MATRIX FORMULATION OF PEBBLE CIRCULATION IN THE PEBBED CODE

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ABSTRACT

The PEBBED technique provides a foundation for equilibrium fuel cycle analysis and optimization in pebble-bed cores in which the fuel elements are continuously flowing and, if desired, recirculating. In addition to the modern analysis techniques used in or being developed for the code, PEBBED incorporates a novel nuclide-mixing algorithm that allows for sophisticated recirculation patterns using a matrix generated from basic core parameters. Derived from a simple partitioning of the pebble flow, the elements of the recirculation matrix are used to compute the spatially averaged density of each nuclide at the entry plane from the nuclide densities of pebbles emerging from the discharge conus. The order of the recirculation matrix is a function of the flexibility and sophistication of the fuel handling mechanism. This formulation for coupling pebble flow and neutronics enables core design and fuel cycle optimization to be performed by the manipulation of a few key core parameters. The formulation is amenable to modern optimization techniques.

Introduction

The PEBBED [1] code is a new tool for analyzing the asymptotic fuel cycle in recirculating pebble-bed reactors. Equations for neutron flux and nuclide distribution in a pebble-bed core are solved self-consistently by an iterative scheme, and the algorithm is shown to converge quickly to a solution unique to the pebble flow pattern. The neutronics solver currently relies on a standard finite difference technique, but more advanced solution methods are planned. The burnup solver uses a semi-analytical method that guarantees convergence with accuracy. A key step in the algorithm is the computation of the entry-plane density of each nuclide of interest in each axial flow channel. These values depend upon the pebble loading and recirculation policy and the burnup accrued by pebbles on successive passes through the core. The current iterate of the flux is used to compute the exit-plane nuclide density in a pebble after one pass through the core in each channel, based on the density of that nuclide in a fresh pebble. Pebbles are then distributed according to the recirculation scheme to generate the entry-plane density in each channel on the next pass. This is repeated until the pebbles exceed the discharge burnup. The exit-plane values are then averaged according to the recirculation scheme in order to produce the actual entry-plane nuclide densities. The entry-plane nuclide flow rate is derived in the next section.

The homogenized entry plane nuclide density of a given nuclide for each flow channel, expressed here as the vector $\vec{N}$, is computed as a weighted average of the contributions from pebbles of various types and trajectories. The symbol ${}^{m}\vec{N}^{p}$ refers to the number density vector (the elements of which correspond to the flow channels) of that nuclide in a pebble of type *p* that has passed through the core *m* times. A *recirculation matrix* **R** stores the weight of each contribution so that

$$\vec{N} = \mathbf{R}\,{}^{m}\vec{N}^{p} \quad . \tag{1}$$

It is shown in this paper that the values of the elements of R are dependent upon basic core parameters and thus can be computed manually or generated using a suitable optimization algorithm.

Theory

The flow rate of a given nuclide in pebble flow channel *i* is composed of contributions from pebbles of different types (*p*) and different prior histories. For a core with *P* pebble types each undergoing an average of M_p passes before discharge, the flow rate (atoms per second) of that nuclide at the entry plane of channel *i* is the sum of of the flow rates of the nuclide in pebbles of all types and pass histories at this location, expressed as

$$\dot{n}_i = \sum_{p=1}^{P} \sum_{m=1}^{M_p} {}^{m}\hat{N}_i^{\,p} f_i \cdot \alpha_i^{\,p} \cdot {}^{m}\alpha_i^{\,p} \quad , \tag{2}$$

where

$\alpha_i^{\,p}$	is the fraction of channel *i* flow that consists of type *p* pebbles,
${}^{m}\alpha_i^{\,p}$	is the fraction of type *p* pebbles in channel *i* flow that are on their m^{th} PASS,
${}^{m}\hat{N}_i^{\,p}$	is the number density of the nuclide of interest within pebbles of type *p*, entering channel *i*, starting their m^{th} pass, and
f_i	is the volumetric flow rate of pebbles through channel *i*.

