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FINITE ELEMENT ANALYSIS OF RANDOM INTERACTING BRANCHED CRACKS

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ABSTRACT

Combination of mechanical loads and aggressive environment causes a development of random interacting branched cracks in materials with grain structure (e.g. Inconel 600 in PWR steam generator tubing). Understanding and predicting behavior of such cracks are important for the safety of nuclear facilities and also for economical reasons in common process industry. Reliable and robust analysis of such cracks is possible only with numerical methods, among which finite element method is the most suitable for the task.

The paper proposes procedure, which enables analysis of large number of random interacting branched cracks for linear elastic materials. The proposed procedure consists of numerical analysis of crack pattern with finite element method (using the general-purpose finite element code ABAQUS with calculation of *J*-integral) and mixed mode decomposition of *J*-integral using displacements at crack surfaces.

Proposed procedure is used to evaluate different patterns of random two-dimensional complex shaped cracks in general biaxial stress field. The accuracy of the numerical results obtained is compared with reference solutions from the literature.

1 INTRODUCTION

Intergranular cracking in materials with grain structure is frequently observed as networks of random interacting branched cracks. Cracks gradually branch and/or grow together into complex patterns that follow grain boundaries [1]. Good example is an intergranular stress corrosion cracking, which has been observed in tubing made of Ni-Cr-Fe alloy Inconel-600 and has caused premature replacement of many steam generators in nuclear power plants [2].

Figure 1 shows typical crack pattern obtained by computer simulation, which is qualitatively in accordance with metalographic analysis of intergranular stress corrosion cracks.

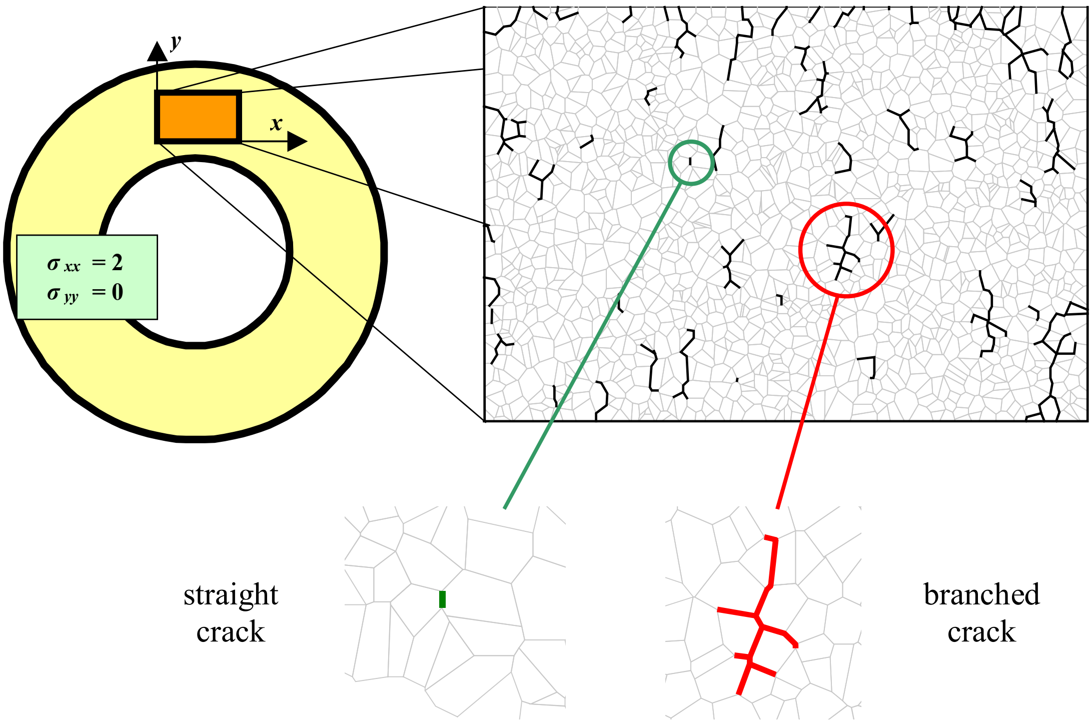

Figure 1: Random crack pattern

Understanding and prediction of initialization and growth of such cracks are exceptionally important for the safety of nuclear facilities and also for economical reasons in common process industry [3, 4]. Complexity and randomness of such networks of cracks are limiting applicability and/or accuracy of available methods for analysis of crack tip loading [1], which is assumed to be responsible for crack growth.

