3. 7 Plasma Transport at Magnetic Axis in Toroidal Confinement System

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The particle orbits, which intersect the magnetic axis, behave differently from banana ones[1, 2], referred to as potato orbits. The potential importance on tokamak transport is emphasized by Politzer[3], Lin, Tang, and Lee[4], and Shaing, Hazeltine, and Zarnstoff[5, 6]. However, there are many problems in the last two papers. For example, the Eq. (48) in Ref. [5] should satisfy the orbit constraint, which guarantees single value of the function g_0, that is, solubility condition, and Eq. (8) in Ref. [6] has the same problem. The con strait comes from the distribution function being an invariant when collision can be neglected for the same order. In this paper, plasma transport at magnetic axis in tokamaks is systematically studied by means of variation principle with orbit constraint instead of the constraint from magnetic differential equation used by Rosenbluth, Hazel tine, and Hinton[7]. The particle trajectory and the magnetic surface are quite different near the magnetic axis.

The potato orbits are characterized by their turning points of the poloidal velocity, r_t and θ_t, which change with the particle pitch angle, $r_{\rm t} = (4q\rho R_0/3)^{1/2}(v_{\phi 0}/v)^{1/2}$ and $\cos\theta_{\rm t} = (6R_0/27q\rho)^{1/2}(v_{\phi 0}/v)^{3/2}$. We define inverse aspect ratio $\varepsilon = r_0/R_0$ by the averaged r_t over the distribution of the trapped particles, where

$$r_0 = \frac{\int f_{\rm t}(\theta_{\rm t}) r_{\rm t}(\theta_{\rm t}) {\rm d}\theta_{\rm t}}{\int f_{\rm t}(\theta_{\rm t}) {\rm d}\theta_{\rm t}} \tag{1}$$

and $f_{\rm t}(\theta_{\rm t})$ is the distribution of the trapped particles, which intersect the magnetic axis. ε is like the inverse aspect ratio far away from the axis, which determines the trapped particle fraction in the order of $\sqrt{\varepsilon}$, potato width w_p in the order of $q\rho/\sqrt{\varepsilon}$, where $\rho = v_{t0}/\Omega$, v_{t0} thermal velocity at axis, Ω the gyrofrequency, and the effective collision frequency, $v_{\rm eff}=v/\varepsilon$. The ion thermal conductivity derived from the random walk process near the magnetic axis is

$$\chi_{\rm i} = \sqrt{\varepsilon} v_{\rm eff} w_{\rm p}^2 = v_{\rm i} q^2 \rho_{\rm i}^2 \varepsilon^{-3/2} \tag{2}$$

The result in Eq. (2) seems quite conventional. Eliminating ε in favour of R_0. It is equal to $v_i q_i R_0$ which has weaker dependence on magnetic field. In Ref.[5], the ion thermal conductivity is also provided, but the step in the random walk is different, $\chi_{\Psi} = 0.8 v_i (I v_t / \Omega_0)^{7/3} (q / IR)^{1/3}$. It is obscure in physics.

The bootstrap current at the magnetic axis is also derived. In the reactor scale, the bootstrap current density at magnetic axis could reach 10% of the total plasma current density. The variational principle[7~9] was proved to be more accurate and effective in evaluation of all neoclassical transport coefficients.

1 Particle dynamics

In tokamak configuration, the Hamiltonian[10] of a charged particle can be expressed as:

$$H = \frac{1}{2M}\left[(P_R - eA_R)^2 + (P_Z - eA_Z)^2 + (P_\phi - eRA_\phi)^2 / R^2\right] + eU \tag{3}$$

where A_R, A_Z, and A_ϕ are the vector potential components of the magnetic field, U the electrical potential which is assumed to be function of Ψ, M the mass of the charged particle which we set equal to unity for simplicity, and e the charge. P_R, P_ϕ and P_Z are the canonical momenta conjugate to R, ϕ, and Z respectively,

