# 3 Plasma Theory

## 3. 1 Lower Hybrid Wave Absorption in a Quasi-stationary Reversed Shear Mode of Operation in HL-2A[1)]

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**Key words:** Lower hybrid wave, Reversed magnetic shear, $n_{//}$–upshift

To demonstrate the viability of steady-state tokamak operation in a high performance regime, which is one of the key physics issues for a viable economic fusion reactor[1], the quasi-stationary reversed shear (RS) mode of operation in the HL-2A tokamak have been modeled with the TRANSP code[2]. In order to sustain the RS operation towards steady state, off-axis current drive with lower hybrid (LH) wave at 2.45 GHz is used to control the current profile. The target plasma is maintained by means of 2.0 MW neutral beam (1.5 MW co-injection and 0.5 MW counter-injection) injected into an ohmic heating discharge of $I_p$=265 kA with a modest peaking density profile [$n_e(0) / \langle n_e \rangle$=1.86, $n_e$ =2. 32× $10^{19}$ $m^{-3}$]. When the LH wave power injects, the profile of LH wave driven current is peaked at off-axis. After a transitional phase that lasts about 0. 35 s, the driven current profile keeps nearly the same shape during the temporal evolution [Fig. 1(a)]. By using off-axis lower hybrid current drive (LHCD) combined with the effect of bootstrap current and beam driven current a steady-state RS discharge is formed and sustained with $x_{min}$≈0. 6 and $q_{min}$≈2. 8 (where $q_{min}$ is the minimum $q$, and $x_{min}$ the location of $q_{min}$) until the LH power is turned off.

The sustainable RS scenario is robust. To examine the robustness, we perform the RS discharge modeling for various different target plasmas:

(a) Target (i): the approximately balanced beam injection (1.0 MW co-injection and 0. 8 MW counter-injection) produces only small beam driven current $I_{NB}$≈10 kA compared with $I_{NB}$≈45 kA in the standard target plasma.

(b) Target (ii): the peaking factor of density profile $n_e(0) / \langle n_e \rangle$ increases by

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about 25%.

(c) Target(iii): the total plasma density increases by 25%.

(d) Target(iv): NBI heating power decreases by about 25%, accordingly the plasma temperature is reduced to $T_{e0} \approx 1.0$ keV, and $T_{i0} \approx 2.2$ keV.

(e) Target(v): in a shaping plasma with $k_{95} = 1.3$ and $\delta_{95} = 0.35$.

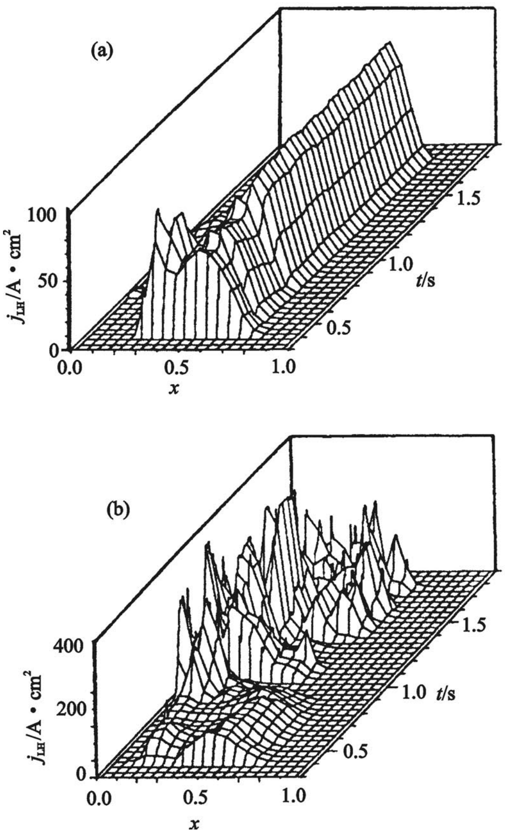


Fig. 1 Temporal evolution of LH wave driven current profile in the case of (a) $I_p$=265 kA, and (b) $I_p$= 300 kA

In all these cases similar off-axis non-inductive driven current profiles as in the standard case are produced during the steady state phase, and sustained RS discharges are achieved. However, when the plasma current increases to $I_p$=300 kA the LH wave driven current profile is centered with higher current drive efficiency after a transitional phase [Fig. 1(b)], indicating that the LH wave penetrates into the plasma center. In this case the $q$ profile evolves approaching $q_{min} < 1.0$ and $x_{min} \approx 0.2$, and the RS discharge would not be sustained.