Analytical solutions for stress fields in the vicinity of crack tips are, in general biaxial stress field, limited to straight, kinked and symmetrically branched cracks. Interaction between neighboring cracks is limited to straight and some symmetrically branched cracks [5]. Therefore the use of numerical methods is necessary [6, 7].

So far only one author dealt with the analysis of loading of random interacting crack patterns [8]. This approach is based on setting up an empirical model, which correlates the actual cracks with simple replacement cracks. The approach was derived within equibiaxial stress field and becomes unpredictable when unknown crack patterns are considered.

The main purpose of this paper is to propose a procedure that can reliably and robustly analyze crack tip loading of random interacting branched cracks. The use of numerical methods is unavoidable because of a large number of cracks; the use of commercially available programs is reasonable in the sense of general applicability.

The paper discuses linear elastic (brittle) materials and two-dimensional crack patterns. Limitation to linear elastic materials is justified with the fact that stress corrosion causes local brittleness in ductile materials [9]. Relatively simple transition towards more complex materials in the future can be achieved by selection of appropriate numerical methods. Incomplete random tessellation is utilized to model random grain structure (as seen in Figure

1) [8, 10]. Incomplete random tessellation is not limited to two-dimensional cases – current limitation to two-dimensional crack patterns is caused by limited computational capacities.

2 PROCEDURE OF NUMERICAL ANALYSIS

The proposed procedure consists of numerical analysis of crack pattern (i.e. calculation of *J*-integral for all cracks in the pattern) and mixed mode decomposition of *J*-integral to the contributions of crack loading modes I and II.

The most appropriate numerical method for evaluation of a large number of random interacting branched cracks is the finite element method [12, 13]. Finite element method enables reliable and robust analysis of crack tip loading and enables relatively simple transition towards the more complex material properties and use of commercially available software. In this case general-purpose finite element code ABAQUS [14] was used. *J*-integral was selected as a measure for crack tip loading, which is assumed to determine crack growth [11]. Main reasons for selecting *J*-integral are numerical stability of calculation and relatively simple transition towards the more complex materials (e.g. elastic-plastic materials).

A decomposition using displacements at crack surfaces was used for mixed mode decomposition of *J*-integral (to contributions of crack loading modes I and II). *J*-integral does not differentiate between crack loading modes, which strongly influence the direction of crack growth [1]. Random cracks can in each branch grow in only limited number of directions (following the grain boundaries). The direction of crack growth depends on crack tip loading and contributions of crack loading modes, respectively. The decomposition using displacements at crack surfaces is based on *J*-integral and two complementary relative displacements of points at each surface close to the crack tip [11, 12, 15]. The accuracy of this approach has been verified elsewhere [12, 13].

Figure 2 shows block scheme of proposed procedure of finite element analysis.

