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Particle Dynamics under Quasi-linear Interaction with Waves

F. Castejón
S. Eguilior

Asociación EURATOM/CIEMAT para Fusión - 85

Departamento de Fusión y Física de Partículas Elementales

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Particle Dynamics under Quasi-linear Interaction with Electromagnetic Waves

Castejón, F. and Eguilior, S.

Laboratorio Nacional de Fusión Asociación EURATOM-CIEMAT para la Fusión.
Av. complutense, 22. 28040 Madrid, Spain.

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Abstract

Langevin equations for quasi-linear wave particle interaction are obtained taking advantage of the unequi vocal equivalence between Fokker-Planck equation and the former ones. Langevin equations are solved numerically and, henee, the evolution of a single particle embedded in an electromagnetic field in momentum space is obtained. The equations are relativistic and valid for any wave. It is also shown that the stochastic part of the equations is negligible in comparison with the deterministic term, except for the momentum cióse to the resonance condition for the main parallel refractive index.

Dinámica de las partículas bajo interacciones cuasi-lineales con ondas electromagnéticas

Castejón, F. and Eguilior, S.

Laboratorio Nacional de Fusión Asociación EURATOM-CIEMAT para la Fusión.
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Resumen

En este trabajo se han obtenido las ecuaciones de Langevin que describen la interacción quasi-lineal entre ondas y partículas partiendo de la equivalencia univoca entre dichas ecuaciones y la ecuación de Fokker-Planck.

Una vez obtenidas, las ecuaciones de Langevin se resuelven numéricamente y así se calcula la evolución en el espacio de momentos de una particular sumergida en un campo electromagnético. Las ecuaciones obtenidas son totalmente relativistas y válidas para cualquier onda. En el trabajo se muestra también que la parte estocástica de las ecuaciones es despreciable en comparación con la parte determinista, excepto cuando el momento de las partículas es próximo a la condición resonante para el índice de refracción paralelo principal.

1.- INTRODUCTION

Several phenomena showing a clear interaction between transport and ECRH appear in present magnetic confinement devices, especially for high absorbed power density. These phenomena have been observed in both tokamaks [1,2] and stellarators [3,4,5]. The heating causes the appearance of peaked temperature profiles, and flat or even hollow density profiles in both types of device. This is explained by assuming that an extra particle flux is created by EC-waves, then the ambipolarity condition gives a positive electric field that is able to diminish the particle flux and reduce the heat transport. In this regime the ECE signals present jumps [6] that can be interpreted as transitions in the transport regime. In the case of FTU and TCV tokamaks a similar phenomenon is observed and is accompanied by sawteeth.

This kind of interaction between transport and heating is associated with the existence of a loss cone in momentum space, therefore its mathematical treatment must be done introducing the real geometry in the kinetic equation. Nevertheless this equation remains difficult to solve numerically for the real geometry of the magnetic confinement devices, especially for stellarators. Moreover, it should be coupled to a similar equation for ions in order that the ambipolarity condition can be established. 5-D Montecarlo calculations have been performed for a weak deviation of the Maxwellian distribution function [7] that simulates such an interaction between transport and heating. The Fokker-Planck (FP) equation is solved in stellarators assuming three electron populations: circulating ones, and trapped in wells with and without direct gyrotron heating [8]. A three dimensional (radial dimension plus 2 dimensions in momentum space) FP equation is solved for a tokamak plasma considering particle diffusion under the effect of anomalous transport [9].

FP and Langevin equations are equivalent for studying the movement of a particle under a stochastic interaction [10, 11]. For instance, the Brownian movement in plasmas has been studied using Langevin equations [12, 13]. Langevin equations can also be obtained for describing quasi-linear wave-particle interaction using the unequivocal correspondence with FP and can be completed with other terms that have influence on the dynamics of the particles: Collisions and drifts due to the actual geometry of the device can be introduced in the equations. This approach could be explored as an alternative way to solve the problem of interaction between transport and heating. In this way, present Montecarlo codes that are based on the estimation of particle trajectories [14] could be completed by adding the wave-particle interaction term. In order that this method could be useful, the problem to solve is that the volume of interaction between waves and particles is small in comparison with the plasma volume: a large number of particles should be followed in order that the estimations are accurate. Further investigation of techniques to accelerate the convergency should be carried out (see e. g. [15]).

In this paper we obtain and solve the Langevin equations for the quasi-linear wave-particle interaction. Section 2 is devoted to the derivation of Langevin equations, the numerical solution is presented in section 3, the comparison between deterministic and stochastic terms is presented in section 4, and, finally conclusions are drawn in section 5.