It turns out that in such plasma (plasma temperature $T_{e0}$=1.4 keV, and $T_{i0}$=2.8

keV) the absorption of high phase velocity LH waves is too weak to ensure that the wave damped directly in the outer part of the plasma. The achievement of off-axis LH wave deposition would rely on wave propagation constrained by the LH wave dispersion relation in plasmas and on multiple reflections in the plasma. Therefore, in order to establish stationary hollow current profile by using LHCD it is necessary to analyze the LH wave absorption regime and choose the plasma parameters properly so as to obtain favorable scenarios for stationary RS operation.

To analyze the LH wave absorption in tokamak plasmas, a simplified LH wave dispersion relation is derived in the specific regime of the plasma parameters. When the WKB approximation is valid, the wave matrix equation is[3]

$$[\boldsymbol{kk}-\boldsymbol{I}k^2+k_0^2K(\boldsymbol{r},\ \boldsymbol{k},\ \omega)]\cdot E=0 \tag{1}$$

where $k_0=\omega/c$ is the free space wave number, and $\boldsymbol{K}$ is the dielectric tensor. For a non-trivial solution,

$$D(\omega,\boldsymbol{k},\boldsymbol{r})\equiv[\boldsymbol{kk}-\boldsymbol{I}k^2+k_0^2K(\boldsymbol{r},\ \boldsymbol{k},\ \omega)]=0 \tag{2}$$

To proceed, it is convenient to choose a local Cartesian system such that $z\times\boldsymbol{B}=0$, and $\boldsymbol{k}$ is contained in the $x$-$z$ plane[3]. $\boldsymbol{k}=k_{/\!/}z+k_{\perp}x$.

For LH waves, the electron cyclotron frequency is much higher than the wave frequency, which is much higher than the ion cyclotron frequency: $\omega_{\rm ci}\ll\omega\ll\omega_{\rm ce}$, the plasma dielectric behaviour is described by the following tensor elements:

$$K_{xx}=K_{yy}=\varepsilon_{\perp}=1+\frac{\omega_{\rm pe}^2}{\omega_{\rm ce}^2}-\sum_j\frac{\omega_{{\rm pi},\ j}^2}{\omega^2} \tag{3}$$

$$-K_{yz}=K_{xy}=-{\rm i}\varepsilon_{xy}={\rm i}\frac{\omega_{\rm pe}^2}{\omega\omega_{\rm ce}} \tag{4}$$

$$K_{zz}=\varepsilon_{/\!/}=1-\frac{\omega_{\rm pe}^2}{\omega^2} \tag{5}$$

where $\omega_{\rm pe}$ is the electron plasma frequency, $\omega_{{\rm pi},j}$ is the ion plasma frequency of the $j$th species. In the dielectric tensor only cold plasma terms are kept since the warm plasma term is dominant only near the lower hybrid resonance.

If $n\equiv kc/\omega\gg 1$, a simplified dispersion relation can be found from the matrix equation by asymptotically expanding

$$E \approx \boldsymbol{E}^{(0)} + \frac{1}{n^2}\boldsymbol{E}^{(1)} + o\left(\frac{1}{n^4}\right) \quad (6)$$

To the lowest order, $\boldsymbol{I}_{\perp} \bullet \boldsymbol{E}^{(0)} = 0$, where $\boldsymbol{I}_{\perp} = \boldsymbol{I} - kk$, so that $\boldsymbol{E}^{(0)} = \boldsymbol{k}E^{(0)}$.

In first order, there arises the solubility condition:

$$\boldsymbol{k} \bullet K \bullet \boldsymbol{k} = 0 \quad (7)$$

This gives the simplified dispersion relation (electrostatic limit)

$$\varepsilon_{\perp} k_{\perp}^2 + \varepsilon_{//} k_{//}^2 = 0$$

For tokamak plasmas, the elements of dielectric tensor are of the order as follows: $\varepsilon_{//} \approx 10^2$, $\varepsilon_{xy} \approx 10^1$, and $\varepsilon_{\perp} \approx 10^0$, so that

$$k_{\perp}^2 = k_{//}^2 (\omega_{\mathrm{pe}}^2 / \omega^2) / \varepsilon_{\perp} \quad (8)$$

The consistency condition

$$\left|\boldsymbol{E}^{(1)}\right| = \left|\boldsymbol{K} \cdot \boldsymbol{E}^{(0)}\right| < \frac{1}{n^2}\left|\boldsymbol{E}^{(0)}\right| \quad (9)$$

is satisfied for $n_{//}^2 > (1 + \omega_{\mathrm{pe}}^2 / \omega_{\mathrm{ce}}^2)$, which is justified in the investigated plasmas.