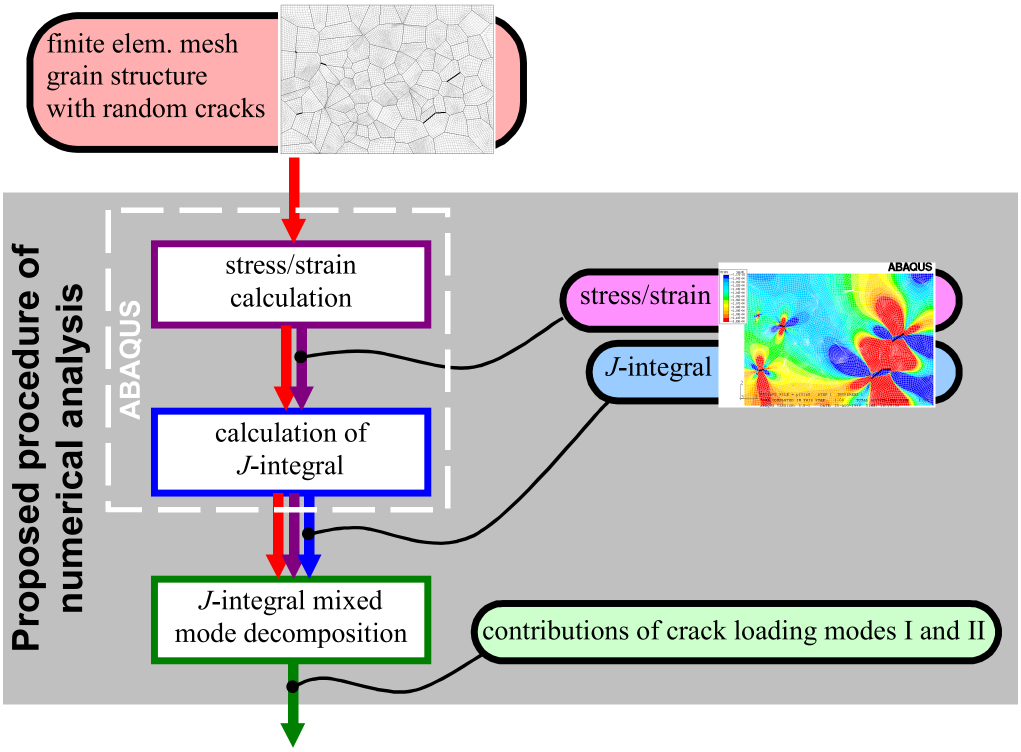

Figure 2: Proposed procedure of numerical analysis

3 NUMERICAL EXAMPLES

A finite element analysis of crack patterns (as shown in Figures 3–5) with increasing number and complexity of cracks was performed [11].

The development of crack patterns and finite element mesh generation for use in numerical analysis is carried out as shown in Figures 3–5. The cracks on the grain boundaries are generated with random processes [16], which simulate crack initialization and propagation. Cracks in crack pattern 0 shown in Figure 3 were obtained by initialization of cracks. Cracks in crack pattern 1 shown in Figure 4 were obtained by initialization of new cracks and growth of cracks from crack pattern 0. Cracks in crack pattern 2 shown in Figure 5 were obtained by initialization of new cracks and growth of cracks from crack pattern 1.

Upper left parts of Figures 3–5 show the generated cracks with thick lines. Thin lines represent the grain boundaries.

Cracks in crack patterns are shown in upper right parts of Figures 3–5.

Finite element meshes with quadrilateral finite elements, which follows grain boundaries, are presented in the bottom left parts of Figures 3–5. Algorithms for automatic mesh generation of random crack patterns are extensively using the mesh generation capabilities of commercially available program CADfix [17].

Von Mises equivalent stress is shown in bottom right parts of Figures 3–5. It shows the influences of the cracks in the uniform biaxial remote stress field. Strengthening (red areas) and weakening (blue areas) of the stress field is especially pronounced in the vicinity of crack tips. Crack interactions are becoming more significant with the increasing number of cracks in the crack pattern (seen as strengthening and weakening of the stress field).