2.- THE LANGEVIN EQUATIONS FOR QUASI-LINEAR WAVE-PARTICLE INTERACTION

Despite of the fact that the interaction of a single charged particle and waves is a deterministic process [16], the resonant wave-particle interaction in the quasi-linear theory can be considered as a stochastic particle diffusion plus a drift in momentum space. The interaction between waves and particles, considered as a collective process, depends on the relative phase between the waves and the particles, which is a random quantity, since the phases of particles are random and collisions are assumed to be able to decorrelate the phase between the interacting particle and waves. This stochastic process can be described by a general FP equation with a convective term plus a diffusive one [17]:

$$\frac{\partial f(\bar{u})}{\partial t}=\frac{\partial}{\partial u_i}\left[-A_i(\bar{u})f(\bar{u})+\frac{1}{2}\frac{\partial}{\partial u_j}\left(B_{ij}(\bar{u})f(\bar{u})\right)\right], \quad i,j=\perp,// \tag{1}$$

Einstein summation convention is assumed for repeated indices in this paper. Parallel and perpendicular signs refer to the direction of the confining magnetic field, and *f* is the distribution function.

There are several expressions for the different plasma regimes (non-relativistic, weakly relativistic and fully relativistic) where the above equation is applied and all of them have the same structure [18]. The formulation for this process in the relativistic regime for any propagation angle and for any resonant wave can be written as [19]:

$$\frac{\partial f(\bar{u})}{\partial t} = \frac{1}{u_\perp}\left(Y_s \frac{\partial}{\partial u_\perp} + u_\perp N_{//} \frac{\partial}{\partial u_{//}} \right) u_\perp D_{cy} \left(Y_s \frac{\partial}{\partial u_\perp} + u_\perp N_{//} \frac{\partial}{\partial u_{//}} \right) f(\bar{u}) \tag{2}$$

Where the quantities are: $N_{//}$ the parallel refractive index of the wave, $Y_s = s\omega_c/\omega$ the harmonic order times the ratio between cyclotron and heating frequencies, $\bar{u} = \bar{p}/mc$ the normalised momentum of the particle, and D_{cy} the resonant cyclotron diffusion coefficient, where:

$$D_{cy} = \int dN_{//}\Gamma(N_{//}) d_{cy}(\bar{u}, N_{//}) \tag{3}$$

The spectral density, $\Gamma(N_{//})$, has been introduced, that satisfies $\int dN_{//}\Gamma(N_{//}) = 1$. The single wave diffusion coefficient is:

$$d_{cy}(\bar{u}, N_{//}) = \delta(\gamma - Y_s - u_{//}N_{//}) \frac{w}{\gamma} |\hat{e} \cdot \bar{\Pi}|^2 \tag{4}$$

Where w is the absorbed power density in phase space and $\gamma = \left(1 + u^2\right)^{1/2}$. The polarization term is:

$$\bar{\Pi} = \left(\frac{sJ_s(\rho)}{\rho}, iJ'(\rho), \frac{u_{//}}{u_\perp} J_s(\rho) \right)$$

and $\hat{e}$ is a unit vector parallel to the electric field. $J_s(\rho)$ are the Bessel functions of order s and argument $\rho = N_\perp u_\perp \omega_c / \omega$, which is the product of the Larmor radius times the perpendicular wave vector.

The solution of Equation (2) is usually obtained numerically but it is expensive in terms of computational time. There is extensive literature on the solution of this equation using different methods, such as the alternate direction one. A useful list of references is given in [20]. This difficulty has meant that the kinetic properties of radio frequency (RF) plasma heating are only studied for homogeneous plasmas or for simple geometries [8, 9], it being difficult to introduce three dimensional real geometry of the plasma confinement devices, especially for those complex ones like TJ-II stellarator [21].

After manipulation, the resonant quasi-linear Equation (2) may be written in a more compact way as:

$$\frac{\partial f(\bar{u})}{\partial t} = \bar{\nabla} \cdot \left(\bar{s} D_{cy} \bar{s} \cdot \bar{\nabla} f(\bar{u}) \right) \tag{5}$$

which is the equation that describes quasi-linear diffusion in momentum space. This diffusion, as can be seen, is along the direction of vector **s**:

$$\bar{s} = Y_s \hat{e}_\perp + u_\perp N_{//} \hat{e}_{//} \equiv s_\perp \hat{e}_\perp + s_{//} \hat{e}_{//} \tag{6}$$

For quasi-perpendicular wave propagation, this vector is nearly in the perpendicular direction.