The LH wave propagation domain is defined as the domain in phase space ($\boldsymbol{r}$, $\boldsymbol{k}$) where the wave phase is real. In tokamak geometry the appropriate canonical coordinates are ($r$, $\theta$, $\varphi$, $k_r$, $m$, $n$). The components of the wave-vector are ($k_r$, $m/r$, $n/R$). By solving the wave dispersion relation $D$ ($m$, $r$, $k_r$, $\omega$)=0 for $k_r$ on each flux surface for a given $n$, the region where the propagation is allowed (i.e. $k_r^2 \geq 0$) is defined. At the boundary of the propagation domain,

$$\bar{n}_{\perp}^2 + \bar{n}_{//}^2 = \bar{n}_{\theta}^2 + \bar{n}_{\phi}^2 \quad (10)$$

As the tokamak equilibrium is toroidal axis-symmetric, the toroidal mode number $n$ is conserved, $n_{\phi} = \dfrac{n}{k_0 R_0} = n_{//0}$, where $n_{//0}$ and $R_0$ refer to the $n_{//}$ launched at the LH antenna located at $R_0$. From the definition of $n_{//} = \dfrac{1}{k_0}\boldsymbol{k} \cdot \boldsymbol{B} / B$,

$$\frac{n_{//}}{n_{//0}} = 1 + \frac{n_{\theta}}{n_{//0}} \frac{\varepsilon x}{q_{\mathrm{cyl}}(x)} \quad (11)$$

By using the dispersion relation (8), at the boundary of the propagation domain,

$$\overline{n}_{//} = n_{//0}\frac{R_0}{R}\frac{\hat{q}^2 \mp \sqrt{1+(1+\hat{q}^2)(\omega_{\rm pe}^2/\omega^2)/\varepsilon_\perp}}{\hat{q}^2 - [1+(\omega_{\rm pe}^2/\omega^2)/\varepsilon_\perp]} \tag{12}$$

where $\hat{q} = \frac{q_{\rm cyl}}{\varepsilon x}$, $\varepsilon$=$a/R$, $x$= $r/a$, $a$ is the plasma minor radius.

In the RS discharges described above, the electron temperature $T_{\rm e0}$≈1.4 keV with $\overline{n}_{\rm e}$=2.32×10$^{19}$ m$^{-3}$. In such conditions there is a spectral gap between the parallel LHW phase velocity and the electron thermal velocity. To achieve LH wave power deposition in the weak damping regime, we rely on the $n_{//}$ upshift to fill the spectrum gap. There are several proposed explanation for filling the spectrum gap, such as the broadening of the $n_{//}$ spectrum caused by anomalous Doppler instability[4, 5], or scattering by density fluctuation[6]. Here, only the toroidal axis-symmetry geometry effect, which affects the evolution of $k_{//}$ on the LH wave trajectories, is used to determine the limit of $n_{//}$ upshift.

$$k_{//} = \frac{B}{B_\phi}k_\phi \pm \frac{B_\theta}{B_\phi}\sqrt{k_\perp^2 - k_r^2} \tag{13}$$

We are interested in the maximum upshift factor of $k_{//}$. Taking $k_r$=0, which applies at

a radial turning point, in tokamak plasma,

$$k_{//} \approx k_\phi + k_\perp \frac{B_\theta}{B_\phi}$$

By using the dispersion relation (8), we obtain

$$n_{//} \le n_{//0}\frac{R_0/R}{1-(\omega_{\rm pe}/\omega\ )(\hat{q}\varepsilon_\perp^{1/2})} \tag{14}$$

If the LH wave phase velocity is higher than 3.5 times the electron thermal velocity, there are too few velocity-resonant electrons to carry driven current density comparable with the ohmic current density. The corresponding required condition for current drive is

$$n_{//} = \frac{k_{//}c}{\omega} \ge \frac{6.5}{\sqrt{T_{\rm e}({\rm keV})}} \tag{15}$$

which is the condition for strong Landau damping.