One can show that the channel-averaged nuclide density at the entry plane of channel *i* is given by

$$N_i = \sum_{p=1}^{P} \left\{ {}^{1}\hat{N}_i^{\,p} \cdot {}^{1}\alpha_i^{\,p} \cdot \alpha_i^{\,p} + \sum_{m=1}^{M_{\max}-1} \sum_{j=1}^{J} {}^{m}\widehat{N}_j^{\,p} \cdot \left[\frac{\alpha_j \cdot \alpha_j^{\,p} \cdot {}^{m}\alpha_j^{\,p} \cdot {}^{m}\alpha_{ij}^{\,p}}{\alpha_i} \right] \right\} \quad , \tag{3}$$

where

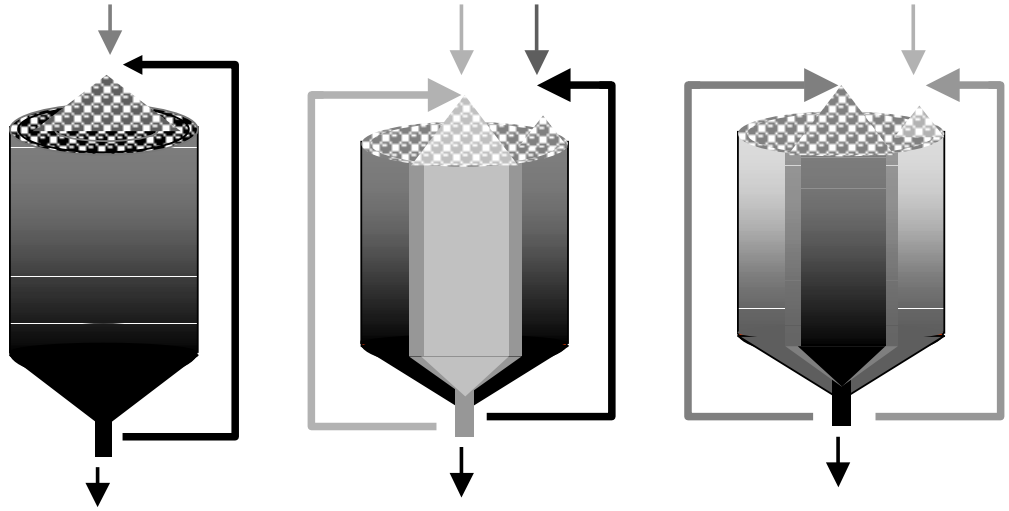
α_i	is the fraction of pebble flow that passes through channel *i*,
${}^{m}\alpha_{ij}^{\,p}$	is the fraction of type *p* pebbles in flow channel *j*, and on pass *m*, that are transferred to channel *i*, following this m^{th} pass,
${}^{m}\widehat{N}_j^{\,p}$	is the number density of the nuclide of interest within pebbles of type *p*, exiting channel *j*, after completing their m^{th} pass, and
f_j	is the flow rate of pebbles through channel *j*.

The values are shown here to be functions of the flow properties of the core and the fuel loading mechanism. Three models are discussed: the HTR Modul 200 [2], the PBMR [3], and an alternative PBMR cycle.

The HTR Modul 200 (Figure 1a) possesses a single loading tube and a single discharge tube. The channel coefficients α_i are determined by the channel boundaries and the radial flow distribution. There is only one pebble type (P = 1), thus $\alpha_j^{\,p} = 1$. The pebbles emerging from the bottom are randomly dropped back onto the bed, so the so-called *transfer* coefficient ${}^{m}\alpha_{ij}^{\,p} = \alpha_i$ for all *i*, *j*, *p*, and *m*. The random recirculation also implies that the burnup classes are equally represented in each zone, i.e. ${}^{m}\alpha_j^{\,p} = (M_{\max})^{-1}$. The recirculation matrix for a core with *J* flow channels is then given by

$$\mathbf{R} = \frac{1}{M_{\max}} \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_J \\ \alpha_1 & \alpha_2 & \cdots & \alpha_J \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1 & \alpha_2 & \cdots & \alpha_J \end{bmatrix} \quad . \tag{4}$$

The value of M_{max} is fixed by the core power, geometry, heavy metal content of pebbles, and discharge burnup. Hence the recirculation matrix elements are also entirely determined by these quantities and the partition of the core flow. The fuel handling mechanism in this design cannot be used to vary the asymptotic core nuclide distribution.