$$P_R = v_R + eA_R \tag{4}$$

$$P_\phi = Rv_\phi + eRA_\phi \tag{5}$$

$$P_Z = v_Z + eA_Z \tag{6}$$

The magnetic field can be expressed as:

$$B = \nabla\phi \times \nabla\Psi + I\nabla\phi \tag{7}$$

where Ψ is related to the poloidal flux of the magnetic field, I related to the poloidal current, R is the major radius. Then, in tokamak, we have

$$A_R = 0, \quad A_Z = -I\ln\frac{R}{R_0}, \quad A_\phi = -\frac{\Psi}{R} \tag{8}$$

We introduce a generating function, the area conserved transformation,

$$F_1 = -\frac{\Omega_0 R_0^2}{2}\exp\left(\frac{X}{\Omega_0 R_0}\right)\left(\ln\frac{R}{R_0} - \frac{X}{\Omega_0 R_0}\right)^2 \mathrm{tg}\alpha - ZX \tag{9}$$

where

$$X = \Omega_0 R_0 \ln\frac{R_\mathrm{c}}{R_0}, \quad R = R_\mathrm{c} \exp\left(-\frac{\rho\cos\alpha}{R_\mathrm{c}}\right) \tag{10}$$

and Ω is the toroidal gyrofrequency, ρ the Larmor radius, α the gyrophase, subscripts 0 and c refer to the values at the magnetic axis and the guiding center respectively. X and α are the new coordinates conjugate to the momenta

$$P_X = Z + \rho\sin\alpha + \frac{\rho^2}{4R_\mathrm{c}}\sin 2\alpha \tag{11}$$

$$P_\alpha = \frac{1}{2}\Omega_\mathrm{c}\rho^2 \tag{12}$$

where P_X is actually the guiding center of Z coordinate, Z_c. The Hamiltonian is rewritten as:

$$\mathrm{H} = \frac{1}{2}\Omega_\mathrm{c} P_\alpha \left[\left(\frac{R_\mathrm{c}}{R}\right)^2 \sin^2\alpha + \cos^2\alpha\right] + \frac{1}{2R^2}\left[P_\phi + e\Psi\right]^2 + eU \tag{13}$$

The canonical transformation makes the Hamiltonian be exact in new coordinates. Now, we introduce a small parameter δ,

$$\delta \approx \rho / R_\mathrm{c} \approx B_\mathrm{p} / B_\phi \tag{14}$$

Since the total energy is conserved during the particle gyromotion, to the first order of ρ / L_n, the magnetic moment could be divided in two parts, $P_\alpha = P_{\alpha 0} + P_{\alpha 1}(\alpha)$. $P_{\alpha 1}$ could cancel the explicitly α-dependent part of the Hamiltonian, that is, the gyrocenter could be shifted to construct the Hamiltonian in the form,

$$\mathrm{H}_0 = \Omega_\mathrm{c} P_{\alpha 0} + \frac{1}{2R_\mathrm{c}^2}\left[P_{\phi 0} + e\Psi_0(X, P_X)\right]^2 + eU \tag{15}$$

where Ψ_0 and U_0 are independent of α. Ω_c and R_c are functions of X through Eq. (10). Both $P_{\alpha 0}$ and $P_{\phi 0}$ are constant of motion conjugate to α. and ϕ respectively.