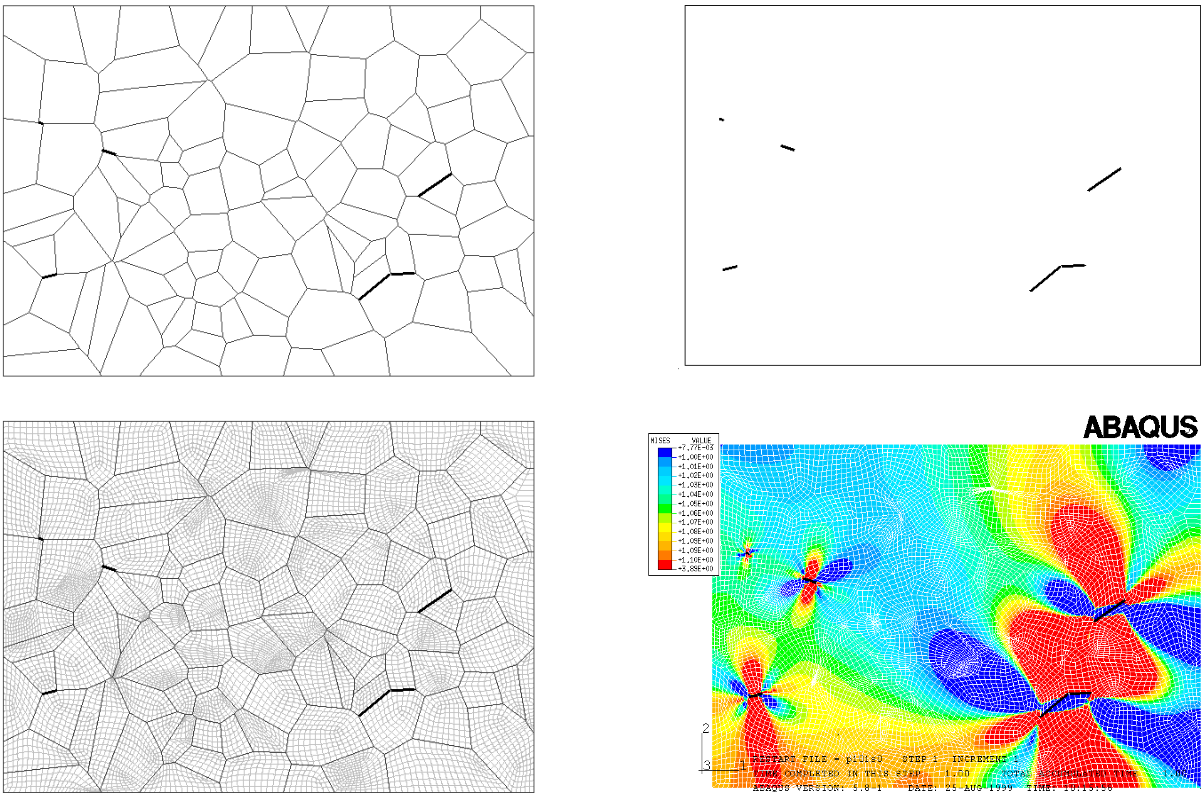

Figure 3: Random crack pattern 0 (with crack initialization only)