As stated above, there is another approach to the study of stochastic processes based on studying the trajectories of single particles in phase space using Langevin equations. The microscopic movement of the particle is described by these equations and solving them is equivalent to knowing the distribution function. The general form of these equations is:

$$\frac{\partial u_i}{\partial t} = F_i(\bar{u}) + D_{ik}(\bar{u})\xi_k, \quad i,k = \perp, // \tag{7}$$

Although F_i and D_{ij} are deterministic functions, the vector $\boldsymbol{\xi}_i$ is a random white noise that satisfies $\langle\xi_i(t)\rangle = 0$, $\langle\xi_i(t)\xi_k(t+\tau)\rangle = \delta_{ik}\delta(\tau)$, which makes the Langevin equations have a non-deterministic nature. The former equation has the problem that its value is not defined when a jump occurs in $\xi_i(t)$. This jump will modify the evolution of $u_i(t)$. Therefore, the value of u_i to estimate the right hand side of Eq. (7) is undetermined by the size of the jump. This problem can be solved by using two conventions to choose the time in which $u_i(t)$ is estimated: one developed by Stratonovich and the other by Itô. Both give rise to different but equivalent algebras. Therefore, one must be coherent when estimating averages and derivatives. We will use the Stratonovich convention since our equation represents an Ornstein-Uhlenbeck process (having zero measurement but finite correlation time). When the correlation time tends to zero we obtain Stratonovich convention. Once one of the algebras has been chosen, there is a unequivocal correspondence between the FP equation and the Langevin ones [22]. Stratonovich algebra gives the following equivalence between coefficients of Equations (1) and (7):

$$\begin{aligned} \overline{D}(\bar{u}) &= \overline{B}^{1/2}(\bar{u}) \\ F_i(\bar{u}) &= A_i(\bar{u}) - \frac{1}{2} D_{kj} \frac{\partial D_{ij}}{\partial u_k} \end{aligned} \tag{8}$$

This mathematical equivalence implies that a physical basis must exist. The drift term of the equations (7), proportional to vector $\boldsymbol{F}$, is a force that acts on the particles, while the random part accounts for the stochasticity of the collective behaviour of particles when interacting with the waves.

Using this correspondence, we can obtain Langevin equations for the quasi-linear resonant diffusion for relativistic interaction between plasma and waves, that correspond to Equation (2). First of all, we write Eq. (2) in the general form of the FP equation:

$$\frac{\partial f(\bar{u})}{\partial t} = \frac{\partial}{\partial u_i}\left(s_i\, D_{cy}\, s_j \frac{\partial}{\partial u_j} f(\bar{u})\right) = \frac{\partial}{\partial u_i}\left(-f(\bar{u}) \frac{\partial}{\partial u_j}(s_i\, D_{cy}\, s_j) + \frac{\partial}{\partial u_j}(s_i\, D_{cy}\, s_j f(\bar{u}))\right) \quad (9)$$

We can identify in equation (9) the terms that appear in equation (1) as:

$$A_i = \frac{\partial}{\partial u_j}(s_i\, s_j\, D_{cy}) \qquad B_{ij} = 2 s_i\, s_j\, D_{cy} \qquad (10)$$

Following Stratonovich algebra, given by the relations (8), it is possible to write the corresponding Langevin equations for Eq. (2). The vector $\boldsymbol{F}$ and the tensor D_{ij} in these equations are obtained for this particular case and can be written as:

$$\begin{aligned} F_\perp &= \frac{1}{2} Y_s \left(Y_s \frac{\partial D_{cy}}{\partial u_\perp} + u_\perp N_{//} \frac{\partial D_{cy}}{\partial u_{//}} \right) = \frac{1}{2} s_\perp (\bar{s} \cdot \nabla) D_{cy} \\ F_{//} &= \frac{1}{2} (u_\perp N_{//}) \left(Y_s \frac{\partial D_{cy}}{\partial u_\perp} + u_\perp N_{//} \frac{\partial D_{cy}}{\partial u_{//}} \right) = \frac{1}{2} s_{//} (\bar{s} \cdot \nabla) D_{cy} \end{aligned} \qquad (11)$$

$$D = \sqrt{\frac{2}{Y_s^2 + (u_\perp N_{//})^2}}\, D_{cy}^{1/2} \begin{pmatrix} Y_s^2 & Y_s\, u_\perp N_{//} \\ Y_s\, u_\perp N_{//} & (u_\perp N_{//})^2 \end{pmatrix} = \sqrt{2} D_{cy}^{1/2} \frac{1}{|\bar{s}|} \begin{pmatrix} s_\perp^2 & s_\perp s_{//} \\ s_\perp s_{//} & s_{//}^2 \end{pmatrix} \qquad (12)$$

