the current drive efficiency is

$$\begin{aligned} \bar{\eta} = \frac{1}{B_0 \eta c k_z} \left\{ \int_0^a r\mathrm{d}r \left(\frac{k_z^2 V_A^2}{\omega} \right) \left(\frac{k^2 \nu}{\omega} \right) \left\{ \frac{4\Omega}{\omega} + \frac{1}{k_z R q} (2 - \hat{s}) + \right. \right. \\ \left. \left. \bar{\delta} (1 - \frac{\omega}{k^2 \nu}) \left(\frac{2\pi V_A^2}{c^2 \eta \omega} \right)^{1/2} \left(\frac{\omega^2}{k_z^2 V_A^2} - 1 \right)^{1/2} \frac{J_0(l_b r)}{J_0'(l_b r)} \right\} J_0'(l_b r) J_0^{*'}(l_b r) \right\} \cdot \\ \left\{ \int_0^a r\mathrm{d}r \left(\frac{k_z^2 V_A^2}{\omega} \right) \left\{ \frac{k^2 \nu}{\omega} J_0'(l_b r) J_0^{*'}(l_b r) + \left(1 - \frac{\omega^2}{k_z^2 V_A^2} \right) J_0(l_b r) J_0^*(l_b r) \right\} \right\}^{-1} \end{aligned} \tag{56}$$

When $\omega^2/(k_z^2 V_A^2) \approx 1$, i.e. it is close to the Alfven resonance, in some reasonable approximations we have

$$\bar{\eta} \approx \frac{1}{c B_0 \eta k_z} \left[\frac{4\Omega}{\omega} + \frac{1}{k_z R q} (2 - \hat{s}) \right] \tag{57}$$

We find that the driven current efficiency due to the shear Alfven wave is not only independent of the plasma density but also independent of the kinematic viscosity, which is distinct from the compressional Alfven wave. However, it is the same as the compressional Alfven wave that the effect of the negative magnetic shear on the driven current is more efficient than the effect of the positive. Thirdly, we consider the drive current due to circularly polarized wave which requires mode mixing, $\delta B_r = i\delta B_\theta$, thus $b_s J_0'(l_b r) = b_c \frac{k_z}{l_z} J_0'(l_z r)$. According to Eqs. (25) and (34)～(38), for the circularly polarized wave we have

$$\delta B_r = \frac{ik_z}{l_z} b_c J_0'(l_z r) e^{ik_z z} \tag{58.1}$$

$$\delta B_\theta = \frac{k_z}{l_z} b_c J_0'(l_z r) e^{ik_z z} \tag{58.2}$$

$$\delta B_z = b_c J_0(l_z r)e^{ik_z z} \tag{58.3}$$

and

$$\delta E_r = \frac{k_z^2 V_A^2}{c\varpi l_z} b_c J_0'(l_z r)\, e^{ik_z z} \tag{59.1}$$

$$\delta E_\theta = -i\frac{\omega}{cl_z} b_c J_0'(l_z r)\, e^{ik_z z} \tag{59.2}$$

$$\delta E_z = -\frac{c\eta l_b}{4\pi l_z} k_z b_c \frac{J_0(l_b r)}{J_0'(l_b r)} J_0'(l_z r)e^{ik_z z} \tag{59.3}$$

For this wave, the first-order perturbation current is given by

$$\delta j_r = -i\frac{c}{4\pi}\frac{k_z^2}{l_z} b_c J_0'(l_z r)e^{ik_z z} \tag{60.1}$$

$$\delta j_\theta = -\frac{c}{4\pi}\left(l_z^2 + k_z^2\right)\frac{1}{l_z} b_c J_0'(l_z r)e^{ik_z z} \tag{60.2}$$

$$\delta j_z = -\frac{c\eta l_b}{4\pi l_z} k_z b_c \frac{J_0(l_b r)}{J_0'(l_b r)} J_0'(l_z r)e^{ik_z z} \tag{60.3}$$

