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The Production and Transmission of Covarience in the Evaluation Processing of Fission Yield Data

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【**abstract**】 *The production and transmission of correlation in the evaluation processing of fission yield data, including average with weight, ratio and sum consistence adjusting, are researched. The variation of the averaged and adjusted yields and/or rations with the correlation coefficient of the input data are investigated. The results obtained are reasonable in physics.*

Introduction

The error, as traditionally given, is only the diagonal elements of the covariance matrix and only describes the total error of the data, nothing about the systematical error or the correlation of the data. The systematical error is more important for the engineering applications, because the statistical one can be counteracted in the calculation by using large amount of data, but the systematical one could not be. For evaluators and experimenters, the information is given completely, only in the case of the covariance matrix is given out.

As well known that the correlation describes the systematical error or medium and long range error[1] in the experiment, for example, due to calibration of detector efficiency, multiple scattering correction and measurement geometry, sample quantification, normalization etc. All of these depend on the experimental conditions. Also the correlation can be produced in the data evaluation processing, which is not only from the mathematical restriction condition, for example, the curve must be smooth in the curve fitting by using least-square method, but also from physical restrictive condition, for instance, the total must be the sum of all partials, absolute cross section or fission yield should be consistent with their concerned ratios measured etc.

For fission yield data, the main processing in the evaluation is

a. average with weight;

b. the consistency adjusting between the absolutely measured yields and the ratios concerned;

c. the consistency adjusting between the sum and its partials, e.g. cumulative yield and its independent yields concerned.

All of these processing can be done by using code ZOTT[2], with the different form of the sensitivity matrix. ZOTT is a general partitioned least squares method code, using for adjusting a combined set of data $Y(I)$ of differential and integral quantities with covariance $CY(I,J)$ to new values $Y'(I)$ with covariance $CY'(I,J)$, which are minimum variance linear unbiased estimator of the true values and satisfy the physical relationship prescribed by the sensitivity matrix $S(K,L)$, where I or J is the total number of differential plus integral quantities, K is the number of the integral quantities and L is the number of differential quantities.

In this paper, the production and transmission of the covariance in the fission yield data evaluation processing, as listed above, are investigated. The first section is for the average with weight, and the second one is for ratio adjusting, the third one is for sum adjusting, a practical example is given in the section

4 and the conclusion remarks are given in section 5 at the last.

1 Averaging with Weight

The data measured at same energy point for the same quantity should be averaged with weight, which can be done with code AVERAG [3] or ZOTT. The former is a special code for averaging, weighted or mathematical average. In the later case, the sensitivity matrix should be defined, as given below.

An example is given for the fission yield of product nuclide ^{103}Ru at thermal energy point. There are 7 sets of absolutely measured data. In this case, the sensitivity matrix is given in the form of (6 rows × 1 column), which means the all of last 6 quantities should be equal to the first one, that is the result should be the average with weight of all 7 quantities.

The calculated results with different correlation coefficients are given in Fig. 1. When the correlation coefficient is 0.0, the results, including the quantity itself and its error, are the same as calculated with code AVERAG. It means that the adjusted value is the mean with weight and its error is the internal error of the mean with weight, only depends on the fission yield error D$Y(I)$, no matter how much the differences among the yields $Y(I)$ are. From Fig.1 it can be seen that the mean increases with the increasing of correlation coefficient ρ. When correlation coefficient ρ varies from 0.0 to 0.999, the mean of the yield varies from 3.043 to 3.0656, that is 0.7% change. The change of the mean error is shown in Fig. 2. It can be seen that when the correlation increases, the error of the mean increases slightly from 0.0299(0.98%) to 0.0373(1.22%) at first, then decreases rapidly and very sharply to 0.0 when the correlation coefficient closes to 1.0 at last.

2 Ratio Adjusting

It is the general case that not only the yields themselves but also their ratios were measured. Both kinds of data are valuable and should be used in the fission yield data evaluation, especially the ratio, which was measured with high accuracy. Due to the measurements could be done at different laboratories and different time, the yields and their ratios are often not consistent with each other. It is the task of the evaluation to get a set of fission yield data, the variance of which and their ratio to the corresponding experimental data are minimum. This can be done by using code ZOTT, taking the form of sensitivity matrix as shown in following example.