Now we know all the elements we need to write the Langevin equations. Taking into account the form of these equations and the definition of vector **s**, the Langevin equations corresponding to quasi-linear diffusion are given by:

$$\frac{d}{dt}\bar{u} = \bar{s} \left[\frac{1}{2} (\bar{s} \cdot \nabla) D_{cy} + \frac{\bar{s}}{|\bar{s}|} \cdot \xi\, D_{cy}^{1/2} \right] \qquad (13)$$

This vector equation shows clearly that the vector **s** is always tangential to the trajectory of the particle in momentum space. This fact can be attributed to the momentum and energy conservation [23]. Both the stochastic and the deterministic parts of the movement are in the same direction, always tangential to **s**, therefore, the plot of microscopic trajectories in momentum space will not present any stochastic feature and this character will only appear when the trajectory is plotted versus time.

3.- PARTICLE TRAJECTORIES IN MOMENTUM SPACE

The diffusion coefficient D_{cy} can be written as:

$$D_{cy}(\vec{u}) = \int dN_{//}\Gamma(N_{//})\delta(\gamma - Y_s - u_{//}N_{//})\frac{w}{\gamma}|\hat{e}\cdot\vec{\Pi}|^2 = \\ = \int dN_{//}\frac{w}{|u_{//}|\gamma}|\hat{e}\cdot\vec{\Pi}|^2\delta(N_{//} - N_{//R}))\Gamma(N_{//}) = \frac{w}{|u_{//}|\gamma}|\hat{e}\cdot\vec{\Pi}|^2\Gamma(N_{//R}) \tag{14}$$

Where the resonant parallel refractive index has been introduced:

$$N_{//R} = \frac{\gamma - Y_s}{u_{//}} \tag{15}$$

In this way we can estimate the derivatives of equation (14):

$$\frac{\partial D_{cy}(\vec{u})}{\partial u_i} = \frac{w}{|u_{//}|\gamma}\left\{-\Gamma(N_{//R})\left[|\hat{e}\cdot\vec{\Pi}|^2\left(\frac{\delta_{i//}}{u_{//}} + \frac{u_i}{\gamma^2}\right) + \frac{\partial|\hat{e}\cdot\vec{\Pi}|^2}{\partial u_i}\right] + \Gamma'(N_{//R})|\hat{e}\cdot\vec{\Pi}|^2\frac{1}{u_{//}}\left(N_{//R}\delta_{i//} + \frac{u_i}{\gamma}\right)\right\} \tag{16}$$

The derivatives of the polarization term that appear in expression (15) are given by:

$$\frac{\partial|\hat{e}\cdot\vec{\Pi}|^2}{\partial u_{//}} = 2\left(\frac{sJ_s(\rho)}{\rho}e_x + \frac{u_{//}}{u_\perp}J_s(\rho)e_{//}\right)\left(\frac{sJ_s(\rho)}{\rho}e_x + \frac{J_s(\rho)}{u_\perp}e_{//}\right)$$

$$\frac{\partial|\hat{e}\cdot\vec{\Pi}|^2}{\partial u_\perp} = 2\left(\frac{sJ_s(\rho)}{\rho}e_x + \frac{u_{//}}{u_\perp}J_s(\rho)e_{//}\right)\rho'\left(J'_s(\rho) - \frac{J_s(\rho)}{\rho}\right)\left(\frac{se_x}{\rho} + \frac{u_{//}e_{//}}{u_\perp}\right) + \\ + 2e_y\rho' J'_s(\rho)J''_s(\rho)$$

Now we can solve equations (13) numerically assuming a Gaussian spectral density:

$$\Gamma(N_{//}) = \frac{1}{\Delta\sqrt{\pi}}\exp\left(-\left(\frac{N_{//} - N_{//0}}{\Delta}\right)^2\right) \tag{17}$$

In this expression Δ is the spectral width, and $N_{//0}$ is the main longitudinal refractive index. The value of Δ will depends on the dispersion of the microwave beam that can be estimated by ray or beam tracing techniques. Δ=0.1 is used in the following calculations.