Together with conservation of the canonical momentum in toroidal direction, we get a set of equations of motion for the guiding center,

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{B_R}{B_\phi}\left(\frac{u}{R} - \frac{R u_E}{R_0^2}\right) \tag{16}$$

$$\frac{\mathrm{d}Z}{\mathrm{d}t}=\frac{B_z}{B_\phi}\left(\frac{u}{R}-\frac{Ru_E}{R_0^2}\right)+v_\mathrm{d} \tag{17}$$

$$\frac{\mathrm{d}u}{\mathrm{d}t}=\varOmega_R R v_\mathrm{d}+\varOmega_0 R_0\frac{E_\phi}{B_\phi} \tag{18}$$

where $u=Rv_\phi$, $u_E=-R_0^2\frac{\partial U_0}{\partial \varPsi}=R_0^2\omega_E$, $v_\mathrm{d}=\frac{1}{\varOmega_0 R_0}\left(\varOmega_\mathrm{c}P_\alpha+\frac{u^2}{R^2}\right)$. Some subscripts are omitted for simplicity. The velocities in R and z directions are easy to change to the radial and poloidal directions through rotating the coordinates. For any tokamak configuration, the particle guiding-center equations of motion are reduced in (R, ϕ, z) coordinates,

$$v_\varPsi=\frac{B_R}{B_\mathrm{p}}v_\mathrm{d} \tag{19}$$

$$v_\mathrm{p}=\frac{B_\mathrm{p}}{B_\phi}\left(\frac{u}{R}-\frac{Ru_E}{R_0^2}\right)+\frac{B_z}{B_\mathrm{p}}v_\mathrm{d} \tag{20}$$

where $v_\varPsi$ is in radial direction, while v_p is in poloidal direction. The Eqs. (19) and (20) are the generalized version of equations of motion obtained by Balescu[11].

Near the magnetic axis where ε is, generally, small, large-aspect-ratio expansion always applies. Eq. (15) could be changed into

$$\mathrm{H}=\varOmega_0 P_\alpha+\frac{1}{2q^2R_0^2}\left(qR_0v_{\phi 0}+\varPhi\right)^2-(\varOmega_0P_\alpha+v_{\phi 0}^2)\frac{\varPhi^{1/2}}{\varPhi_0^{1/2}}\cos\theta \tag{21}$$

where $\varPhi=\frac{1}{2}\varOmega_0 r^2$, the longitudinal magnetic flux conjugate to θ, $\varPhi_0=\frac{1}{2}\varOmega_0R_0^2$, $q=\varPhi/e\varPsi$ the safety factor, $v_{\phi 0}$ is the toroidal velocity at magnetic axis and the electrical potential in Eq. (15) neglected for simplicity. The Hamiltonian equations for the guiding center motion in toroidal cylindrical coordinates (r, θ, ϕ) are given by

$$\frac{\mathrm{d}r}{\mathrm{d}t}=-v_\mathrm{d}\sin\theta \tag{22}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t}=\frac{1}{qR_0}v_\phi-\frac{v_\mathrm{d}}{r}\cos\theta \tag{23}$$

Where $v_\mathrm{d}=\frac{\varOmega P_\alpha}{\varOmega_0R_0}+\frac{v_{\phi 0}^2}{\varOmega_0R_0}$. It is easy to verify that $v_{\phi 0}^2/2=\mathrm{H}-\varOmega_0P_\alpha$. We define

$x=\sqrt{\Phi}$, then Eq. (21) is turned to be cubic equation,

$$x^3+2qR_0v_{\phi0}x-2q^2R_0^2(\Omega_0P_\alpha+v_{\phi0}^2)\cos\theta/\Phi_0^{1/2}=0 \quad (24)$$

The solutions of Eq. (24) given in Refs. [1, 2] and [5], have following form,

$$x=2(2\sigma qR_0v_{\phi0}/3)^{1/2}Y \quad (25)$$

where Y is one of the functions: sin (π/6±β/3), cos (β/3), sin h (β/3), or cos h (β/3) depending on range of parameter $\sigma\kappa$; where β is a new angle, $\sigma=v_{\phi0}/|v_{\phi0}|$, and

$$\kappa=(\sigma v_{\phi0}/F_\mathrm{a}v)^3 \quad (26)$$

where $F_\mathrm{a}=(27q\rho_\mathrm{a}/16R_0)^{1/3}=1.3\sqrt{\Phi}$, the fraction of trapped particles, $\rho_\mathrm{a}=v_\mathrm{a}/\Omega_0$, the Larmor radius, a stands for either election or ion. We take v_a=v_ta for simplicity, where v_ta is the thermal velocity of species a.