There are lot of measured data for ^{140}Ba yield, 9 set of data for the yields at thermal energy (Y_t), 3 sets of data at fission spectrum (Y_f), 2 sets data around 14 MeV (Y_h), and 2 sets data for the ratios of the yield at fission spectrum over the yield at thermal ($R_{f/t}$), 1 set for the ratio of around 14 MeV over thermal($R_{h/t}$). To simplify the processing with code ZOTT, they were averaged with code AVERAG (the same result with ZOTT as discussed above) for the yields at thermal, fission spectrum, around 14 MeV and ratios of fission over thermal. As results, 3 yields at each energy point and 2 ratios and their errors were got. It was supposed that there is no correlation among the absolutely measured yields and their ratios and between two ratios, but there is correlation among the absolutely measured yields. The sensitivity matrix of this example for ratio adjusting is $\begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$ where "-1" in the matrix means at denominator, and "1"means at numerator, so the first row, corresponding to the first ratio in the combined vector, in the matrix means the second yield over first one, that is the yield at fission spectrum over the yield at the thermal one. In the same sense, the second row means the yield around 14 MeV over the yield at thermal.

The variation of adjusted yields ratios and their errors with correlation coefficient were calculated. The corresponding ratios of the adjusted ones over the input ones are given in Fig. 3 and 4 respectively. It can be seen that the $Y_1(Y_t)$, $Y_3(Y_h)$, which were adjusted smaller to make consistence with the measured ratios, increases with the correlation coefficient increasing, and opposite $Y_2(Y_f)$, which was adjusted larger, decreases. As a result they were made closer when the correlation becomes stronger. This is reasonable in physics. As regards to the ratios, they change only as the comprehensive results of Y_2/Y_1, Y_3/Y_1 due to no correlation between them were supposed. It can be seen from Fig. 4 that all errors are smaller than input ones, especially for the ratios $R_{1/2,}$ $R_{1/3,}$ this is the statistically processing result. When the correlation increases, the error of all yields Y_1, Y_2, Y_3 increases a little at first, and then decreases, the only difference is decreasing faster for Y_1. The error of all two ratios decreases a lot with correlation increasing.

The correlation matrices of the adjusted quantities were investigated. In the case of the input data ρ=0.0, there is no correlation for the input data, the correlation of the adjusted values was completely induced in the processing due to the restrict condition of ratios. Also the correlation between $Y_{2,}$ Y_1 and R_1 as well as $Y_{3,}$ Y_1 and R_2, is quite large. The other elements are quite small, due to the correlation among them was only induced indirectly. When the correlation coefficient ρ=0.9 of the input data, the correlation among the adjusted quantities is quite strong, not only among the yields themselves Y_1, Y_2, Y_3, which was transferred from the input data correlation, but also between the ratios R_1, R_2 and between yield and ratios, which was induced in the processing through the ratios. As an example, in Fig. 5 is given the correlation between yield Y_1 and yields Y_1~Y_3, Ratios R_1, R_2.

3 Sum Adjusting

In the fission yield evaluation processing, sometimes it is needed to make the sum of some quantities equal a value. For example, the sum of independent yields in a decay chain equals its chain yield, the chain yield sum of all mass number equals 2.0 etc. This can also be done by using code ZOTT, taking the sensitivity matrix in the form as given in the following example.

Taking the mass chain A=99 as an example, there are fission product nuclides $_{35}$Br, $_{36}$Kr, $_{37}$Rb, $_{38}$Sr, $_{39}$Y, $_{40}$Zr, $_{41}{}^{m}$Nb, $_{41}$Nb, $_{42}$Mo. Among them, the independent yields of the first three are very small ($<10^{-5}$), the independent yields of other 6 are 0.133158, 1.94933, 3.58394, 0.406579, 0.0299649, 0.0428140 respectively, but the recommended value based on the measured data of cumulative yield for ^{99}Mo is 6.15000, they are not consistent with their sum and should be adjusted. For this, the sensitivity matrix was constructed as only one low, and all of the 6 elements are 1. That means the sum of first 6 quantities should be equal to the seventh one. In this example, the correlation coefficient was changed from 0.0 to 0.999 in the calculation to study the effect of the correlation on the sum adjusting result.