The particle trajectories in momentum space are plotted in Figure 1 for a downshifted incident frequency, taking the spectral density given by (17). Several starting positions in momentum space are considered. The resonance condition for $N_{//0}$ is plotted as a guide for the eye. It is seen that the resonant curve in momentum space is an invariant curve for the particles. It can be also seen that the particles located outside the resonance loss perpendicular momentum in the electromagnetic filed. This because the relative velocities of the waves and the particles is such that particles give energy to the waves. Figure 2 shows the same for an upshifted heating frequency. The behaviour of the particles is qualitatively the same as in the former case, but the evolution of the particles near the right "leg" of the resonance is weaker. Again particles that give energy to the wave are found.

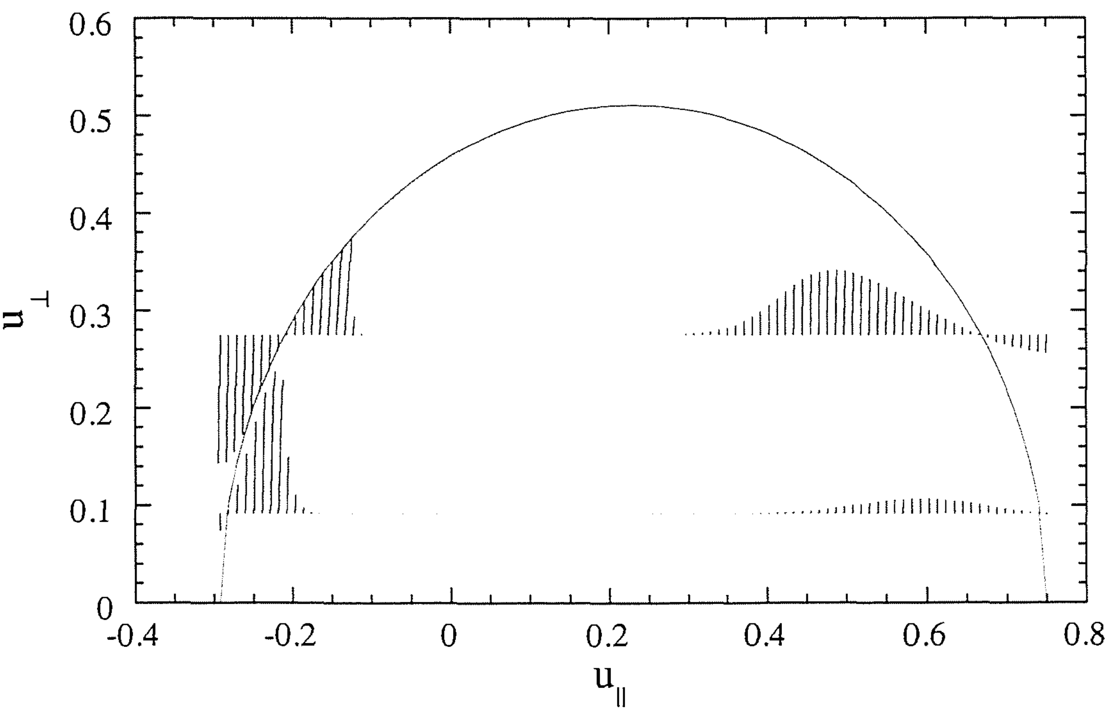


Figure 1: *Particle trajectories in momentum space for w=10, $N_{//}$=0.2, Y_s=1.1, and two values of perpendicular momentum $u_\perp=0.2\ u_0$ and $u_\perp=0.6\cdot u_0$, where $u_0=(Y_s^2-1)^{1/2}$ the resonant perpendicular momentum for $u_{//}=0$ for the main parallel refractive index. The resonance condition is plotted for the main parallel refractive index (grey curve).*

