3. 6 The Poloidal Rotation Caused by LH Waves in Tokamak Plasma Edge[1)]

JIAO Yiming GAO Qingdi

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Understanding the improved confinement still remain one of the major goals of the tokamak fusion program, and most important research schemes in tokamak relate to plasma confinement with auxiliary heating and current drive. When auxiliary power exceeds a threshold value, L-H transition will take place [1]. Theoretics and experiments show that this transition is concerned with the sheared plasma poloidal rotation in the edge [2]. When an external electromagnetic wave is injected into plasma by antenna, a powerful wave field will be induced, this field changes as $e^{-i\omega t}$, and the particles in plasma edge will make rotation poloidally by the so-called ponderomotive force[3], this mechanism is different from the $\boldsymbol{E}\times\boldsymbol{B}$ shift, the rotation velocity is proportional to $\frac{\mathrm{d}|E|^2}{\mathrm{d}r}$.

The purpose of this paper is to present a theoretical study of the ponderomotive force effects on the plasma which induced by injecting LH waves. The phased waveguide arrays[4] are the presently accepted method for exciting LH waves in toroidal plasmas.

1 Wave excitation by waveguide array

The waveguide structure is illustrated as Ref. [4]. The waveguide opens to the wall defined by the y-z plane, with the x axis pointing toward the plasma (corresponding to $-r$ direction in tokamak). The toroidal magnetic field (assumed to be straight) is parallel to the z axis. Along this direction there is a series of the same N ports, with the edges of the p-th port located at $z = z_p$ and $z = z_p + \mathrm{b}$. The ports are separated by a series of perfectly conducting slabs with a width $d=z_{p+1} - (z_p + \mathrm{b})$ in z direction and length l in y direction (corresponding to θ direction in tokamak), here $l \gg \lambda$, where $\lambda = 2\pi c/\omega$ is the wavelength.

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For tokamak experimental situation, it is likely that the plasma density near the waveguide aperture greatly exceeds the critical density defined by $n_c \approx \omega^2 \varepsilon_0 m_e/e^2$, and the coupling properties of LH waves depend strongly on the density and the density gradient which occurs very near the waveguide aperture. One can model the plasma edge profile as a density plateau plus a density linear ramp in front of the waveguide aperture. Here we shall put our waveguide aperture in the density ramp region, and this density profile can be defined as:

$$n_e(x) = n_0 + \nabla n_e x, \quad x \geqslant 0 \tag{1}$$

where ∇n_e is the density gradient.

As a first approximation, because of $b < \pi c/\omega$, only the fundamental mode in the waveguide is considered, the evanescent modes and the fast wave is neglected.

For $x \leqslant 0$ within the waveguide, the sum fields of the fundamental mode with a real amplitude α_p, a phase factor Φ_p, and a complex reflection coefficient β_p in the p-th waveguide can be taken as:

$$E_z(x, z) = -E_0 \sum_{p=1}^{N} \theta_p(z) \exp(\mathrm{i}\phi_p)[\alpha_p \exp(\mathrm{i}k_0 x) + \beta_p \exp(-\mathrm{i}k_0 x)]$$

$$B_y(x, z) = -\frac{E_0}{c} \sum_{p=1}^{N} \theta_p(z) \exp(\mathrm{i}\phi_p)[\alpha_p \exp(\mathrm{i}k_0 x) - \beta_p \exp(-\mathrm{i}k_0 x)]$$

$$E_x(x, z) = -\frac{\mathrm{i}c}{k_0} \frac{\partial B_y(x, z)}{\partial z} \tag{2}$$

where $\theta_p(z)$ is unity for z within the p-th waveguide and zero elsewhere, and E_0 can be defined by the total power:

$$E_0^2 = \frac{2\mu_0 c P_{\mathrm{total}}}{Nbl} \tag{3}$$

The total power delivered by the waveguide array to the plasma is:

$$P_{\mathrm{in}} = \frac{blE_0^2}{2\mu_0 c} \sum_{p=1}^{N} (\alpha_p^2 - |\beta_p|^2) \tag{4}$$

2 Electric field in the plasma periphery

The field in the plasma is given as a superposition of plane waves in the z direction:

$$\boldsymbol{E}(X,\ Z)=\int_{-\infty}^{\infty}\mathrm{d}k_z\boldsymbol{E}(X,\ k_z)\exp(\mathrm{i}k_z z) \tag{5}$$