The variation of the yields with correlation coefficient was calculated. The ratio of adjusted yields over input ones is given in Fig. 6 as a function of correlation coefficient. It can be seen that the adjusting for the yield is very small, the maximum one is 0.1% for Y_3, whose error is the largest one in the input data. This is because there is no large discrepancy in the input data. Another feature can be seen from Fig. 6 is that the adjusting increases with increasing the correlation for all yields but Y_3, whose adjusting decreases. The stronger correlation makes them stronger influence with each other, as a result, the yields with smaller errors were also adjusted a little more.

The errors of the adjusted values were also calculated. The variation of their ratio of the adjusted one over the input one is shown in Fig. 7. It can be seen that the ratios of all yield errors decrease in a similar way from 1.0 to closing 0.0 with the correlation coefficient increasing from 0.0 to 0.999, except for the ratio of yield Y_3, whose input error is relatively large, so it was adjusted more (see Fig. 6) and its error was reduced a lot (about 80%) even in the case of the correlation coefficient is 0.0, as a result, even the correlation become stronger, it only can be reduced more slowly.

The correlation matrices of the adjusted yields were studied for correlation coefficient ρ of input data from 0.0 to 0.999. It can be seen that the correlation is not large when the input correlation is small, even the correlation coefficients ρ=0.9 of the input yields, the ones of the adjusted yields are about 0.4~0.5, which is much smaller than the input one. Another feature of the matrices is that the correlation between the yield Y_3 and Y_1, Y_2, Y_4~Y_6 is negative and the correlation between the sum yield Y_7 and other 6 yields Y_1~Y_6 is positive, except for the case of input ρ=0.9, where the correlation of the input data is quite strong. These are reasonable in physics. Increasing of any one from the partial yields Y_1~Y_6 should make the increasing of the sum yield Y_7. Keeping the sum yield Y_7 unchanged, increasing of some partial yields, the others should decrease, for this example all of partial yields change in the same direction, but Y_3, whose error of the input data is quite large comparing to others, in opposite direction. In Fig.8 is shown the variation of the correlation coefficients of yield Y_1 with Y_1~Y_7. It can be seen that all of them increase with the increasing of the input correlation coefficient ρ(c in the Fig.), except for ρ_{13}, which is decreases.

4 Practical Example

In the following a practical example is given to show how to get the input covariance matrix and how the covariance is transferred and produced in the practical fission yield data evaluation.

For the yields of fission products ^{147,149,151,152}Sm at thermal energy, there are 5 sets of measured data for yields themselves and their ratios. All of them were measured with mass spectroscopy method absolutely or relatively. They are listed in Table 1. The quantities were measured for each isotope is listed in Table 2.

Table 2 The measured quantities for each product nuclides

Number	Nuclide	Yield Y	Ratio R
1	^{147}Sm	Ref. [1], [2]	Ref. [4]
2	^{151}Sm	Ref. [1], [2]	Ref. [4]
3	^{152}Sm	Ref. [1], [2]	Ref. [4], [5]
4	^{149}Sm	Ref. [1], [3]	

Based on the total and systematical error given in the Table 1 for each reference and the measured quantities for each product listed in Table 2, the input data and their covariance matrix were constructed. There are all together 12 quantities in the input vector, including 2 sets of yield data for each product, 4 sets

Table 1 Measured data for the fission yields of ^{147,149,151,152}Sm at thermal energy

Ref.	EXFOR	Method (Year)	Error given	Error adjusted	Syst.Error estimated	Note
1	13270003	Mass (1970), Absolutely	⩽1.0%	1.5%, (10% discre. with ^{152}Sm)	0.9%	1)
2	13384005	Mass (1955), Relatively (^{149}Sm)	No	2.0%	1.7%	2)
3	13386002	Mass (1955), Absolutely	No	3.0%	1.7%	1)
4	13384006	Mass (1955), Relatively (^{149}Sm)	No	1.5% (Ratio)	1.3%	3)
5	13352003	Mass (1950), Relatively (^{149}Sm)	No	1.5% (Ratio)	1.3%	3)

Note: 1) The fission rate, which is the main source of the systematical error, was measured also by mass spectroscopy method. The contribution to the total error is about 1/3, so the systematical error is $(1.5^2 \times 1/3)^{1/2}=0.9$.

