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NUMERICAL ANALYSIS OF INTERACTING CRACKS IN BIAXIAL STRESS FIELD

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ABSTRACT

The stress corrosion cracks as seen for example in PWR steam generator tubing made of Inconel 600 usually produce highly irregular kinked and branched crack patterns. Crack initialization and propagation depends on stress state underlying the crack pattern. Numerical analysis (such as finite element method) of interacting kinked and branched cracks can provide accurate solutions. This paper discusses the use of general-purpose finite element code ABAQUS for evaluating stress fields at crack tips of interacting complex cracks. The results obtained showed reasonable agreement with the reference solutions and confirmed use of finite elements in such class of problems.

INTRODUCTION

Intergranular cracks develop in random and complex patterns [1] (e.g., cracks caused by outer diameter stress corrosion cracking in PWR steam generator tubing made of Inconel 600). The shape of cracks depends strongly on the shape of grains. A significant degree of crack branching and coalescence may be observed. The methods currently implemented in similar problems have limitations with respect to crack shape (e.g., degree of branching) and level of details implemented in crack interaction modeling [2].

The aim of this paper is to validate the use of finite element for analysis of stress fields around tips of randomly shaped complex interacting cracks [3, 4]. Crack shapes and interaction effects are limited only with the shape of underlying random grain structure. No limitations are due to the analysis method.

Stress fields around tips of complex shaped cracks in crack pattern are calculated by finite element method. Finite element method also accounts for the influences of crack shape and interactions between two or more shapes and supports use of general biaxial stress field. As such, finite element method is more suitable for stress field analysis than previously adopted empirical models [2], which correlate the actual crack shape in equibiaxial stress field with a simple equivalent replacement crack. This approach is unpredictable, when unknown crack forms are considered.

The paper discusses the mathematical model, followed by the numerical example and results. Conclusions are added at the end.

MATHEMATICAL MODEL

Finite element models of random crack patterns for finite element analysis were evaluated as follows:

1. *Selection of crack shapes*

   Generate model of random grain structure
   Incomplete random tessellation as one of methods of stohastic geometry tessellation is utilized to model randomly distributed grain boundaries. Model of randomly distributed grain boundaries is currently limited to 2-D, which can be directly compared with results of metalographic analyses [1, 2].

   Crack initialization and propagation
   Cracks are generated on randomly selected grain boundaries. The initialization and propagation of cracks depends, among others, on the evaluated stress field at crack tip. Details are given in [1, 2].

2. *Automatic generation of finite element mesh*
   Current meshing strategy takes grain boundaries fully into account [5]. Grains are irregularly shaped [2] (e.g. acute or obtuse angles, polygons of high order) and as such can not be used as finite elements in most commercialy available finite element codes. Therefore, the grains are further subdivided in finite elements. Grain structure was automatically meshed by CADfix [6] and then exported for evaluation to finite element code ABAQUS [7].

3. *Calculation of J-integral and comparison with reference solution.*
   J-integral was selected as a measure of stress field around the crack tips. It provides stability of results obtained using coarse meshes and therefore reduces computational efforts [3, 4].

   Results of *J*-integral by finite element method for each of the crack tips were compared to the reference solutions. Reference solutions for straight cracks were obtained analytically, but without accounting for interaction between the neighboring cracks. Reference solutions for kinked and branched cracks were obtained by empirical model [2]. The interaction between neighboring cracks is only approximately accounted for in reference solutions.

   Expected accuracy of used finite element code ABAQUS [7] was obtained from extensive evaluation of simple shaped cracks with analytical *J*-integral solutions [8]. Expected accuracy depends on the finite element mesh size.

NUMERICAL EXAMPLE

Evaluation of the stress field around crack tips within a plate is presented here as a numerical example. The plate is divided into 101 randomly shaped grains and loaded with remote biaxial stress ($\sigma_x = 2$ N/m^2, $\sigma_y = 1$ N/m^2, E = 1 N/m^2, $\nu = 0.3$).

Figure 1 presents random grain structure (thin lines), which was generated using incomplete random tessellation with cracks (wider lines) generated on randomly selected grain boundaries. The cracks in Figure 1 are an outcome of both crack initialization and crack propagation. With further propagation complexity of crack shape usually increases.