To show the off-axis LH wave power deposition in HL-2A, we draw the electron Landau damping (ELD) limit (Eq. 15), the $n_{//}$ upshift boundary (Eq. 14), and the boundary of wave propagation domain (Eq. 12) in a ($x$, $n_{//}$) plane (See Fig. 2). It is shown that the LH wave absorption by strong electron Landau damping is bounded in the region above the ELD limit and below the boundary of wave propagation domain. For the quasistationary RS operation obtained with $I_p$=265 kA, it is shown in Fig. 2(a) that the spatial region of power deposition is limited to 0.5< $x$ < 0. 8, and it is off-axis. Nevertheless, when the plasma current increases to $I_p$ =300 kA, the intersection between the boundary of propagation domain and the ELD limit is located at $x$ = 0. 15 [See Fig. 2 (b)]. In this case the LH power can deposit near the plasma center, which would increase the central electron temperature allowing LH wave penetrate further into the center, and the central peaking driven current is generated. According to the analysis above, it is concluded that the sensitivity of the LH driven current profile to the variation of the total plasma current is due to the constraint imposed by the wave propagation domain.

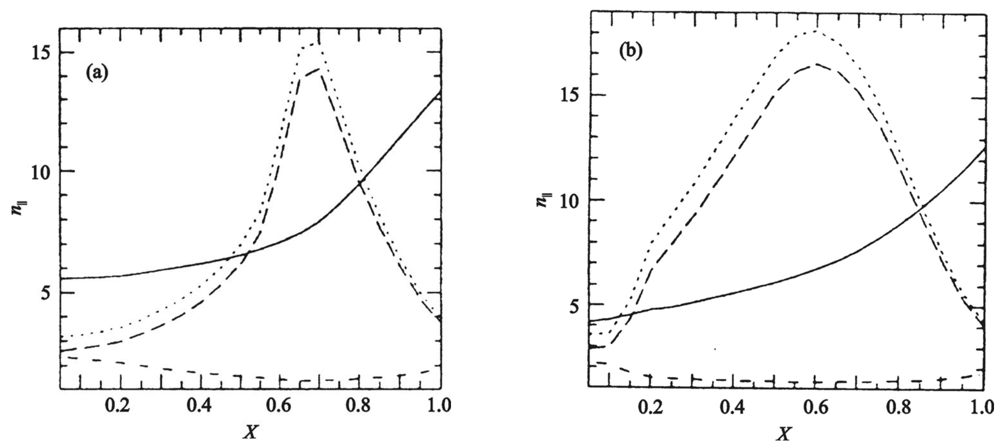


Fig. 2 Region of LH power absorption by strong electron Landau damping (at $t$= 1.0 s)

Electron Landau damping limit (full line), $n_{//}$-upshift boundary (dotted line),

and boundary of the wave propagation domain (dash line). (a) $I_p$ = 265 kA, (b) $I_p$ = 300 kA.

In summary, as the initial parallel phase velocity of LH wave is sufficiently high compared to the thermal electron velocity in the plasma center, the wave absorption is weak and it makes many passes through the plasma until the initial launched wave spectrum is sufficiently broadened to be absorbed. Nevertheless, the constraint imposed by the wave propagation condition limits the maximum allowed $n_{//}$ upshift. Taking into account the Landau damping condition, an off-axis LH power deposition region is generated by choosing the plasma parameters properly, which would direct the way for current profile control with LHCD in establishing stationary RS operation.

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# 3. 2 Instability Driven by Anisotropy Ion Temperature Gradient in Toroidal Plasmas[1)]


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**Key words:** Ion temperature gradient, Instability, Anisotropy


Stimulated by ion cyclotron resonance frequency (ICRF) heating and neutral beam injection (NBI) heating which can generate anisotropy in temperature and temperature gradient of ions in tokamak experiments, Migliuolo first investigated effects of ion temperature anisotropy on ion temperature gradient (ITG) driven modes in a shearless slab configuration[1]. He found that the anisotropy in the direction of $T_{\perp} > T_{/\!/}$ gives an overall stabilizing effect. Later, Mathey and Sen investigated effect of an anisotropy in ion temperature gradient on ITG driven instability in same geometry to explain the experiments on the Columbia Linear


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