2) The error mainly comes from the standard of ^{149}Sm, about 70%.

3) Only the ratio was used, which was calculated from fission yield divided by the yield of the standard ^{149}Sm. The error mainly comes from the standard of ^{149}Sm, about 70%.

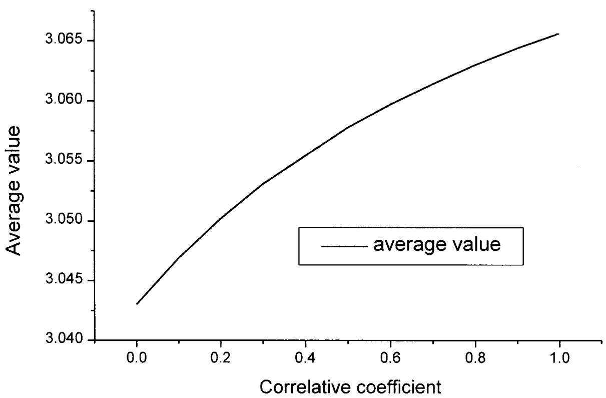

Fig. 1 The variation of the mean with the correlation coefficient

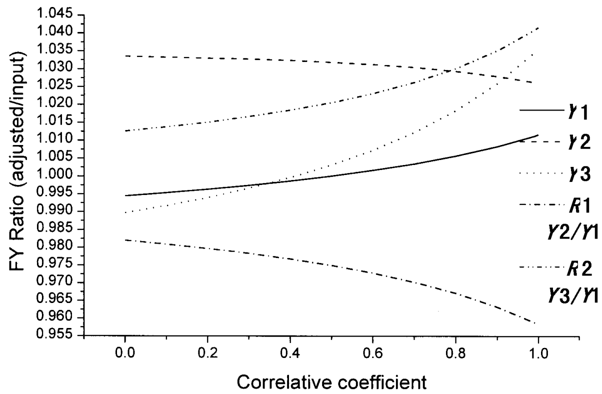

Fig. 3 The variation of the adjusted yields and ratios with the coefficient in the ratio adjusting

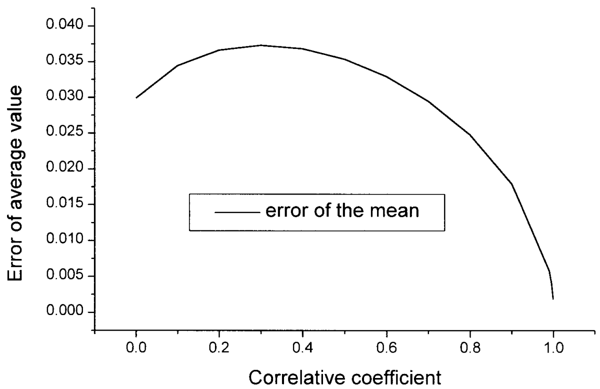

Fig. 2 The variation of the mean error with the correlation coefficient

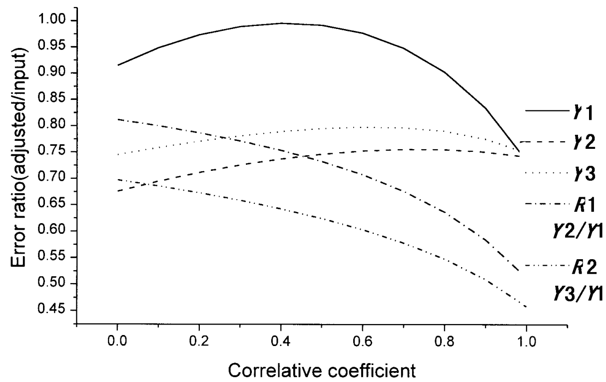

Fig. 4 The variation of the error of the adjusted yields and ratios with the correlation coefficient in the ratio adjusting