(a) HTR Modul 200 (b) PBMR with Graphite Inner Reflector (c) PBMR with OUT-IN Fuel Cycle

Figure 1. Modular Pebble-Bed Cores with Different Fuel Cycles

The Pebble-Bed Modular Reactor (PBMR) design under consideration by the South African utility Eskom uses two pebble types (graphite and fuel) flowing in separate regions of the core (Figure 1b). The graphite pebbles are dropped onto the bed via a central loading tube. The fuel pebbles are loaded by a number of loading tubes evenly spaced near the core periphery. There is no barrier between the two regions, so there is a zone between them in which the pebble types are mixed. The asymptotic core consists of a central reflector region composed entirely of graphite and a surrounding annulus of fuel pebbles. The size of the graphite region is regulated by the relative flow rates in the central and peripheral loading tubes. Many models of the PBMR core feature five concentric flow zones with roughly equivalent flow rates[4]. The volume of the graphite reflector is about 25% of the pebble bed volume. Thus the innermost channel is composed of only graphite, the second channel contains roughly equal portions of fuel and graphite pebbles, and the outer three channels consist only of the fueled type. The radial placement and discharge of pebbles is not burnup (pass) dependent, so the burnup classes are equally represented in all channels. Though slightly more complicated than the HTR Modul 200, the partition coefficients are again simple functions of core flow properties, power, and discharge burnup. The recirculation matrix (**R**) can be expressed as two submatrices; one each for fuel (*f*) and graphite (*g*):

$$\mathbf{R}^f = \frac{1}{M_{\max}\alpha^f}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & .25\alpha_2 & .5\alpha_3 & .5\alpha_4 & .5\alpha_5 \\ 0 & .5\alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 \\ 0 & .5\alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 \\ 0 & .5\alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 \end{bmatrix} \quad , \quad \mathbf{R}^g = \frac{1}{M_{\max}\left(1-\alpha^f\right)}\begin{bmatrix} 0 & .5\alpha_2 & 0 & 0 & 0 \\ .5\alpha_1 & .25\alpha_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & .0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad . \quad (5)$$

The coefficient α^f refers to the fraction of total core flow consisting of fuel pebbles. This value is a simple function of the relative flow rates through the central and peripheral loading tubes and thus offers a degree of freedom in core design not available in the HTR Modul 200. By "tuning" the tube flow rates, one alters the sizes of the central reflector and active core annulus.

The fuel loading mechanism of the PBMR also allows for another type of two-region core. An "OUT-IN" cycle [5] is possible in which fresh fuel pebbles are loaded via the peripheral tubes but no graphite pebbles are used (Figure 1c). The fuel circulates in the outer region until an intermediate burnup threshold is exceeded. This occurs after a specified number of passes M_T, after which the pebbles are then loaded via the central tube. At equilibrium, the central region then consists of highly depleted elements while the annulus is composed of relatively fresh elements. Like the HTR Modul, there is only one pebble type ($\alpha_j^p = 1$), but the transfer coefficients vary with the pass number *m*. To conserve pebble flow, only a fraction α_T of the pebbles completing pass M_T are diverted to the inner region; the remainder are circulated once more and diverted on the following pass. Defining α_j^o as the fraction of flow in channel *j* that is in the outer region, one can derive the following expressions for the transfer coefficients:

$$ {}^m\alpha_{ij} = \frac{\alpha_i^o \alpha_i}{\sum_{\text{all } i} \alpha_i^o \alpha_i} \qquad m < M_T \tag{6a} $$

$$ {}^m\alpha_{ij} = \alpha_T \alpha_j^o \frac{\left(1-\alpha_i^o\right)\alpha_i}{1-\sum_{\text{all } i} \alpha_i^o \alpha_i} + \left(1-\alpha_T\right)\alpha_j^o \frac{\alpha_i^o \alpha_i}{\sum_{\text{all } i} \alpha_i^o \alpha_i} + \left(1-\alpha_j^o\right)\frac{\left(1-\alpha_i^o\right)\alpha_i}{1-\sum_{\text{all } i} \alpha_i^o \alpha_i} \qquad m = M_T \tag{6b} $$

$$ {}^m\alpha_{ij} = \frac{\left(1-\alpha_j^o\right)\alpha_i}{1-\sum_{\text{all } i} \alpha_i^o \alpha_i} \qquad m > M_T \quad . \tag{6c} $$