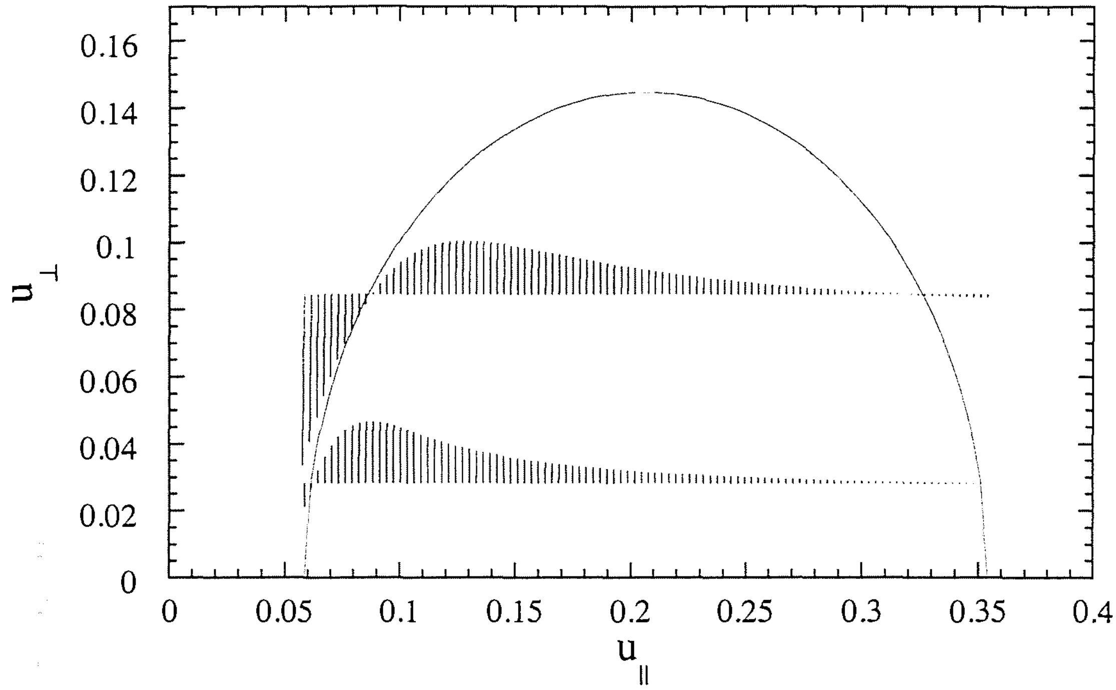


Figure 2: *The same as in figure 1, but for Y_s=0.99 (upshifted frequencies).*

4.- DETERMINISTIC APPROXIMATION

It is possible to estimate the relative size of both terms of Eq. (7) in order to know if there is an energy range in which one of them is dominant. A deterministic equation will govern the movement of electrons in the momentum range in which the deterministic part is dominant. This will be accomplished if:

$|\vec{F}(\vec{u})| >> |\overline{D}(\vec{u})\xi|$

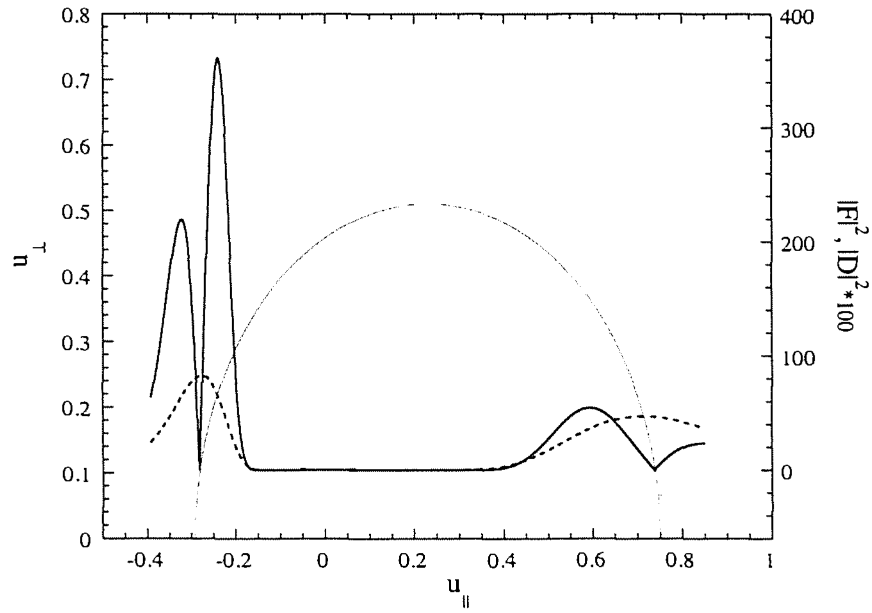
Equivalently it must also hold that:

$$F_\perp{}^2 + F_{//}{}^2 >> D_{11}^2 + D_{22}^2 + 2D_{12}(D_{11} + D_{22} + D_{12}) \qquad (18)$$

When this condition is fulfilled, the stochastic movement in momentum space of particles embedded in an electromagnetic field will be a small perturbation of the deterministic movement.

Figures 3a and 3b show the comparison of the deterministic part with the stochastic one for two perpendicular momentums and for downshifted frequencies. The characteristic of the wave are: spherical polarization, i.e., E_z=E_x=E_y, and oblique propagation. For Y_s=1.1 the resonant energy is as large as 50 keV. Figures 4a and 4b show the same as in Figs. 3a and 3b for upshifted frequencies. Both terms are smaller in the right "leg" of the resonance in the case of upshifted frequencies.