In the plasma periphery, cold plasma dispersion relation can be used. From the two-fluid, cold plasma model, we have the equations for the field in a two dimensional cold plasma with a constant magnetic field along z:

$$\begin{aligned}&\frac{\mathrm{d}^2E_z}{\mathrm{d}x}-k_0^2(n_z^2-\varepsilon_\perp)\varepsilon_{//}E_z=0\\&E_x=\frac{\mathrm{i}n_z}{k_0(n_z^2-\varepsilon_\perp)}\frac{\mathrm{d}E_z}{\mathrm{d}x}\end{aligned} \tag{6}$$

where $n_z=ck_z/\omega$, and in the range of lower hybrid frequency, we have the following expressions for the elements of the dielectric tensor:

$$\begin{aligned}&\varepsilon_{//}=1-\frac{\omega_{\mathrm{pe}}^2}{\omega^2}\\&\varepsilon_\perp=1+\frac{\omega_{\mathrm{pe}}^2}{\Omega_{\mathrm{ce}}^2}\end{aligned} \tag{7}$$

where ω_{pe} is the electron plasma frequency, and Ω_{ce} is its gyrofrequency. With our exciting structure, for the fields inside the plasma, we have the inequality $|E_x|>>|E_z|>|E_y|$, and here we have assumed that E_y=0. This simplifies the problem, and we obtain only a slow wave solution. In low densities, with $\varepsilon_\perp\approx$ and $\varepsilon_{//}=1-n_e/n_c$ the equation for E_z becomes:

$$\frac{\mathrm{d}^2E_z}{\mathrm{d}x^2}-k_\perp^2(x/\mathrm{L}_n+n_0/n_\mathrm{c}-1)E_z=0 \tag{8}$$

where $\mathrm{L}_n=\left(\frac{1}{n_\mathrm{c}}\frac{\mathrm{d}n_\mathrm{e}}{\mathrm{d}x}\right)^{-1}$ denoting $\bar{x}=x/\mathrm{L}_n$, $a=\mathrm{L}_n^{2/3}\left|k_n^2-k_{//}^2\right|^{1/3}$, we can write the slow wave solution as:

$$E_z=\begin{cases}C_1(k_{//})A\mathrm{i}[\alpha(\bar{x}+n_0/n_c-1)] & k_0^2\geq k_{//}^2\\ C_2(k_{//})\{A\mathrm{i}[-\alpha(\bar{x}+n_0/n_c-1)]+\mathrm{i}B\mathrm{i}[-\alpha(\bar{x}+n_0/n_c-1)]\} & k_0^2<k_{//}^2\end{cases} \tag{9}$$

Imposing the continuity of fields at the waveguide throats, we obtain that:

$$\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \sum_{p=1}^{N} \exp(\mathrm{i}\phi_p)(\alpha_p + \beta_p) F_p(k_{//}) \begin{pmatrix} \{Ai[\alpha(n_0/n_c - 1)]\}^{-1} & k_0^2 \ge k_{//}^2 \\ \{Ai[-\alpha(n_0/n_c)] + \mathrm{i}Bi[-\alpha(n_0/n_c)]\}^{-1} & k_0^2 < k_{//}^2 \end{pmatrix} \tag{10}$$

where $F_p = \frac{\mathrm{i}}{2\pi k_{//}}[\exp(\mathrm{i}k_{//}b) - 1]\exp(-\mathrm{i}k_{//}z_p)$, and β_p satisfies following linear system:

$$\beta_q \exp(\mathrm{i}\phi_q) + \sum_{p=1}^{N} \exp(\mathrm{i}\phi_p) K(q,\ p)\beta_p = \alpha_q \exp(\mathrm{i}\phi_q) - \sum_{p=1}^{N} \exp(\mathrm{i}\phi_p) K(q,\ p)\alpha_p \quad q = 1,\ N \tag{11}$$

where $K(q,\ p) = \frac{2\pi k_0}{b}\int_{-\infty}^{\infty} \mathrm{d}k_{//} \frac{F_p F_q^*}{k_\perp Z_p}$, and the plasma impedance is:

$$Z_p = \begin{cases} \mathrm{i}a^{1/2} \dfrac{Ai[a(n_0/n_\mathrm{c} - 1)]}{Ai[a(n_0/n_\mathrm{c} - 1)]} & k_0^2 \ge k_{//}^2 \\ a^{1/2} \dfrac{Ai[-a(n_0/n_\mathrm{c} - 1)] + \mathrm{i}Bi[-a(n_0/n_\mathrm{c} - 1)]}{Ai[-a(n_0/n_\mathrm{c} - 1)] + \mathrm{i}Bi[-a(n_0/n_\mathrm{c} - 1)]} & k_0^2 < k_{//}^2 \end{cases} \tag{12}$$

3 The ponderomotive force and poloidal rotation induced by LHW

In the investigation of the nonlinear interaction of the intense rf field with magnetized plasma, the time-averaged ponderomotive force has attracted considerable attention. This ponderomotive force drives the plasma poloidal rotation in the tokamak edge and has been invoked to interpret the observed stabilization of low-frequency modes in tokamak.