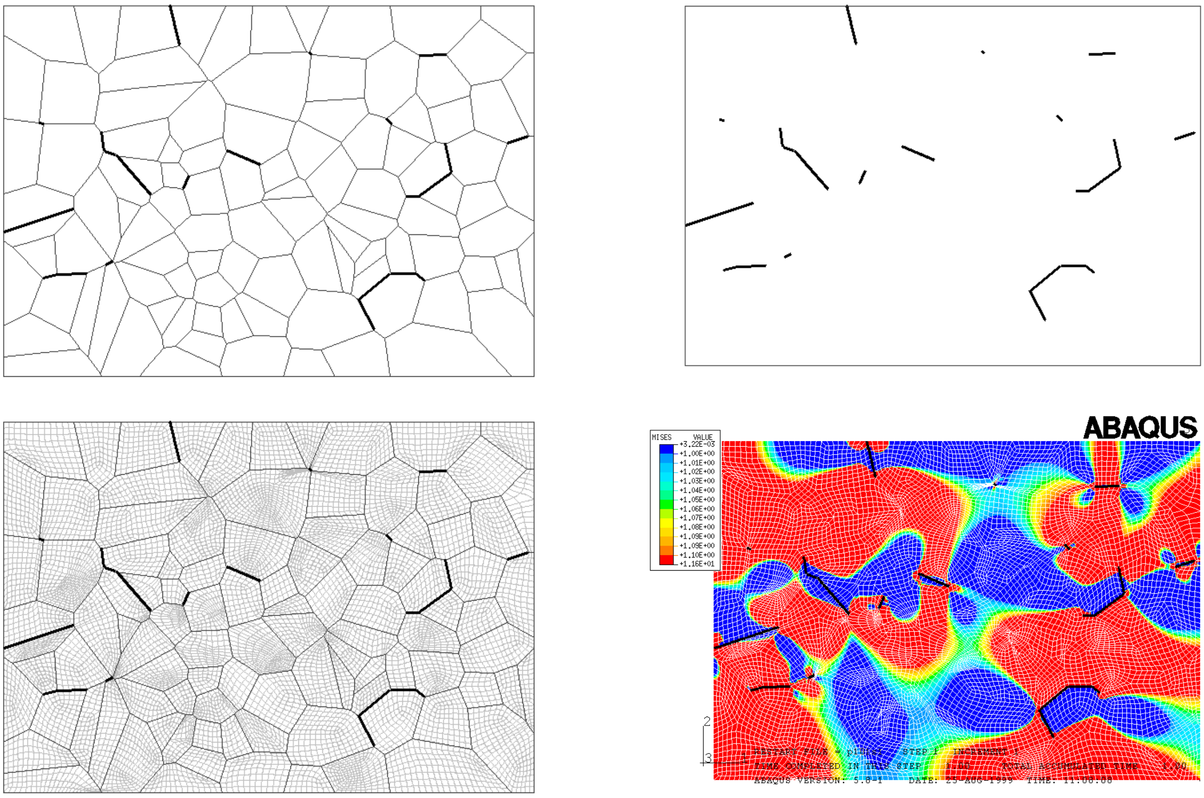

Figure 4: Random crack pattern 1 (random crack pattern 0 with crack growth and initialization of new cracks)

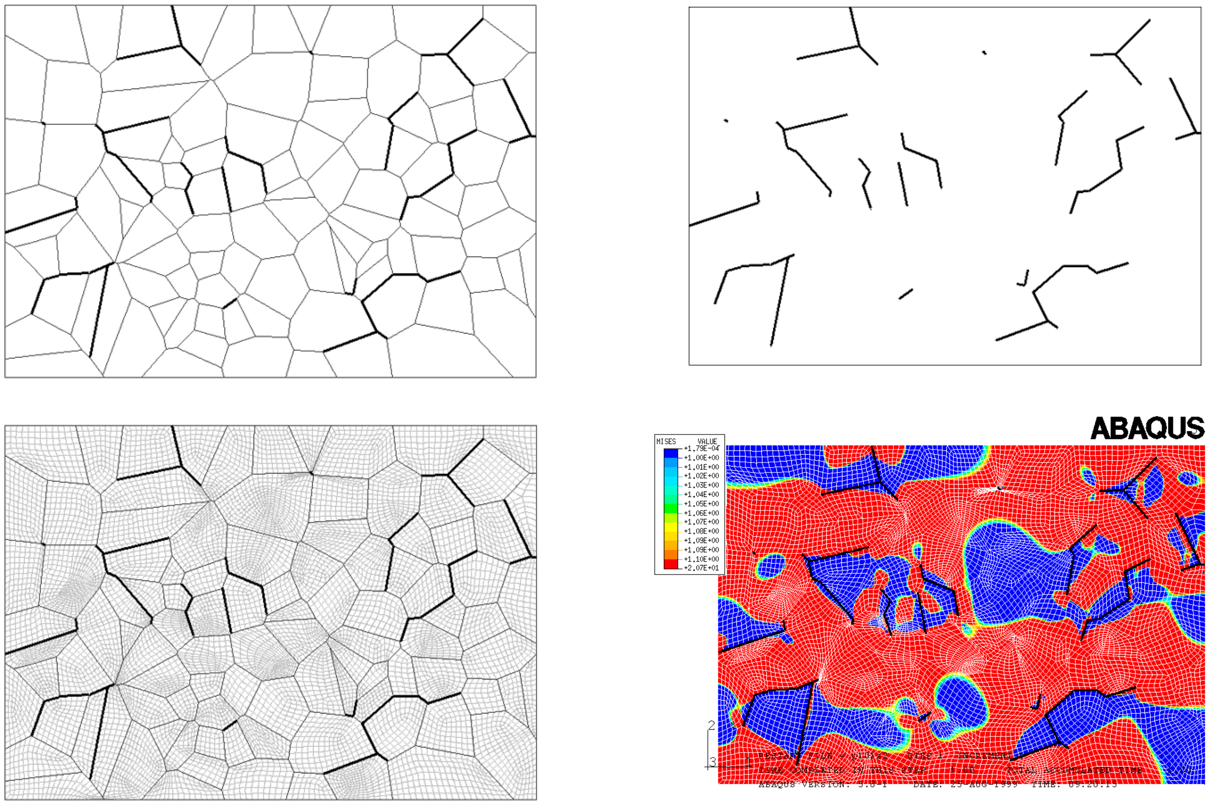

Figure 5: Random crack pattern 2 (random crack pattern 1 with crack growth and initialization of new cracks)

Random crack patterns and large number of cracks limits effective control over automatically generated finite element mesh [10, 16]. Mesh density (i.e. element size compared to the crack length) varies strongly from one crack tip to another in all analyzed patterns. In general, shorter cracks are modeled with fewer elements, which also increases the pure numerical error of finite element analysis. Such numerical errors are expected to be significantly smaller than errors caused by inadequate modeling of the interaction between neighboring cracks [15].