2 Variational principle

Calculation and variation of the entropy source provides a particularly efficient means of evaluating transport coefficients, which are found from the Onsager relations.

To establish the variational formalism, first, we look at the drift kinetic Eq. (12).

$$\frac{\mathrm{d}F}{\mathrm{d}t}=C(F) \quad (27)$$

where $C(F)$ is the Fokker Planck collision operator. F can be expressed as:

$$F=F_\mathrm{m}^*(H,P_\phi)+g \quad (28)$$

where the first term is the Maxwellian form with H in the place of kinetic energy, and P_ϕ in the place of Ψ, the second term is the correction due to collision. At magnetic axis, P_ϕ is small compared with $e\Psi_\mathrm{b}$ where Ψ_b is the plasma boundary position. For the particles, which intersect the magnetic axis, F_m^* can be expanded as follows:

$$F_\mathrm{m}^*(H,\Psi)=F_\mathrm{m}(H,0)+\Psi\frac{\partial F}{\partial\Psi}-\frac{\partial F_\mathrm{m}}{\partial\Psi}\frac{Rv_\phi}{e} \quad (29)$$

Near the magnetic axis, for the parabolic distribution, $\frac{\partial F_\mathrm{m}}{\partial\Psi}$ is finite and constant. For any position r, we have

$$\frac{\partial F}{\partial \Psi}\frac{Rv_\phi}{e}=\frac{\partial F}{\partial r}\frac{v_\phi}{\Omega_\mathrm{p}} \tag{30}$$

where Ω_p is the poloidal gyrofrequency. If r_0 in Eq. (1) is chosen as expansion position, the resulting thermodynamic force and fluxes are relevant to the neoclassical transport problem near the magnetic axis. Using H, P_ϕ, r, t, ϕ, θ coordinates, the drift kinetic equation near the magnetic axis is

$$\omega\frac{\partial g}{\partial \theta}+r\frac{\partial g}{\partial r}+eE_\phi v_\phi\frac{\partial F}{\partial H}=C(F) \tag{31}$$

where $\omega=\mathrm{d}\theta/\mathrm{d}t$ is given in Eq. (23). Then, we have, from Eq. (31), the orbit constraints

$$\int\frac{C(F-v_\phi E_\phi f_\mathrm{s})}{\omega}\mathrm{d}\theta=0 \qquad \text{(untrapped region)} \tag{32}$$

$$\int_{\theta_1}^{\theta_2}\frac{C(\sigma=+1)+(\sigma=-1)}{\omega}\mathrm{d}\theta=0 \qquad \text{(trapped region)} \tag{33}$$

where f_s is given by Spitzer and Harm[13]. Finally, we observe, in the trapped region, that

$$g(+1)=g(-1) \tag{34}$$

since $\omega(\theta_{1,2})=0$ by definition.

Thus, our problem is reduced to determination of the function g, which satisfies Eqs. (31)～(33). We now wish to derive a variational formulation of this problem. For simplicity, the temperatures of electron and ion are assumed to be equal. From the four perturbed distributions,

$$f_\mathrm{a}=f_\mathrm{Ma}(1+\hat{f}_\mathrm{a}),\quad g_\mathrm{a}=f_\mathrm{Ma}(1+\hat{g}_\mathrm{a}) \qquad \mathrm{a}=\mathrm{i,e} \tag{35}$$

one may construct a bilinear form[7] from

$$K(f,g)=-\sum_\mathrm{a}\int\mathrm{d}^3\bar{v}\hat{f}_\mathrm{a}C_\mathrm{a}(g) \tag{36}$$

where

$$C_\mathrm{a}(g)=C_\mathrm{aa}(\hat{g}_\mathrm{a},\hat{g}_\mathrm{a})+C_\mathrm{ab}(\hat{g}_\mathrm{a},\hat{g}_\mathrm{b}) \tag{37}$$