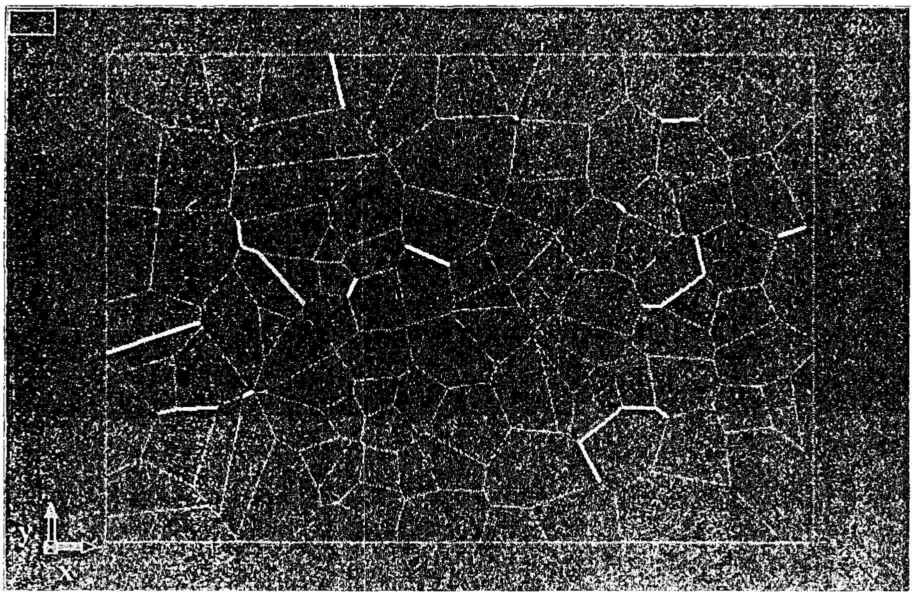

Figure 1: Grain structure with crack pattern

Figure 2 presents propagation of a selected single crack (thick lines) over adjacent grain boundaries (thin lines) during several simulation steps.

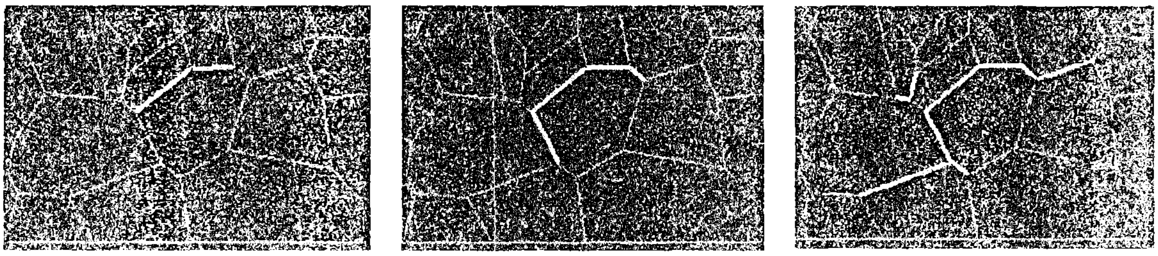

Figure 2: Enlarged cutout from the grain structure with crack pattern

Figure 2 clearly shows the processes of crack initialization in step 0 and propagation in steps 1 and 2.

Model meshing is an essential part of finite element method. In particular case, automatic generation of finite element mesh with quadrilateral isoparametrical elements was used.

Figure 3 shows finite element mesh generated over the random grain boundaries and crack pattern (presented in Figure 1). Cracks are denoted as thick bright lines and mesh as thin gray ones.

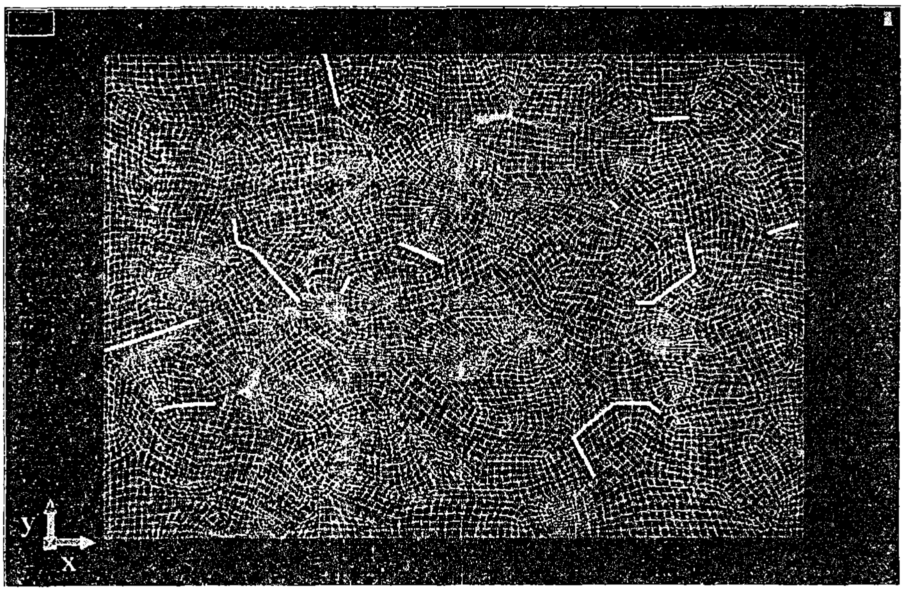

Figure 3: Finite element mesh of grain structure with crack pattern

Stress field around each of the crack tips of the crack pattern was calculated using general-purpose finite element code ABAQUS.

