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3.6 Design of the Air-core Transformer in Sperical Tokamak

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Key words Spherical tokamak Air-core transformer

Spherical tokamak has not been paid much attention until START successfully operated. There are two unpredicted features in the spherical tokamak. First, there is natural diverter configuration. Secondly, there is no major disruption in first 30 000 ohmic discharges.

A modified variational principle combined with climbing mountain method is used for the design of air-core transformer in the Chinese first spherical tokamak, SUNIST. The stray field by air-core transformer in the plasma region is less than 0.1% of toroidal magnetic field. Integer turn in each coil is convenient for power supply.

1 Modified variational method

The magnetic energy of stray field by air-core transformer in plasma region is written as:

$$J = \int_{\Gamma} \frac{B^2}{2\mu_0} \mathrm{d}\tau \qquad (1)$$

where

$$\boldsymbol{B} = \nabla \Psi \times \nabla \varphi \qquad (2)$$

Since there is no coil in plasma region, Ψ satisfies

$$\nabla \cdot \frac{\nabla \Psi}{r^2} = 0 \qquad (3)$$

J can be rewritten as:

$$J = -\frac{\pi}{\mu_0} \oint_{\Gamma} \Psi \boldsymbol{B} \cdot \mathrm{d}l \qquad (4)$$

where Γ is the boundary of the plasma region, l is clockwise.

The air-core transformer is composed of coils $\{c_i\}$ as seen in Fig. 1. $\{r_i, z_i\}$ are the centers of coils. Minimization of magnetic energy J can obtain current density in each coil, $j_i = I_i / s_i$.

The flux Ψ in Eq. (2) can be expressed as:

$$\Psi = \sum_{i=1}^{N} j_i \iint_{s_i} \Psi_i \, \mathrm{d}x \, \mathrm{d}y \qquad (5)$$

where

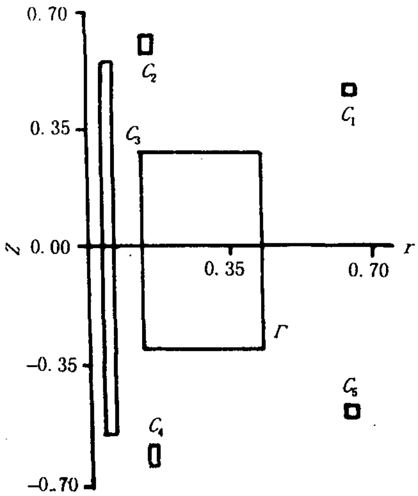

Fig. 1 $\{c_i\}$ are air-core coils
Γ is boundary of plasma region.

$$\Psi_i = \frac{\mu_0}{\pi}\sqrt{\frac{r r_i'}{k_i}}\left\{\left(1 - \frac{k_i^2}{2}\right)K(k_i) - E(k_i)\right\} \tag{6}$$

where $K(k_i)$ and $E(k_i)$ are the first and second elliptic sanctions respectively. The tangential magnetic field in Eq. (4) is expressed as:

$$B_l = \sum_{j=1} j_j \iint_{s_i} b_{lj}(r,z,r_j',z_j')\,dx\,dy \tag{7}$$

where b_{lj} is tangential magnetic field on boundary of plasma region generated by j^{th} coil.

$$b_{rj} = \frac{\mu_0}{2\pi r}\frac{z - z_j'}{\sqrt{(r + r_j')^2 + (z - z_j')^2}}\left[-K(k_j) + \frac{r_j'^2 + r^2 + (z - z_j')^2}{(r_j' - r)^2 + (z - z_j')^2}E(k_j)\right] \tag{8}$$

$$b_{zj} = \frac{\mu_0}{2\pi}\frac{1}{\sqrt{(r + r_j')^2 + (z - z_j')^2}}\left[K(k_j) + \frac{r_j'^2 - r^2 - (z - z_j')^2}{(r_j' - r)^2 + (z - z_j')^2}E(k_j)\right] \tag{9}$$

$$r_j' = r_j + x\,,\quad z_j' = z_j + y$$

$$k_j^2 = \frac{4 r r_j'}{(r_j' + r)^2 + (z - z_j')^2} \tag{10}$$

J can be expressed by quadratic form.

$$J = \sum_i A_{ij} j_i j_j \tag{11}$$

$$\text{where } A_{ij} = -\frac{\pi}{\mu_0}\oint_{\Gamma}\left\{\iint_{s_i}\Psi_i(r, z, r_i, z_i')\,dx\,dy\iint_{s_i} b_l(r, z, r_i', z_i')\,dx\,dy\right\}dl \tag{12}$$

J is positive and symmetric.