The quantity may be recognized as the rate of irreversible entropy production. With the abbreviation

$$U_{\alpha\beta} = u^{-3}(u^3\delta_{\alpha\beta} - u_\alpha u_\beta) \tag{38}$$

we have

$$K(f,g) = \pi e^4 \ln\Lambda \sum_{\mathrm{a,b}} \int \mathrm{d}^3 v_{\mathrm{a}} \mathrm{d}^3 v'_{\mathrm{b}} f_{\mathrm{Ma}} f'_{\mathrm{Mb}} V_{\alpha\beta} \times$$

$$\left(m_{\mathrm{a}}^{-1} \frac{\partial \hat{f}_{\mathrm{a}}}{\partial v_{\mathrm{a}\beta}} - m_{\mathrm{b}}^{-1} \frac{\partial \hat{f}_{\mathrm{b}}}{\partial v'_{\mathrm{b}\beta}} \right) \times \left(m_{\mathrm{a}}^{-1} \frac{\partial \hat{g}_{\mathrm{a}}}{\partial v_{\mathrm{a}\alpha}} - m_{\mathrm{b}}^{-1} \frac{\partial \hat{g}_{\mathrm{b}}}{\partial v'_{\mathrm{a}\alpha}} \right) \tag{39}$$

where u in Eq. (38) is the relative velocity of the two colliding particles. Furthermore, it is clear that $K(f, g)=0$ whenever $\partial \hat{f}_{\mathrm{a}} / \partial v_{\alpha\beta} = c m_{\mathrm{a}} v_{\mathrm{a}\beta} + d_\beta m_{\mathrm{a}}$ with c and d_β constant, where $d_\beta = B_\beta / B$.

The first two terms in Eq. (29) have no contribution to $K(f, g)$. We conclude

$$f_{1\mathrm{a}} \equiv -\frac{v_\phi}{\Omega_{\mathrm{pa}}}\left[A_{1\mathrm{a}} + A_{2\mathrm{a}} H\right] - v_\phi A_3 \hat{f}_{\mathrm{sa}} - D m_{\mathrm{s}} v_\phi + \hat{g}_{\mathrm{s}} \tag{40}$$

where

$$A_{1\mathrm{a}} = \frac{n'}{n} - \frac{3}{2}\left(\frac{T'_{\mathrm{a}}}{T_{\mathrm{a}}}\right), \qquad A_{2\mathrm{a}} = \frac{T'_{\mathrm{a}}}{T_{\mathrm{a}}^2}, \qquad A_3 = E_\phi$$

We define

$$\dot{S} = -\sum_{\mathrm{a}} \left\lfloor \int \mathrm{d}^3 v \left(\hat{f}_{\mathrm{a}} - v_\phi E_\phi \hat{f}_{\mathrm{sa}}\right) C\left(f - v_\phi E_\phi f_{\mathrm{sa}}\right) \right\rfloor \tag{41}$$

where $\langle A \rangle = \oint A \mathrm{d}\theta / \oint \mathrm{d}\theta$, the orbit integral. The convenient velocity variables are

$$w = \frac{1}{2} v^2, \quad \mu = \Omega_0 P_{\mathrm{a}} \tag{42}$$

If μ defined in Eq. (42) and H change position in Eq. (21), $-\mu$ acts like Hamiltonian with Φ and θ conjugate, that is

$$\frac{\partial(-\mu)}{\partial \Phi} = \omega, \quad \frac{\partial(-\mu)}{\partial v_\phi} = q R_0 \omega, \quad \mathrm{d}^3 v = \sum_\sigma 2\pi v \mathrm{d}v \mathrm{d}\mu / q R_0 |\omega| \tag{43}$$