Considering a single species fluid with charge q, mass m and density n (either ions or electrons) immersed a uniform external magnetic field B_0 which along with Z. Here we have ignored the poloidal magnetic field. The fluid is subjected to an electromagnetic field having an electric field component.

$$\boldsymbol{E}(x,\ k_{//},\ t) = [E_x(x,\ k_{//})\boldsymbol{e}_x + E_z(x,\ k_{//})\cos\omega t \tag{13}$$

The total force acting on a fluid element can be written in the form:

$$\boldsymbol{F}(x,\ k_{//},\ t) = qn\boldsymbol{v}\times\boldsymbol{B}_0 - n\nabla\phi + \boldsymbol{B}_0\times(\nabla\times\boldsymbol{M}) \tag{14}$$

Here, because the assumption of cold plasma approach, we have dropped the pressure-gradient term, and the pondermotive potential ϕ and the induced magnetization $\boldsymbol{M}$ are given by

$$\phi = \frac{1}{16\pi n}(\delta_{\alpha\beta} - \varepsilon_{\alpha\beta})E_{\alpha}^{*}E_{\beta}$$
$$\boldsymbol{M} = \frac{1}{16\pi}\frac{\partial \varepsilon_{\alpha\beta}}{\partial B_0}E_{\alpha}^{*}E_{\beta} \tag{15}$$

where α, β= x, y, z and $\varepsilon_{\alpha\beta}$ is the element of the dielectric tensor for a cold magnetized fluid.

For deriving an explicit expression of the poloidal rotation velocity, we average Eq. (14) over a time period. Because of the periodicity, we can obtain that in a steady state, the ponderomotive effects give rise to an azimuthal (poloidal) component of the momentum density:

$$m_{\mathrm{i}} n v_{\theta} = \frac{1}{\Omega_{\mathrm{ci}}}\left(n\frac{\mathrm{d}\phi}{\mathrm{d}x} - B_0 \frac{\mathrm{d}M_{\mathrm{z}}}{\mathrm{d}x}\right) \tag{16}$$

where Ω_{ci} is the cyclotron frequency of ions. Because of $m_{\mathrm{e}}/m_{\mathrm{i}}$, we have neglected the poloidal momentum density of the electron-fluid element. From Eq.(15) for the LHW we obtain:

$$\phi = \frac{1}{16\pi n}\left(\frac{\omega_{\mathrm{pe}}^2}{\omega^2}|E_z|^2 - \frac{\omega_{\mathrm{pe}}^2}{\Omega_{\mathrm{ce}}^2}|E_x|^2 \right)$$
$$M_z = \frac{-1}{8\pi B_0}\frac{\omega_{\mathrm{pe}}^2}{\Omega_{\mathrm{ce}}^2}|E_x|^2 \tag{17}$$

Substituting Eq. (17) into Eq. (16), we can obtain:

$$m_{\mathrm{i}} n v_{\theta} = \frac{1}{16\pi\Omega_{\mathrm{ci}}}\left(\frac{\omega_{\mathrm{pe}}^2}{\omega^2}\frac{\mathrm{d}|E_z|^2}{\mathrm{d}x} + \frac{\omega_{\mathrm{pe}}^2}{\Omega_{\mathrm{ce}}^2}\frac{\mathrm{d}|E_x|^2}{\mathrm{d}x} \right) + \frac{1}{8\pi\Omega_{\mathrm{ci}}}\frac{\nabla n}{n}|E_x|^2 \tag{18}$$

In the right hand of Eq. (18), the first term is generally expressed by some authors when n is constant, and the second term added comes from the induced magnetization by a fluid approaches treating when n is not constant.

4 Results and discussions

For a ‘grill’ used on the HL-1M tokamak with a total input power about 500 kW, and N=8, b=0.85 cm, d =0.15 cm, $\Delta\phi$=π/2, the peak value of the power spectrum is at about $\langle N_{//}\rangle$=2. 8. The figures $|E_z|^2$ and $|E_x|^2$ versus $N_{//}=\frac{ck_{//}}{\omega}$ and x (or − r in toroidal system) show that $|E_z|^2$ and $|E_x|^2$ both have a peaking at about $\langle N_{//}\rangle$=2. 8, $|E_z|^2$ decreases with x, and $|E_x|^2$ increases with the wave inwards and their derivations on x show that the rapidly changes of $|E_z|^2$ and $|E_x|^2$ will occur in the range of 0 cm < x < 4 cm in front of the 'grill' aperture.

From Eq. (18) one can see that within 0 cm<x < 4 cm, because $\frac{\omega_{\text{pe}}^2}{\omega^2}\Big/\frac{\omega_{\text{pe}}^2}{\Omega_{\text{ce}}^2}\approx 10^4$, in the first term of Eq. (18), the v_θ mainly determined by $\frac{\mathrm{d}|E_z|^2}{\mathrm{d}x}$, and the second term in Eq. (18) is positive, but it is also small comparing to the absolute value of the first term, near the plasma edge, the v_θ induced by the injecting LH wave mainly determined by $\frac{\mathrm{d}|E_z|}{\mathrm{d}x}$, and $|v_\theta|$ decreases along with the wave inwards. This result is fit with the result measured in HL-1M with Mach probes[5].

The main results of this work are that the injecting lower hybrid waves (in LHCD or LHH) shall drive a plasma poloidal rotation in the plasma edge by a so called 'ponderomotive force', this rotation mainly caused by the decay of $|E_z|^2$, and reversal to the plasma density.

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