Figure 4 shows Von Mises equivalent stress within grain structure with crack pattern. Cracks are denoted as thick white lines.

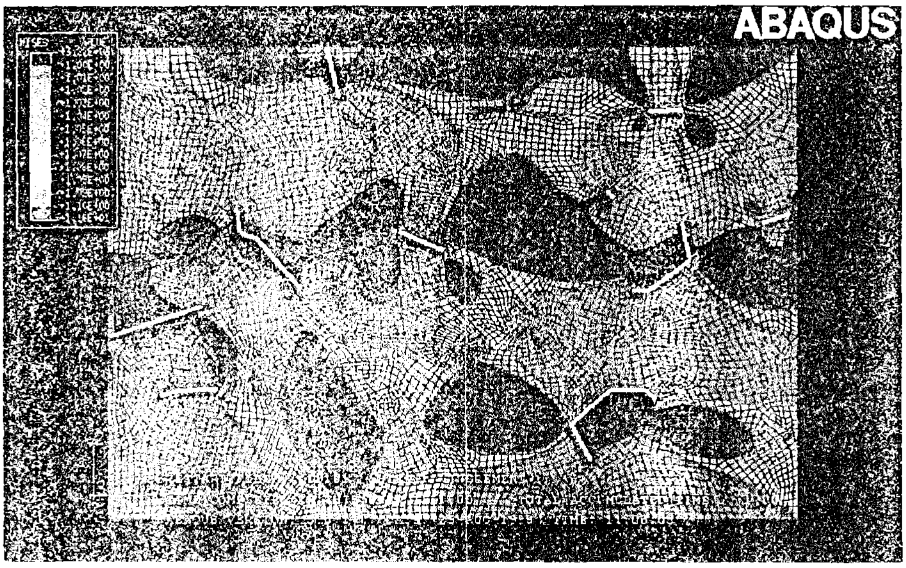

Figure 4: Von Mises equivalent stress of grain structure with crack pattern

Figure 4 shows the influences of the cracks in the uniform remote stress field. Strengthening and weakening of the stress field is especially pronounced in the vicinity of crack tips.

RESULTS

J-integral was selected as a measure of stress field around the crack tips. Results of J-integral for each of the crack tips in the crack pattern were compared to the reference solutions. Results of J-integral were studied as a function of relative size of typical element, which is a measure of a finite element mesh density: size of a typical element at the crack tip divided by the crack length. Smaller relative size of the typical element means more dense mesh and higher accuracy of the results [3, 4].

Results of J-integral were obtained for patterns with increasing complexity and number of cracks. With larger number of cracks in the pattern the impact of interactions between cracks becomes more important and causes larger discrepancies of the results with the reference solutions, which do not model the interaction very accurately.

In further discussion the cracks were divided into straight, kinked and complex cracks.

Straight cracks

Reference solutions for straight cracks were obtained analytically [8]. These solutions however do not account for interactions between neighboring cracks.

Figure 5 shows the accuracy of J-integral estimates for straight cracks as a function of typical element. Thin black line (0%) represents reference solution, thick black lines represent expected numerical accuracy of finite element method [4].

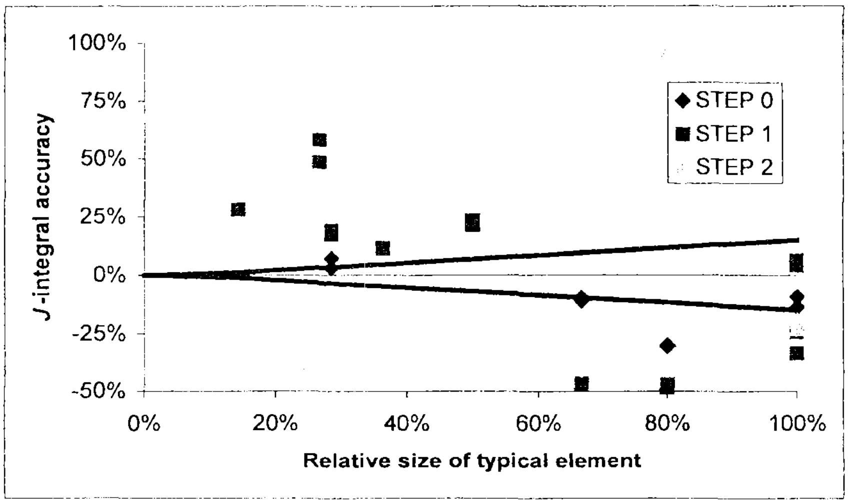

Figure 5: Accuracy of J-integral compared with analytical results for straight cracks

The discrepancy of results obtained is mainly due to the interaction between neighboring cracks, which are not accounted for in reference solutions. More interactions are produced by more cracks in the crack pattern. Crack pattern of step 2 has, in principle, more crack interactions than crack pattern of step 1 and the latter more than crack pattern of step 0, which is in accordance with observed discrepancy.

