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3.5 Neoclassical Kinetic Theory for the Shaped Tokamaks

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Key words Neoclassical kinetic theory Shaped Tokamaks Transport coefficients

Recent experiment studies on START, the low-aspect-ratio tokamak, have shown significant changes [1] in physics behaviour: the natural exhaust system because of the presence of X points on the plasma boundary and the freedom from the major disruption may offer significant advantages to the reactor concept [2]. The new phenomena in the low-aspect-ratio tokamak must be related to equilibrium, instabilities, and transport. Traditionally, neoclassical kinetic theory is studied under the condition of the circular plasma and the large-aspect-radio approximation[3] . In this paper, the kinetic theory for the shaped tokamaks is investigated for a set of Soloveev's configurations[4]. Using the Hamiltonian formalism, the diffusion coefficient is derived for plateau regime. The diffusion coefficient is inversely proportional to connection length. Near the plasma boundary where X points exist, the connection length is much longer compared to the one of a circular cross-section plasma. So, the diffusion coefficient is greatly reduced. However, the diffusivity defined by Solano and Hazeltine[5] does not change considerably, which is slightly increased with toroidality and reduced by elongation and triangularity. Both toroidal and poloidal rotation speeds are calculated. For the low-aspectradio tokamak, for example, START the formalism may be valid in a narrow range of collisionality.

1 Hamiltonian formalism of orbit theory

Using the area-conserved transformation,

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we get the following gyroaveraged Hamiltonian for particle motion in a tokamak configuration[6]:

$$H = \Omega_0 P_\alpha exp\left(-\frac{x}{\Omega_0 R_0}\right) + \frac{1}{2R_0^2} \exp \left(-\frac{2x}{\Omega_0 R_0}\right) [P_\phi + e\Psi(X,P_x)]^2 + e\,\Phi \quad (1)$$

where the momenta

$$P_\alpha = \frac{1}{2}\,\Omega\rho^2\ ,\ P_\phi = Rv_\phi - e\,\Psi\ ,\ P_x = Z_c$$

are conjugate to α, the gyrophase, ϕ, the toroidal angle, and X, expressed as:

$$X = \Omega_0 R_0 \ln \frac{R_c}{R_0} \quad (2)$$

where R_c and Z_c are the coordinates of the guiding center in a cylindrical system, ρ is the Larmor radius, Ψ is the poloidal flux, Φ is the electric potential, and Ω is the toroidal gyrofrequency. Subscripts 0 refer to the magnetic axis. Because there is no α and ϕ dependence in the Hamiltonian, P_α and P_ϕ are constants of motion. The particle mass is taken to be an unity for simplicity.

Arguing with the conservation of the canonical momentum in toroidal direction, we get a set of equations of motion for the guiding center:

$$\frac{dR}{dt} = \frac{B_R}{B_\phi}\left(\frac{u}{R} - \frac{R u_E}{R_0^2}\right) \quad (3)$$

$$\frac{dZ}{dt} = \frac{B_Z}{B_\phi}\left(\frac{u}{R} - \frac{R u_E}{R_0^2}\right) + \frac{1}{\Omega_0 R_0}\left(\Omega P_\alpha + \frac{u^2}{R_0^2}\right) \quad (4)$$

$$\frac{du}{dt} = \frac{B_R}{B_\phi}\left(\Omega P_\alpha + \frac{u^2}{R_0^2}\right) + \Omega_0 R_0 \frac{E_\phi}{B_\phi}$$

$$u = Rv_\phi\ ,\quad u_E = -R_0^2 \frac{\partial \Phi}{\partial \Psi} \quad (5)$$

where the subscript c, indicating the guiding center, is omitted for simplicity.

Eqs. (3) ~ (5) are the extension of the equations of motion derived by Balescu[7].

2 Transport coefficients for a tokamak configuration

The drift kinetic equation is given[8] by

$$\frac{dF}{dt} = C(F) \quad (6)$$

where $C(F)$ is the Fokker-Flanck collision operator. F can be expressed as:

$$F = F_m^*(H,\ P_\phi) + g \quad (7)$$

where the first term is the Maxwellian form with $H\text{-}e\Phi$ in the place of kinetic energy and P_ϕ in the place of ψ, the second term is the correction for collisions.

Now, we introduce a real tokamak configuration given by Soloveev[4]:

$$\Psi = \Psi_0\left[(R^2+\gamma)\frac{Z^2}{b^2} + \frac{1}{4}(R^2 - R_0^2)^2\right]$$

$$b = \frac{E}{(1-Q)^{1/2}}\ ,\quad \gamma = \frac{Q}{1-Q} \quad (8)$$

where E and Q are related to the elongation and triangularity. Now a set of coordinates related to the configuration can be constructed in the form:

$$X_1 = (\Psi/\Psi_0)^{1/2}$$

$$X_2 = -\text{arc tg}\frac{2Z(R^2+\gamma)^{1/2}}{b(R^2-R_0^2)}$$

$$X_3 = \phi \quad (9)$$

and the Jacobian for the transformation from $(R,\ \phi,\ Z)$ to $(X_1,\ X_2,\ X_3)$ is

$$J = \frac{bX_1}{(R_0^2 + 2X_1 \cos X_2 + \gamma)^{1/2}} = \frac{X_1}{R_0} J_0 \quad (10)$$