It is easy to prove that the requirement, $\delta \dot{S} = 0$, is equivalent to Eqs. (32) and (33). Keeping to lowest order in F_{a}, the fraction of trapped particles, since the cross terms is small in the ordering we may approximate, we have

$$K(f_1, f_2) = 2\pi e^4 \ln \Lambda \sum_{a,b} m_a^{-2} \int d^3 v_a F_{Ma} \frac{\partial f_{1a}}{\partial v_{a\alpha}} \frac{\partial f_{1a}}{\partial v_{a\beta}} \times \int d^3 v_b F_{Mb} U_{\alpha\beta} \tag{44}$$

From locality of the trapped particles, we expect

$$\frac{\partial f_1}{\partial \mu} \geq \frac{\partial f_1}{\partial w} \quad \text{and} \quad \frac{\partial f_1}{\partial v_\alpha} \approx -qR_0 \omega \frac{\partial f_1}{\partial \mu} \hat{e}_\xi \tag{45}$$

where $\cos\xi = v_\phi / v$, ξ the poloidal angle in spherical coordinates (v, ξ, α) in velocity space. The following variational process is similar to the one used in Ref. [7], except that v_ϕ is replaced by $qR_0\omega$ and magnetic surface average replaced by orbit average in Eq. (41). There is no need repeating. Finally, we get

$$\dot{S} = 4\pi^2 e^2 \ln \Lambda \sum_a m_a^{-2} I_a \int v^2 dv f_{Ma}(v)\, G_a^2(v) \sum_b F_b(v) \tag{46}$$

where G_a and F_b have the same meaning as in Ref. [7] and with $t = v_{\phi 0}/v$,

$$I_a = \int dl \left[\left\langle \frac{1}{|\hat{\omega}|} \right\rangle - \frac{1}{\langle |\hat{\omega}| \rangle} \right] (1 - l^2) = \left(\frac{q\rho_a}{2R_0} \right)^{1/3} \int \frac{d\kappa}{\kappa^{2/3}} \left[\left\langle \frac{1}{4Y^2 - 1} \right\rangle - \frac{1}{\langle 4Y - 1 \rangle} \right] \times$$

$$[1 - F_a^2(\kappa^{2/3})] \cong 2.2 \left(\frac{q\rho_a}{R_0} \right)^{1/3} \tag{47}$$

which is typical of the neoclassical transport at magnetic axis.

3 Summary and conclusion

We collect our results. Using gradient of n and T, we can write the fluxes Γ, Q, and J

$$\Gamma = F_e \frac{\rho_{pe}^2}{\tau_e} n \left(-2.12 \frac{n'}{n} + 0.59 \frac{T'}{T} \right) - 2.31 F_e \frac{nE_\phi}{B_p} \tag{48}$$

$$\widetilde{Q} = Q_i + Q_e - 5\Gamma T = F_i v_i \rho_{pi} nT \left(-0.46 \frac{T'}{T} \right) +$$

$$F_e \frac{\rho_{pe}}{\tau_e} nT \left(-1.99 \frac{T'}{T} + 2.94 \frac{n'}{T} \right) + 1.66 F_e \frac{nE_\phi}{B_p} T \tag{49}$$

and

$$J_\phi - J_{\phi s} = F_e \frac{nT}{B_p}\left(-4.62\frac{n'}{n} - 0.26\frac{T'}{T}\right) - 3.60F_e \frac{e^2 n}{m_e}\tau_e E_\phi \tag{50}$$

where ρ_p, B_p, n', and T' are the values at the position r_0, F_e and F_i are the fractions of the trapped particles for electrons and ions respectively, and we have used the classical result[13]

$$J_{\phi s} = 1.98\frac{e^2 n}{m_e}E_\phi \tag{51}$$

The ion thermal conductivity could be derived from Eq. (49).