Kinked cracks

Reference solutions for kinked cracks were obtained by empirical model [2]. The interaction between neighboring cracks is only approximately accounted for in reference solutions.

Figure 6 shows the accuracy of J-integral estimates for kinked cracks as a function of typical element. Thin black line (0%) represents reference solution, thick black lines represent expected numerical accuracy of finite element method [4].

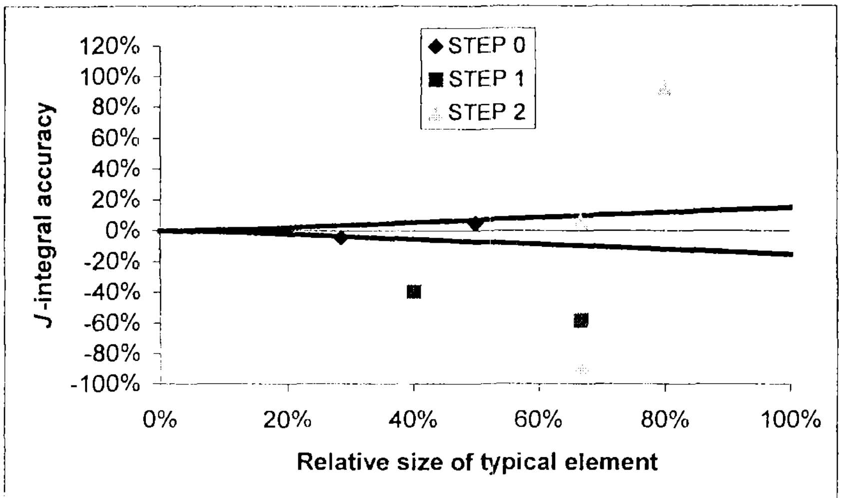

Figure 6: Accuracy of J-integral compared with results of an empirical model for kinked cracks

The discrepancy of results obtained – more pronounced in steps 1 and 2 – is mainly due to the interaction between neighboring cracks, which is only approximately accounted for in reference solutions.

Complex cracks

Reference solutions for complex cracks were obtained by empirical model [2]. The interaction between neighboring cracks is only approximately accounted for in reference solutions.

Figure 7 shows the accuracy of J-integral estimates for complex cracks as a function of typical element. Thin black line (0%) represents reference solution, thick black lines represent expected numerical accuracy of finite element method [4].

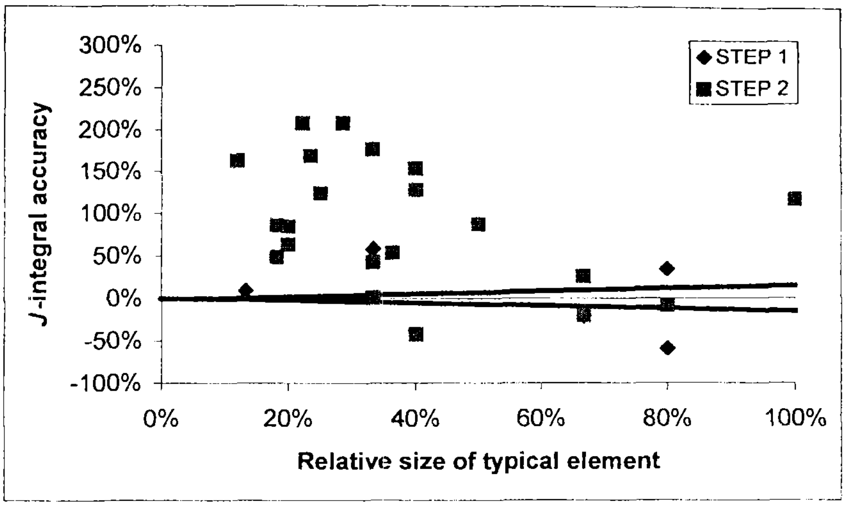

Figure 7: Accuracy of J-integral compared with results of an empirical model for complex and branched cracks

The discrepancy of results obtained is mainly due to the interaction between neighboring cracks, which is only approximately accounted for in reference solutions. As in Figures 5 and 6, the discrepancy is more noticeable in latter steps.

CONCLUSIONS

This paper discusses the use of general-purpose finite element code ABAQUS to evaluate crack tip stress fields in crack patterns with multiple interacting complex shaped cracks. The main conclusions are:

- Reasonable agreement with the reference results.
- Discrepancies between results obtained and reference results are mainly due to different assumptions in available reference cases.

Accuracy of results supports use of finite element analysis in simulations of developing intergranular crack patterns.

LITERATURE

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