Furthermore, conservation of flow also fixes the values of the fraction of outer flow transferred (α_T) and the transfer pass number M_T according to

$$ \alpha_T = 1 + M_T - \frac{F \sum_{all\ j} \alpha_j^o \alpha_j}{{}^1F}, \qquad M_T = INT\left(\frac{F \sum_{all\ j} \alpha_j^o \alpha_j}{{}^1F} \right), \tag{7} $$

in which *F* is the total core pebble flow and ${}^1F$ is the total fresh fuel injection rate.

Obtaining *pass* coefficients ${}^m\alpha_j^p$ is less straightforward. The burnup-dependence of this recirculation scheme means that the burnup classes are not equally represented in each channel. Here, flow conservation is exploited to obtain a system of algebraic equations that represents the flow balance of all the channels. The flow of pebbles commencing their m^{th} pass in channel *i* consists of contributions from pebbles having completed *m-1* passes in all channels. This fact yields *m-1* equations for each channel *i* and pebble type *p* of the form

$$ \alpha_i \cdot \alpha_i^p \cdot {}^m\alpha_i^p = \sum_{j=1}^{J} \alpha_j \cdot \alpha_j^p \cdot {}^{m-1}\alpha_j^p \, {}^{m-1}\alpha_{ij}^p \quad . \tag{8} $$

The final equation required to determine the system completely is a direct consequence of flow conservation; i.e., the pass partition coefficients must sum to unity. Since all channels are coupled, the

solution to this system involves inverting a matrix of order M_p*J, the product of the number of channels and the total number of passes traversed by each pebble type p.

The values of α_j^o are computed in the same manner as the type coefficients in the previous example, They are a function of the size of the inner region and thus can be "tuned" by adjusting the relative flow rates in the loading tubes. Tuning the size of the inner region in this way also changes the value of the intermediate threshold burnup above which the fuel is transferred from the outer to the inner channel. Although computing the coefficients in a burnup-dependent recirculation scheme such as this is rather more complicated than in the other core types, all of the coefficients can be easily computed from the basic core parameters of power, fuel content in the pebbles, discharge burnup, flow velocity, core height and radius, and loading tube flow rates.

Results

Figure 2 illustrates the results of PEBBED calculations for the nominal PBMR (with graphite pebbles) and the same core with the OUT-IN fuel cycle described above. No attempt was made to optimize either core for a particular characteristic. Rather, the flow rate of the OUT-IN core was adjusted so that the high-burnup inner region was the same size as the graphite reflector region in the PBMR. This sets the transfer burnup threshold at about 62 MWD/kg_{hm} which is attained during the tenth pass through the core. Each pebble then circulates four times in the inner region.