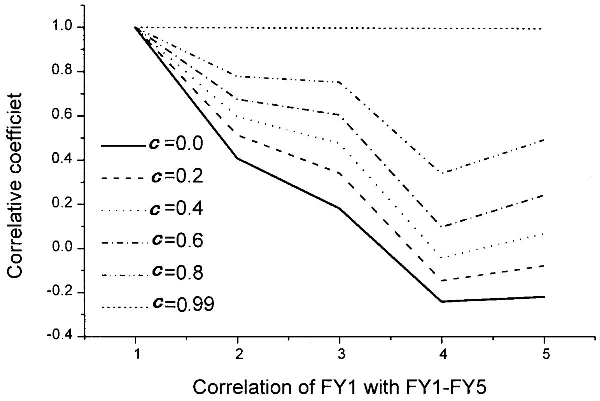

Fig. 5 The variation of the correlation coefficient between the Y_1 and $Y_{1\sim3}$, $R_{1,2}$ with the input correlation coefficient in the ratio adjusting

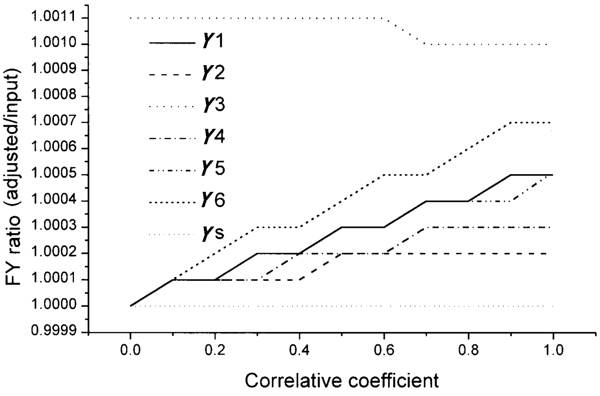

Fig. 6 The variation of the adjusted yields with the input correlation coefficient in the sum adjusting

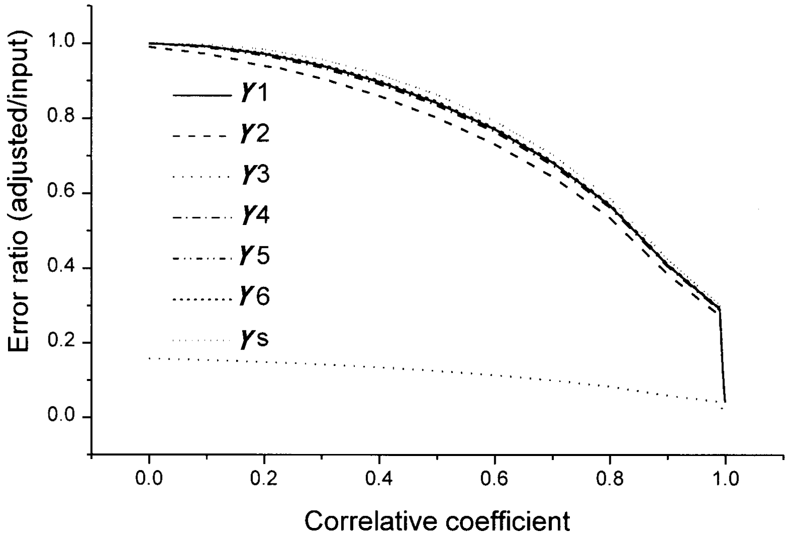

Fig. 7 The variation of the error of adjusted yields with the input correlation coefficient in the sum adjusting