To obtain an ideal current distribution in the coil position (see in Fig. 2), a coil is as-

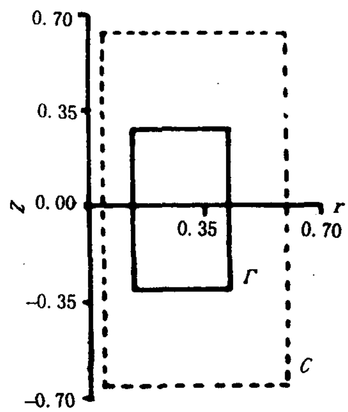

Fig. 2 Virtual closed curve is the position of the coils

sumed to occupate one point, then

$$A_{ij} = \oint_{\Gamma}\left\{\sqrt{\frac{r r_i}{k_i^2}}\left[\left(1 - \frac{k_i^2}{2}\right)k(k_i) - E(k_i)\right] b_l(r, z, r_j, z_j)\right\}dl$$

where $k_i^2 = \dfrac{4 r r_i}{(r + r_i)^2 + (z - z_i)^2}$

The air-core transformer must supply center amount of magnetic flux, which is a constraint in the variational process.

$$2\pi\sum_{i=1}^{N} j_i \iint_{s_i} \Psi_i\,dx\,dy = \Psi_0 \tag{13}$$

where Ψ_0 is magnetic flux needed for generation of plasma current.

If the coil distribute in the curve c (see in Fig. 2), the varitional is

$$J^{\alpha}=\frac{1}{2}\oint_c \oint_c A(s,s')j(s)j(s')\,ds\,ds' + \lambda\oint_c j(s')\,\Psi ds' + \frac{\alpha}{2}\oint_c\left(\frac{dj(s')}{ds'}\right)^2 ds' \quad (14)$$

where λ Lagrangian factor, α is the regulation coefficient.

$$\delta J^{\alpha}=0 \quad (15)$$

We get

$$\oint_c A(s,s')\,ds + \lambda\Psi(s') - \alpha\frac{d^2 j}{ds'^2}=0 \quad (16)$$

Eq. (16) is discreted:

$$\sum A_{ij}j_i + \lambda\Psi_j - \frac{\alpha}{h^2}(j_{j-1}+j_{j+1}-2j_j)=0 \quad (17)$$

where h is the discreted step, λ can be obtained from constraint condition. α is free parameter. If α is too large, the error is large. If α is too small, it may cause paretic oscillation.

2 Climbing-mountain method

Current distribution can be obtained by the variational principle. According to the ideal current distribution, layout of the coil system can be chosen, shown in Fig. 1. Change the position and dimension of the coils to make $\bar{B}$ minimum is called as climbing mountain method.

$$\bar{B}=\frac{\sum_{mk}(B_{Rmk}^2+B_{Zmk}^2)^{1/2}}{MK} \quad (18)$$

$$B_{Rmk}=\sum_{i=1}^{N} j_i\iint_{s_i} b_{ri}\,dx\,dy$$

$$B_{Zmk}=\sum_{i=1}^{N} j_i\iint_{s_i} b_{zi}\,dx\,dy \quad (19)$$

where (m, k) is a net point in the plasma region Γ.

It is required that the magnetic flux Ψ_0 be 0. 036 T vs. 7 727 A, 77 270 A, 3 399 880 A, 77 270 A, 7 727 A currents are obtained in the coils c_1, c_2, c_3, c_4, c_5 respectively. The averaged stray field is 1. 7 mT, 0. 1% of the toroidal magnetic field. Integer turn in each coil is convenient for power supply.

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