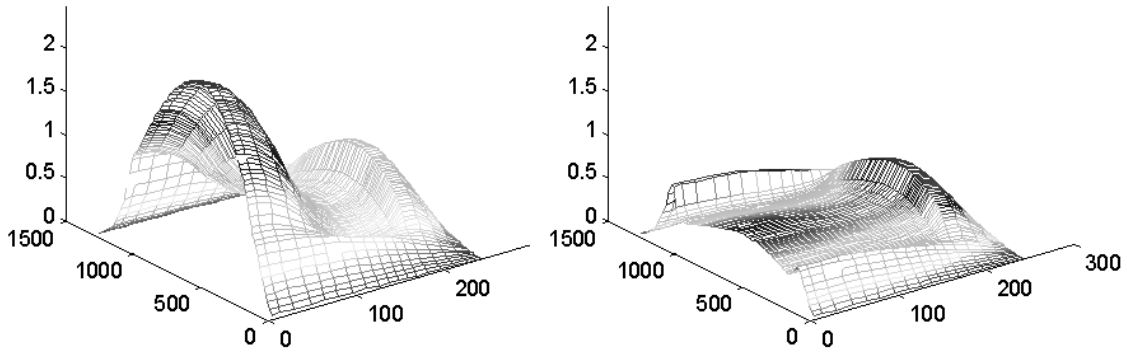


Figure 2. Thermal Flux in Nominal PBMR (left) and PBMR with OUT-IN Cycle
(the origin corresponds to the top and center of the pebble bed)

The thermal flux peak in the graphite inner reflector clearly distinguishes the two cases. For the nominal PBMR, the peak fission power density is computed to be 5.57 W/cm^3. For the OUT-IN core, the peak fission power density is computed to be 4.78 W/cm^3. The lower peak power density was achieved at the expense of neutron economy. Compared to the nominal PBMR fuel enrichment of 8%, the OUT-IN core required an fresh pebble enrichment of 10% to maintain criticality.

Although an advanced optimization algorithm has yet to be added to the code, PEBBED has been used for some simple applications. In a study described elsewhere in these proceedings[6], the code was used to assess some of the proliferation characteristics of a pebble bed reactor. The ability to model and track pebbles of different types and trajectories was also exploited to develop a fuel testing and qualification plan for PBMR fuel at the INEEL[7]. Small quantities of fuel pebbles were restricted to specific flow channels to determine the extreme conditions and operating envelope of the fuel. Figure 3 shows the accumulated burnup and fluence for average pebble and for pebbles restricted to channels 3 and 5 (all fuel).

Conclusions and Further Work

The matrix formulation for coupling nuclide flow to neutronics in the PEBBED code provides an efficient method for accurately modeling all types of pebble bed cores and for performing advanced core design and fuel management. This work reveals how generalized flow coefficients are derived from basic core parameters so that different pebble types and trajectories can be modeled with ease. With this tool, advanced techniques such as genetic algorithms can be applied to perform rapid, accurate, and comprehensive scoping studies and core optimization.

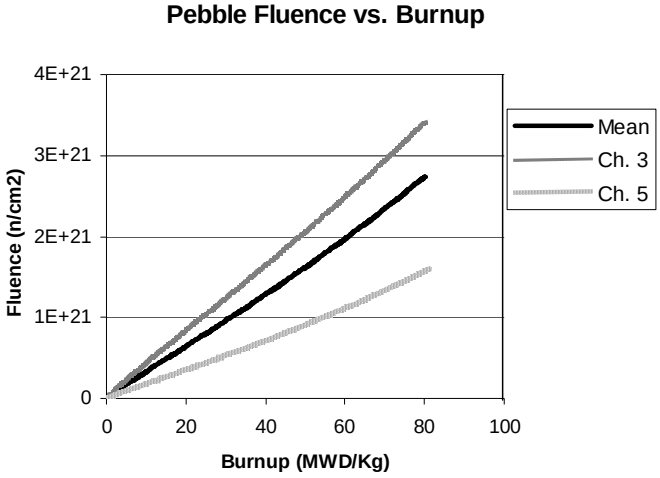


Figure 3. Fast Fluence vs. Burnup for PBMR Fuel

Currently, the code assumes that pebble flow is strictly axial. For an accurate model of the discharge conus region, the pebble flow grid must be decoupled from the diffusion grid and generalized to two or three dimensions. This is not anticipated to invalidate the matrix approach. Future work will also include the development of a three-dimensional nodal diffusion solver and the matrix formulation will be expanded to allow for azimuthal variation in pebble placement. Advanced cross-section generation and thermal feedback parameterization must also be incorporated to capture spectral and thermal effects. All of these improvements are part of the work scope of PEBBED development.

Acknowledgement

Work supported by the U.S. Department of Energy, Assistant Secretary for Environmental Management, under DOE Idaho Operations Office Contract DE-AC07-99ID13727.

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