Substituting Eqs. (58) and (60) into Eq. (17) and only retaining the terms up to the first-order of $2\Omega/\varpi$ or $\bar{\delta}$, we have

$$J_z = -\frac{|b_c|^2}{2B_0\eta c}\mathrm{Re}\left\{\left(\frac{\omega k_z}{|l_z|^2}\right)\left(\frac{ik_z^2 V_A^2}{\omega\varpi}\right)\left[\left(1+\frac{2\Omega}{\varpi}\right)(2+\frac{l_z^2}{k_z^2})J_0'(l_z r)J_0^{*\prime}(l_z r) + \frac{2}{k_z Rq}(2-\hat{s})J_0'(l_z r)J_0^{*\prime}(l_z r) - \frac{\bar{\delta}}{k_z} l_b \frac{J_0(l_b r)}{J_0'(l_b r)} J_0'(l_z r)J_0^{*\prime}(l_z r)\right]\right\} \tag{61}$$

where $l_b = \frac{l}{\sqrt{2}}(1+i)$ and $l \approx \left(\frac{4\pi k_z^2 V_A^2}{c^2\eta\omega}\right)^{1/2}\left(\frac{\omega^2}{k_z^2 V_A^2} - 1\right)^{1/2}$. Substituting Eqs. (59) and (60) into Eq. (44) yields

$$P = 2\pi R|b_c|^2\int_0^a \left(\frac{\omega k_z^2}{|l_z|^2}\right)\left(\frac{k_z^2 V_A^2}{\omega^2}\right)\left[\frac{k^2\nu}{\omega}\left(1+\left(\frac{\omega^2}{k_z^2 V_A^2}\right)^2\right) + \left(\frac{\omega^2}{k_z^2 V_A^2} - 1\right)\frac{J_0(l_b r)J_0^*(l_b r)}{J_0'(l_b r)J_0^{*\prime}(l_b r)}\right] J_0'(l_z r)J_0^{*\prime}(l_z r) r\,dr \tag{62}$$

When $\omega^2/(k_z^2 V_A^2) \approx 1$, we can obtain an approximate formula for the current drive efficiency due to the circularly polarized wave

$$\eta \approx \frac{1}{B_0\eta c k_z}\left[1 + \frac{4\Omega}{\omega} + \frac{1}{k_z Rq}(2-\hat{s})\right] \tag{63}$$

It is thus clear that even if there are no rotation and shear, the circularly polarized wave can drive current, which is in agreement with Chan et al.'s result (see Eq. (54)

in Ref. [5]). In the present paper, we find that the efficiency of the driven current by the circularly polarized wave is independent of the plasma density even though the effects of the rotation and magnetic shear are taken into account. The effects of the rotation and the negative magnetic shear make the efficiency higher.

3 CONCLUSIONS

In this paper, we have studied the driven current of the Alfven waves in tokamak rotating plasma with magnetic shear. In cylindrical geometry, an expression of the driven current is obtained by means of single-fluid MHD model taking the plasma rotating and the magnetic shear into account. For the poloidal mode number $m= 0$ we have considered current drive due to the plane polarized compressional Alfven wave, the shear Alfven wave and the circularly polarized wave. If neglecting both the rotation and shear, our result reduces to the corresponding result in Ref. [5]. When only the magnetic shear is not taken into account, the present result reduces to the corresponding result in Ref. [15]. In the presence of the magnetic shear, the efficiency of the current drive due to the plane polarized compressional Alfven mode is dependent on the plasma density, but the driven efficiency due to the shear and the circularly polarized Alfven waves is independent of the density.

Our analysis shows that the negative magnetic shear is favorable to the efficiency of the driven current by the shear Alfven wave. The plasma rotating is always a good effect on the current drive efficiency with or without the magnetic shear. Consequently, the Alfven wave current drive may be an excellent candidate of current drive for high confinement advanced tokamaks.

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