The drift kinetic equation is

$$\omega_b \frac{\partial g}{J_0\, \partial X_2} - \frac{v_d R_0}{X_1}\frac{\partial g}{J_0\, \partial X_2}\cos X_2 -$$

$$\frac{v_d R_0}{J_0}\frac{\partial g}{\partial X_1}\sin X_2 = C(F) - \frac{\partial F}{\partial t} \quad (11)$$

where, $\omega_b = \frac{u}{q_0 R_0^2}$

$$q_0 = \frac{r B_{\phi 0}}{R_0 B_{p0}} = \frac{I}{2\Psi_0 R_0^2}$$

$B_{p0} = 2\Psi_0 X_1$, $r = X_1 / R_0$

We can expand F_m^* in the form:

$$F_m^* = F_m - \frac{\partial F_m}{\partial r}\frac{u}{R_0 \Omega_{p0}} \quad (12)$$

where Ω_{p0} is the poloidal gyrofrequency related to B_{p0}. The Krook collision operator is used with a shift. We divide g into two parts:

$$g = g_0 + \tilde{g} \quad (13)$$

whrer g_0 is independent of X_2, and $\tilde{g}$ satisfies the following equations:

$$\langle \tilde{g} \rangle = \frac{1}{2\pi}\oint \tilde{g}\frac{J_0 dX_2}{hL} = 0$$

$$L = \frac{1}{2\pi}\oint \frac{J_0 dX_2}{h}, \quad h = R/R_0$$

$$\frac{\partial F}{\partial t} + \frac{\omega_b h}{J_0}\frac{\partial(\tilde{g}/h)}{\partial X_2} - \frac{h}{rJ_0}\frac{\partial(v_d\tilde{g}/h)}{\partial X_2}\cos X_2 - \frac{1}{J_0}\frac{\partial(v_d\tilde{g}/h)}{\partial r}\sin X_2 -$$

$$\frac{v_d}{J_0}\frac{\partial g_0}{\partial r}\sin X_2 = -v\left(g_0 - g_m - \frac{Pu}{T}F_m + g\right)$$

$$g_m = \frac{\partial F_m}{\partial r}\frac{u}{R_0\Omega_{p0}} \quad (14)$$

Averaging Eq. (14), we get

$$\frac{\partial F}{\partial t} - \frac{1}{2rL}\frac{\partial}{\partial r}(r v_d g_s) - v\left(g_0 - g_m - \frac{Pu}{T}F_m\right) \quad (15)$$

$$g_s = \frac{1}{\pi}\oint \sin X_2 v_d \tilde{g}/h dX_2$$

From the zero order of Eq. (15), we get

$$g_0 = g_m + \frac{Pu}{T}F_m \quad (16)$$

Then, from Eqs. (14) and (16), we have, for the next order,

$$\frac{L_0 \partial(\tilde{g}/h)}{J_0 \partial X_2} + \Lambda(\tilde{g}/h) = \frac{v_d}{\omega_b}\frac{\partial g_0}{\partial r}\frac{L_0 \sin X_2}{J_0 h}$$

$$\Lambda = \frac{\nu L_0}{\omega_b} \text{ and } L_0 = \frac{1}{2\pi}\oint J_0 dX_2 \quad (17)$$

We set

$$\theta = \frac{1}{L_0}\int J_0 dX_2 \quad (18)$$

The solution of Eq. (17) is

$$\frac{\tilde{g}}{h} = \frac{v_d}{\omega_b}\frac{\partial g_0}{\partial r}\sum_k \frac{s_k^1[\Lambda\sin(k\theta) - k\cos(k\theta)]}{\Lambda^2 + k^2} \quad (19)$$

with the Fourier coefficients

$$\frac{L_0 \sin X_2}{J_0 h^n} = \sum_{k=1}^{\infty} s_k^n \sin k\theta \quad (20)$$

In the plateau regime, $\frac{1}{\omega_b}\frac{\Lambda}{\Lambda^2 + k^2} \to \frac{\pi}{k}\delta(\omega_b)$. The solution in Eq. (19) is put into Eq. (15), then we get

$$g_s = \pi v_d \frac{\partial g_0}{\partial r}\sum \frac{s_k^1 s_k^0}{k}\delta(\omega_b) \quad (21)$$

Integrating over H and P_ϕ which are independent coordinates with X_1 and X_2, then, we get

$$\frac{\partial n}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\Gamma) = 0 \quad (22)$$

$$\Gamma = -D\left(\frac{n'}{n} + \frac{3}{2}\frac{T'}{T} - \frac{eE_r}{T} + \frac{P\Omega_{p0}}{T}\right)$$

$$D = \frac{\sqrt{\pi}}{4} q_0^2 \rho^2 \omega_t n G$$

$$\omega_t = \frac{v_t}{L_c}, \quad L_c = L q_0 R_0, \quad G = \sum_{k=1}^{\infty}\frac{s_k^0 s_k^1}{k} \quad (23)$$

where G has been calculated. If $A = 1.20$, $E = 2$, and $Q = -0.1$, we get $G = 1.25$ for $\Psi/\Psi_0 = 0.95$, when $A = 5$, $E = 2$, and $Q = -0.5$, $G = 1.03$ on plasma boundary.