In order to verify proposed procedure the accuracy of *J*-integral is compared with analytical solutions for straight cracks [6] and results of the empirical model with replacement cracks for all other crack shapes [8]. The interaction between neighboring crack is not appropriately accounted for in reference solutions.

Comparisons were performed for three crack patterns (crack pattern 0, 1 and 2 as shown in Figures 3, 4 and 5 respectively) with increasing complexity and number of cracks. The case of remotely loaded plate ($E = 1$ N/m^2, $\nu = 0.3$) is composed from 101 random grains within biaxial stress field ($\sigma_x = 1$ N/m^2, $\sigma_y = 1$ N/m^2).

Figure 6 shows the relative accuracy of *J*-integral compared to reference results for straight cracks as a function of relative element size regarding the crack length. Thin line represents reference result, thick line represents expected numerical accuracy of finite element method due to differences in mesh density [18].

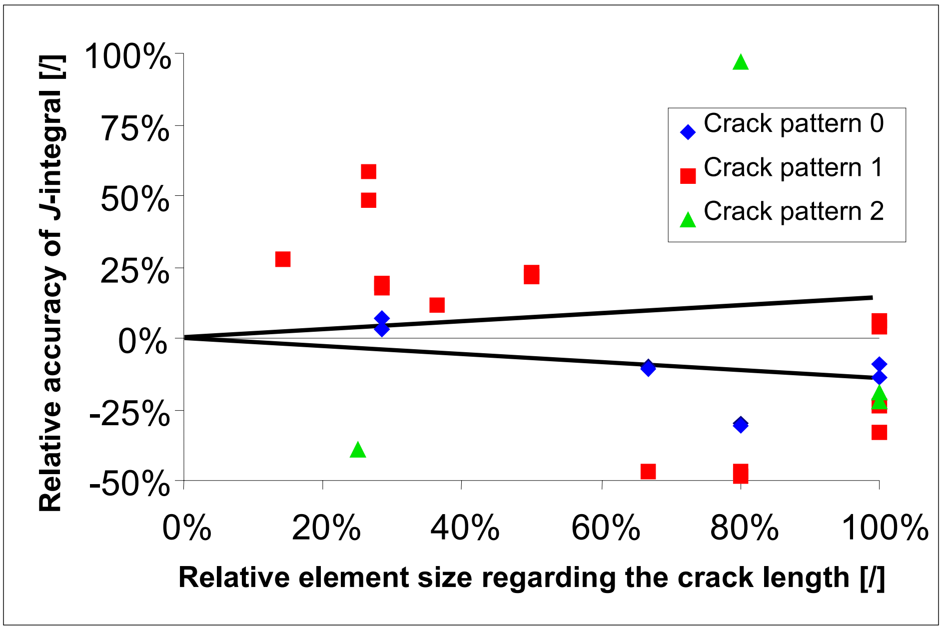

Figure 6: Relative accuracy of *J*-integral compared to reference results for straight cracks

J-integrals obtained for crack pattern 0 (Figure 3) are within expectations. This shows acceptable accuracy of the proposed procedure and verifies the reliability and robustness of proposed procedure. Two crack tips which are off expected numerical accuracy (approximately –30% at 80% relative element size) belong to the shortest crack in the crack pattern 0. The extremely short crack is especially sensitive to interaction, even with far away cracks.

Larger scatter was observed for *J*-integral of cracks in crack pattern 1 (Figure 4). Crack interaction, which is not appropriately accounted for in reference solutions, is becoming more significant with larger number and higher concentration of cracks.

The largest scatter was – as expected – observed for *J*-integral of cracks in crack pattern 2 (Figure 5). Scatter is mainly due to the even larger number of cracks in crack pattern 2 and therefore increasing influence of interaction between neighboring cracks.

Relative accuracy of *J*-integral varies from 25% for crack pattern 0 (with lowest density of cracks), to 60% for crack pattern 1 and 100% for crack pattern 2 (with highest density of cracks).