$$\chi_i = 0.6 v_i q^2 \rho_i^2 \varepsilon^{-3/2} \tag{52}$$

which is the same as the one in Eq. (2) derived from random walk process except for numerical factor. Eliminating ε in favour of R_0, we get

$$\chi_i = 0.78 v_i q \rho_i R_0 \tag{53}$$

The ion thermal conductivity turns to be weaker dependence on the magnetic field, which we may call Bohm-like diffusion. However, high temperature is favourable in confinement. The onset condition of banana regime is determined by $v_i < (q\rho_i / R_0)(v_t / qR_0)$. Therefore, the maximum ion thermal conductivity in the banana regime is

$$\chi_i = 0.78 q^2 \rho_i^2 \omega_{it} \tag{54}$$

where ω_{it} is the ion transit frequency.

The bootstrap current can be obtained in Eq. (50),

$$J_b = F_e \frac{nT}{B_p}\left(-4.62\frac{n'}{n} - 0.26\frac{T'}{T}\right) \tag{55}$$

where

$$F_e = 1.19(q\rho_e / R_0)^{1/3} \tag{56}$$

In the reactor scale for small-aspect-ratio tokamaks, T_e=10 keV, R_0=1 m, B_0=1 T, we get, ρ_e=3.37×10^{-4} m, F_e=0.0828, and the bootstrap current

$$J_b \approx 0.10R\frac{\mathrm{d}p}{\mathrm{d}\Psi} \tag{57}$$

where p is the total kinetic pressure of the plasma. It is possible that the bootstrap current density could reach 10% of total plasma current density at the magnetic axis.

In the Refs. [14] by Hirshman and [15] by Shaing, bootstrap current in an ultra-low aspect ratio tokamak has been calculated. However, the fluid model[16] does not fully apply to the banana regime. And, Shaing's paper has mistakes. For example, it says that when aspect ratio approaches to unity, $(\boldsymbol{n}\cdot\nabla B)^2$ goes to infinity. This conclusion is not true. Using an exact solution of Glad-Shafranov equation which could form spherical tokamak configuration[17] with aspect ratio equal to unity,

$$\Psi = \Psi_0 \frac{R^2}{R_0^4}(2R_0^2 - R^2 - 4\gamma^2 Z^2) \tag{58}$$

where R_0, Ψ_0 and γ are constant, we get

$$\boldsymbol{n}\cdot\nabla B = \frac{\boldsymbol{B}_\mathrm{p}}{B}\cdot\nabla B = \frac{\boldsymbol{B}_\mathrm{p}}{2B^2}\cdot\nabla B^2 = \frac{1}{2B^2}\left(\frac{1}{R}\frac{\partial\Psi}{\partial R}\cdot\frac{\partial B^2}{\partial Z} - \frac{1}{R}\frac{\partial\Psi}{\partial Z}\frac{\partial B^2}{\partial R}\right) \tag{59}$$

where

$$B^2 = \frac{I_0^2}{R^2} + \frac{1}{R^2}\left(\frac{\partial\Psi}{\partial R}\right)^2 + \frac{1}{R^2}\left(\frac{\partial\Psi}{\partial Z}\right)^2 \tag{60}$$

We can see that $\langle\boldsymbol{n}\cdot\nabla B\rangle$ never goes to infinity.

Finally, we conclude as follows. First, the ion thermal conductivity at magnetic axis is larger than outer part of the plasma by $(a/q\rho_\mathrm{i})(a/R)^{1/2}$, where a is radius of plasma boundary and it has weaker dependence on the magnetic field, which we may call Bohm-like diffusion. This accounts for the fast heat diffusion and Bohm-like behavior at magnetic axis in tokamaks. Secondly, for the long-term operation of tokamak reactor, especially for the low aspect ratio tokamaks bootstrap current may play an important role. Ten percent of bootstrap current density at magnetic axis could reduce the need of noninductive current drive. High temperature is favourable in confinement in our scaling. Hopefully the prediction could be verified by the future experiments in tokamak.

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