According to equilibrium Eq. (8), q_0 is proportional to $(1 + b^2)/2b^2$. For a large elongation, q_0 is reduced to half, so is the diffusivity, that seems to be in agreement with Solano and Hazeltine[5].

The parameter P can be determined from collisional momentum conservation, which guarantees the ambipolarity:

$$P\Omega_{pi} - eE_r = -\left[D_i\left(\frac{n_i'}{n_i} + \frac{3}{2}\frac{T_i'}{T_i}\right) - D_e\left(\frac{n_e'}{n_e} + \frac{3}{2}\frac{T_e'}{T_e}\right)\right] \Big/ \left(\frac{D_i}{T_i} + \frac{D_e}{T_e}\right) \quad (24)$$

According to Eqs. (23) and (24), $D_i \gg D_e$, we get

$$\Gamma = -D_e\left[\frac{n'}{n}\left(\frac{T_e + T_i}{T_e}\right) + \frac{3}{2}\left(\frac{T_e' + T_i'}{T_e}\right)\right] \quad (25)$$

3 Rotation speed

Since we have calculated the distribution function, the rotation speeds are easily obtained by integrating over velocity space, considering the equations of motion (3) and (4), and the diamagnetic drift:

$$v_\Phi = -\frac{hT_i}{eB_{p0}}\left[\frac{n_i'}{n_i} + \frac{3}{2}\frac{T_i'}{T_i} - \frac{eE_r}{T_i}\right] \quad (26)$$

$$v_p = \frac{hB_p\rho_i'}{4B_{p0}L_T} v_{ti} + \frac{B_Z}{B_p}\frac{\rho_i}{R_0} v_{ti}$$

$$L_T = [-(\ln T)']^{-1} \quad (27)$$

where the second term in Eq. (27) arising from particle drift is significant only near an X point where B_p goes to zero. v_Φ and v_p are essentially the same as the ones given by Hazeltine and Meiss[9].

4 Conclusion and comments

The neoclassical kinetic theory for the shaped plasma has been investigated for the plateau regime. The radial flux and the rotation speeds are calculated. A set of Soloveev's configurations is used, which can facilitate the procedure of rigorous derivation and demonstrate the effects of toroidality, elongation and triangularity. For the low-aspect-ratio tokamak, the trapped particle fraction, $[2\varepsilon/(1+\varepsilon)]^{1/2}$, is more than 90% at $\Psi/\Psi_0 = 0.95$. In spite of this, there is a narrow parameter range in which the formalism may be valid. For START the aspect ratio, $A = 1.3$, the plateau regime is located in the range of $0.8 < \nu_{*i} < 1$, where, $\nu_{*i} = \nu/\omega_t$, ν is the 90° deflection frequency and ω_t is the transit frequency. In this regime, the detrapping rate is high, but does not reach Pfrisch-Schluter regime. The paper deals with the shaped plasma and there is no assumption on the aspect ratio, therefore, it might be helpful to understand the behaviour of the low-aspect-ratio tokamak. For a banana regime, the kinetic theory will be considered in a coming paper.

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3.6 Design of the Air-core Transformer in Sperical Tokamak

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Key words Spherical tokamak Air-core transformer

Spherical tokamak has not been paid much attention until START successfully operated. There are two unpredicted features in the spherical tokamak. First, there is natural diverter configuration. Secondly, there is no major disruption in first 30 000 ohmic discharges.

A modified variational principle combined with climbing mountain method is used for the design of air-core transformer in the Chinese first spherical tokamak, SUNIST. The stray field by air-core transformer in the plasma region is less than 0. 1% of toroidal magnetic field. Integer turn in each coil is convenient for power supply.

1 Modified variational method

The magnetic energy of stray field by air-core transformer in plasma region is written as:

$$J = \int_{\Gamma} \frac{B^2}{2\mu_0} \mathrm{d}\tau \qquad (1)$$

where

$$\boldsymbol{B} = \nabla \Psi \times \nabla \varphi \qquad (2)$$

Since there is no coil in plasma region, Ψ satisfies

$$\nabla \cdot \frac{\nabla \Psi}{r^2} = 0 \qquad (3)$$

J can be rewritten as:

$$J = -\frac{\pi}{\mu_0} \oint_{\Gamma} \Psi \boldsymbol{B} \cdot \mathrm{d}l \qquad (4)$$

where Γ is the boundary of the plasma region, l is clockwise.

The air-core transformer is composed of coils $\{c_i\}$ as seen in Fig. 1. $\{r_i, z_i\}$ are the centers of coils. Minimization of magnetic energy J can obtain current density in each coil, $j_i = I_i / s_i$.

The flux Ψ in Eq. (2) can be expressed as:

$$\Psi = \sum_{i=1}^{N} j_i \iint_{s_i} \Psi_i \, \mathrm{d}x \, \mathrm{d}y \qquad (5)$$

where