Figure 7 shows relative accuracy of *J*-integral compared to reference results for branched cracks as a function of relative element size regarding the crack length. Thin line represents reference result, thick line represents expected numerical accuracy of finite element method due to differences in mesh density [18].

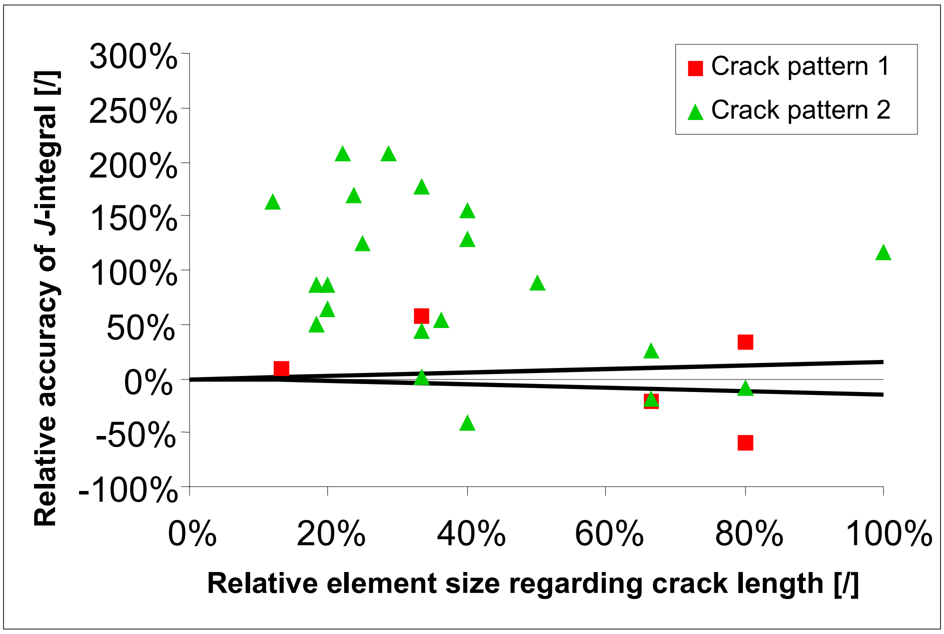

Figure 7: Relative accuracy of *J*-integral compared to reference results for branched cracks

Results obtained for *J*-integral of cracks in crack pattern 1 (Figure 4) are close to expectations. The reason for scatter of the results is crack interaction, which is not appropriately accounted for in reference solutions and is becoming more significant with larger number and higher density of cracks.

The largest scatter was observed for *J*-integral of cracks in crack pattern 2 (Figure 5). Scatter is mainly due to the very high density of cracks in crack pattern 2 and therefore very strong influence of interaction between neighboring cracks.

Results obtained show acceptable accuracy of the proposed procedure. However, its practical application may largely depend upon crack topology (e.g. straight, branched), number of cracks in the pattern and relative size of the finite elements used to model cracks.

4 CONCLUSIONS

Intergranular cracking in materials with grain structure is frequently observed as networks of random interacting branched cracks. Cracks gradually grow following the grain boundaries and form complex patterns. Understanding and predicting the initialization and growth of these cracks are exceptionally important for the safety and economical reasons in nuclear facilities and common process industry. Complexity and randomness of branched cracks are limiting applicability and/or accuracy of available methods for analysis of crack tip loading, which is considered to be responsible for crack growth.

The main objective of research was a development of a procedure that can reliably and robustly analyze crack tip loading of random interacting branched cracks. The use of numerical methods and commercially available programs is necessary because of a large number of cracks and in the sense of general applicability.

The proposed procedure includes finite element analysis of crack pattern (i.e. calculation of *J*-integral for all cracks in the pattern) and mixed mode decomposition of *J*-integral to the contributions of crack loading modes I and II.

The proposed procedure of finite element analysis of crack patterns has been verified with different crack patterns. Based on the numerical examples we can conclude that proposed procedure robustly and reliably analyzes loading of random interacting branched cracks. The procedure is limited to linear elastic (brittle) materials and two-dimensional crack patterns. Current limitation to two-dimensional cracks is mainly due to limitations in available computational capacities.

Proposed procedure enables relatively simple transition towards three-dimensional crack patterns and more complex material properties in the future.

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