(a)



(b)

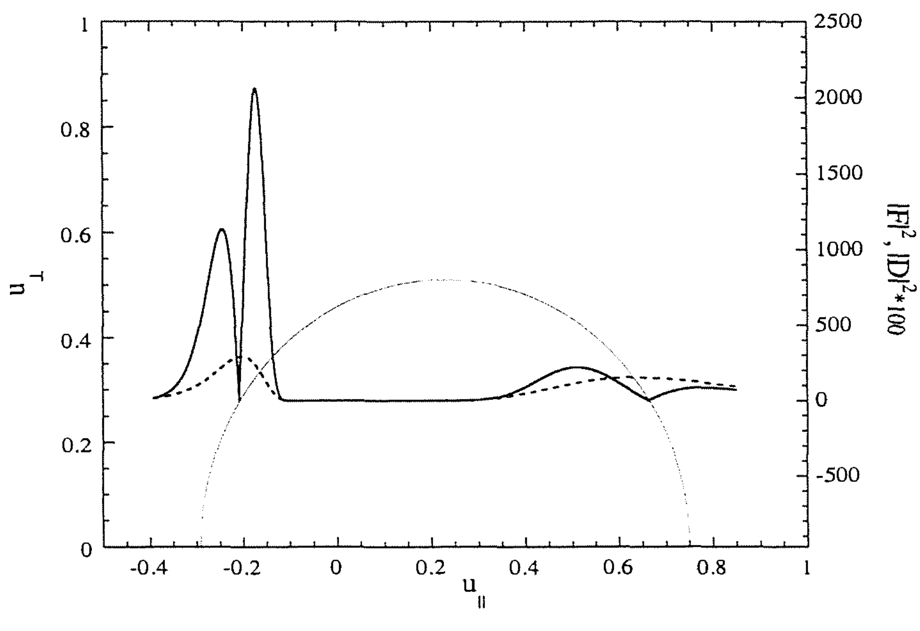
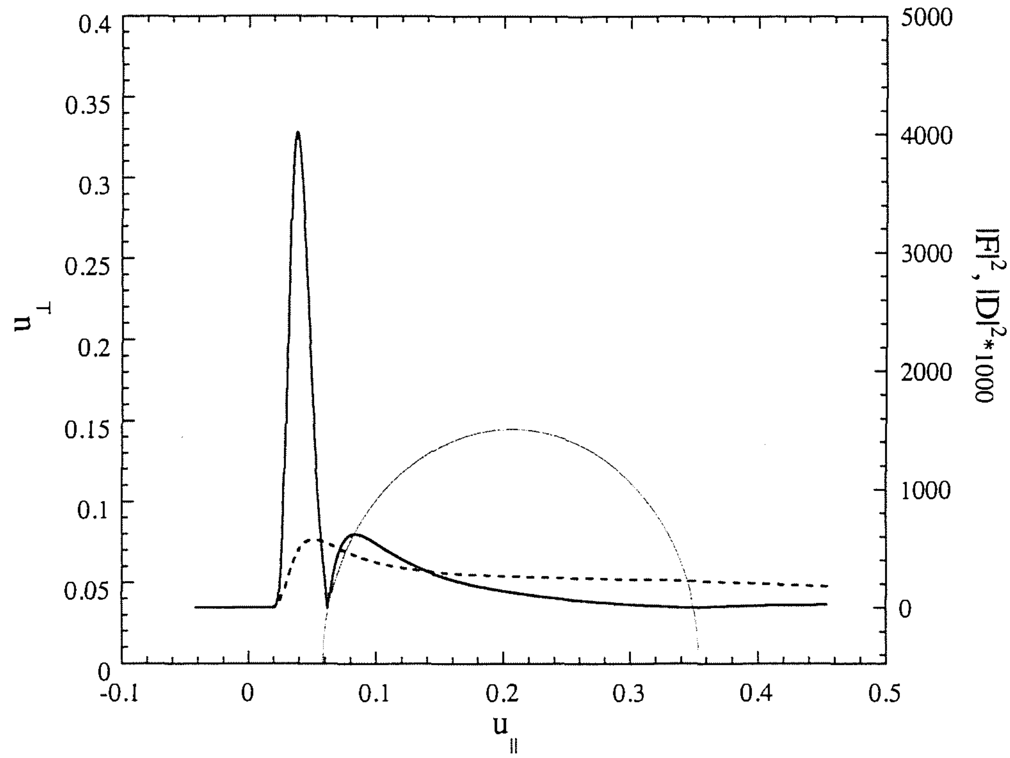