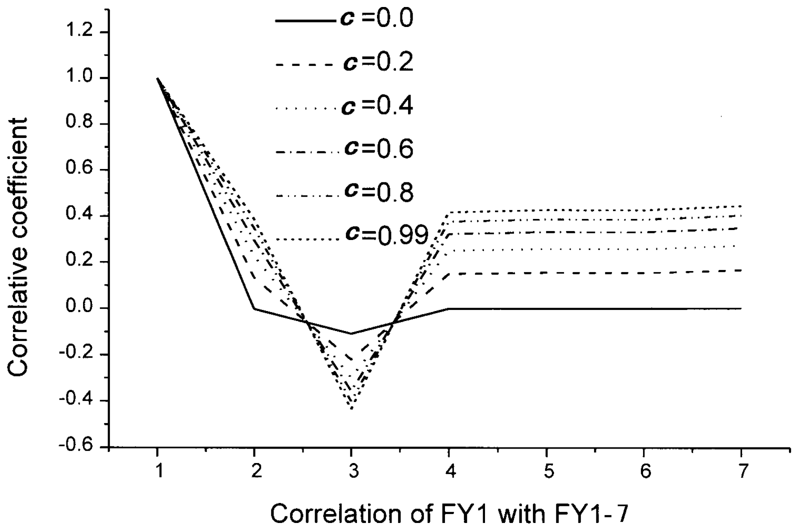

Fig. 8 The variation of the correlation coefficient between Y_1 and $Y_{1\sim7}$ with the input correlation coefficient in the sum adjusting

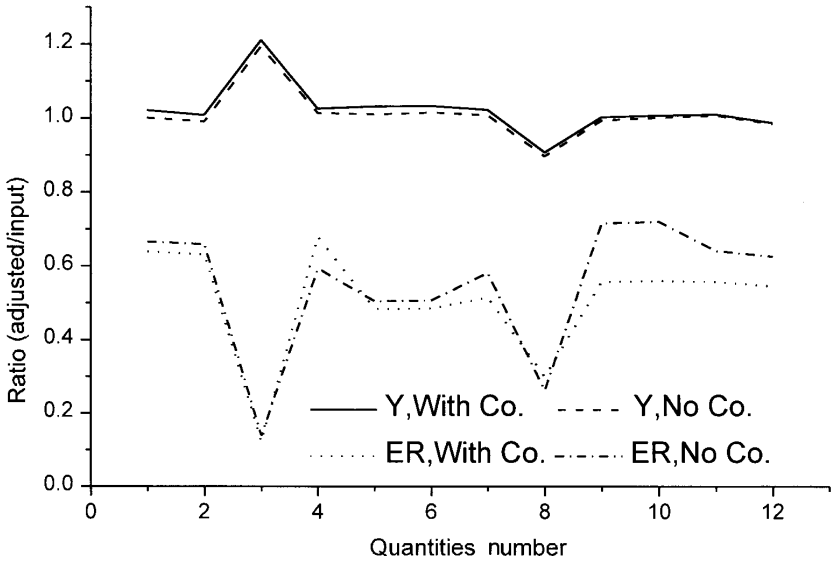

Fig. 9 Comparing between the adjusted data with and without correlation for ^{147,151,152,149}Sm yield data and their errors

of ratio data. The relationships among them were defined by the sensitivity matrix, which is somewhat complicated.

processing results (the ratio over the input data) are given in Fig.9 with the comparing to the one without correlation. The following features can be seen from the figure:

1) All of the quantities, including yields themselves and their ratios, were changed a little, except for the third one (the first yield of ^{152}Sm), and the 8-th (the second yield of ^{149}Sm), both were adjusted about +20% and –10% respectively, for they have larger error of the input data

2) All of the quantities, including yield themselves and their ratios, adjusted somewhat larger with correlation than without.

3) The errors of all quantities were reduced to about 50% comparing to input ones, except to about 10% for Y_3, and about 30% for Y_8, the errors of both input dada are relatively larger, as discussed above.

4) There are no large differences between the errors of all quantities adjusted with and without correlation, although they are a little lower with correlation than without correlation.

5 Conclusion Remarks

From above working over, the following can be concluded:

1) The code ZOTT can be used for the commonly evaluation processing of the fission yield data, including average with weight, ratio adjusting and sum adjusting, just taking the different form of the sensitivity no matter the data have or not have correlation;

2) The correlation of the evaluated fission yield data (also other nuclear data) can be produced in the data evaluation processing, which depends on the processing method, as a result, the processed data have the correlation, even though the input data have no correlation (the non-diagonal elements of the covariance matrix are zero);

3) The correlation of the input data, which depends on the experimental conditions, can be transmitted in the data evaluation processing, like the error propagation in the traditional data processing;

4) The variations of the adjusted yields, ratios and their errors with the correlation coefficient, either in the case of average, ratio adjusting or sum adjusting, are reasonable in physics, although some of them are difficult to be imaged and explained directly.

Reference

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