Figure 3a y b: *Comparison of $|F|^2$ (full curve) with $|D|^2$ (broken curve) for w=10, $N_{//}$=0.2, Y_s=1.1 (downshifted frequencies). a) $u_\perp$=0.2·u_0, and b) $u_\perp$=0.6·u_0, with u_0 =(1-Y_s^2)$^{1/2}$. The resonance condition is plotted for the main parallel refractive index and the stochastic term is multiplied by 100.*

(a)



(b)

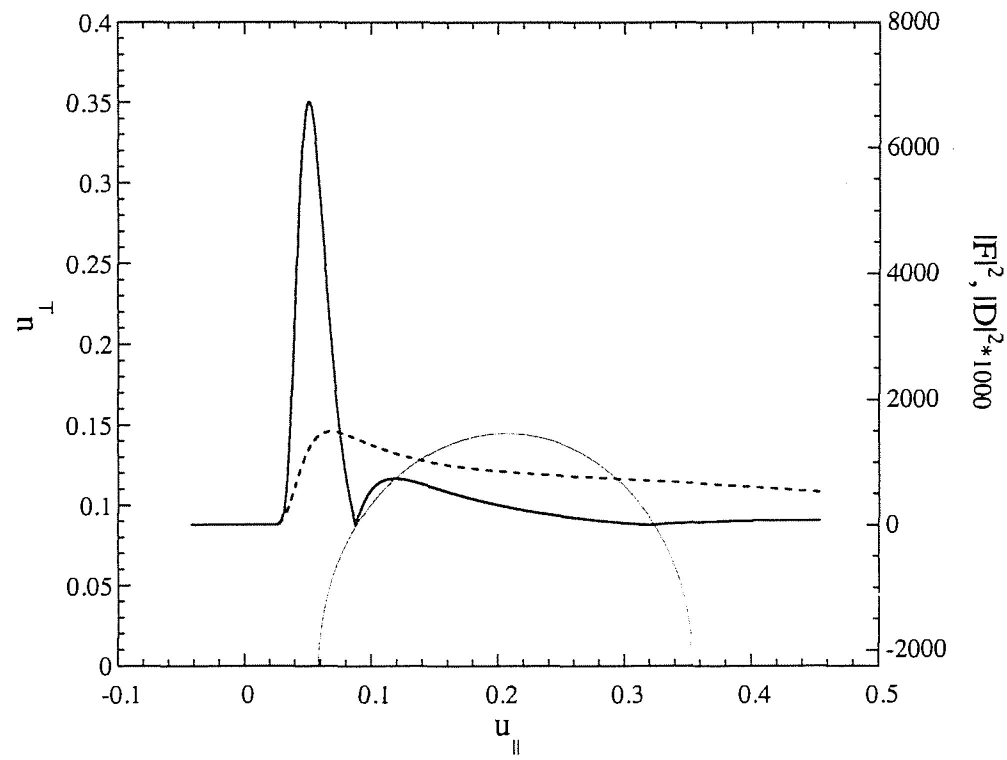


Figure 4a y b: *The same as in figures 3a) and 3b) for Y_s=0.99 (upshifted frequencies). The stochastic term is now multiplied by 1000.*

It is seen that the deterministic equation is a good approximation for these particular Langevin equations, except for the momentum close to the resonance that corresponds to the main refractive index, where the deterministic part of the equations tends to zero.

5.- CONCLUSIONS

Langevin Equations have been obtained for the quasi-linear wave-particle interaction, taking advantage of the fact that this interaction can be considered as a stochastic process, since the relative phase between the wave and the particles are random. Stratonovich algebra allows one to establish the correspondence between the microscopic equations for a test particle and the Fokker-Planck equation.

The microscopic equations obtained for this quasi-linear diffusion in momentum space can now be completed to take into account the geometry of the device and the collisions in the movement of the test particle. In this way, these equations could be useful to study the interaction between transport and RF heating in magnetic confinement devices.

Moreover, these equations can be used to estimate the wave-induced particle and energy flux through any surface in momentum space, as suggested in reference [24]. Therefore, as practical examples, the outward particle flux produced by the particles that enter into the loss cone can be estimated and also the runaway source due to RF heating. These calculations are outside the scope of this work and will be presented elsewhere.

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