On the Fission Spectrum Uncertainty

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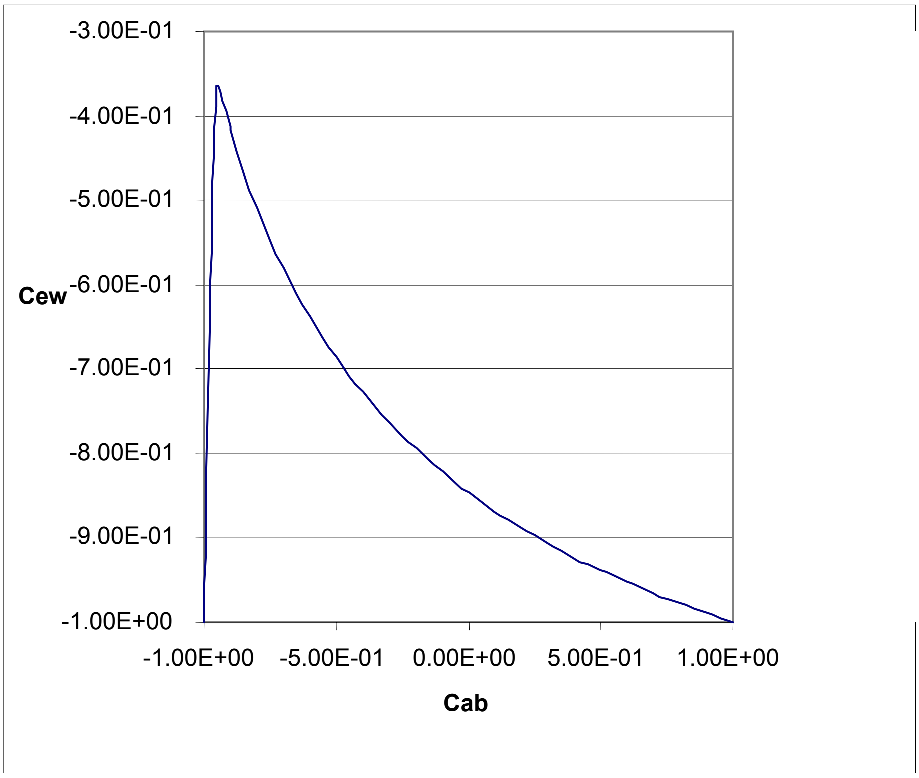
The fission spectrum can be analytically represented in several different forms, such as a simple Maxwellian, the Watt fission spectrum, or the more modern and physically sound Madland-Nix representation. A quantitative estimate of the uncertainty matrix associated with the fission spectrum is needed in many nuclear data applications. Criticality safety, neutron cross section adjustment and reactor pressure vessel surveillance dosimetry are just a few examples. A simple way to estimate the relative uncertainty in the fission spectrum, in any representation, is to represent it in terms of the relative uncertainties in certain characteristics of the fission spectrum, such as the mean energy of the secondary fission neutrons, $\overline{E}$, and w , the relative root-mean-square width about the mean energy, which is $w = (1/\overline{E})\sqrt{\overline{E^2} - \overline{E}^2}$, or "width" for short[1]. These two characteristics are correlated and their correlation has to be estimated[2]. The following figure depicts the correlation between the mean fission energy $\overline{E}$ and the fission width w, C_{ew}, as a function of the correlation between the Watt fission spectrum parameters, C_{ab}. Even if the parameters of the Watt spectrum are not correlated, i.e. even if C_{ab}=0, the mean fission energy $\overline{E}$ and the fission width w are strongly anti correlated.



As already mentioned, our purpose is to derive the relative uncertainty (or covariance) matrix associated with a fission spectrum, utilizing the relative uncertainties in the mean fission energy $\overline{E}$ and in the fission width w. These relative uncertainties can for instance be derived from the raw experimental data or from the uncertainties in the parameters of a Watt fission spectrum representation of the measured fission spectrum[3].

The Watt fission spectrum is given in ENDF-102 Data Formats and Procedures[4] as

$$f(E \to E') = \sqrt{4/(\pi a^3 b)}e^{-ab/4}e^{-E'/a}\sinh\left(\sqrt{bE'}\right),$$

where the parameters *a* and *b* are dependent on E, the energy of the neutron inducing fission, and E' is the energy of the secondary neutrons. The parameters *a* and *b* are usually determined by fitting the Watt formula to the measured fission spectrum data. Although the Watt formula has no real physical foundation, it represents the experimental data fairly well over most of the energy range of the secondary fission neutrons, it depends on more than a single parameter and is still rather simple to handle.

The relative variation of the fission spectrum in terms of the relative variations of $\overline{E}$ and of w is

$$\delta f / f = \left[(\overline{E}/f)\partial f/\partial\overline{E}\right](\delta\overline{E}/\overline{E}) + \left[(w/f)\partial f/\partial w\right](\delta w/w).$$

However, the sensitivities in the square brackets cannot be directly calculated since f is not represented explicitly as a function of $\overline{E}$ and w. In order to calculate these sensitivities, we utilize the simple Watt fission spectrum representation. The relative variations in $\overline{E}$ and in *w*, as functions of the relative variations in the Watt fission spectrum parameters *a* and *b* are

$$\delta\overline{E}/\overline{E} = (a/\overline{E})(\partial\overline{E}/\partial a)(\delta a/a) + (b/\overline{E})(\partial\overline{E}/\partial b)(\delta b/b)$$
$$\delta w/w = (a/w)(\partial w/\partial a)(\delta a/a) + (b/w)(\partial w/\partial b)(\delta b/b).$$

Inverting the 2x2 matrix relating the relative variations of $\overline{E}$ and w to the relative variations of *a* and *b* we get

$$(\delta a/a) = 1/DET\left[(b/w)(\partial w/\partial b)\delta\overline{E}/\overline{E} - (b/\overline{E})(\partial\overline{E}/\partial b)\delta w/w\right]$$
$$(\delta b/b) = 1/DET\left[-(a/w)(\partial w/\partial a)\delta\overline{E}/\overline{E} + (a/\overline{E})(\partial\overline{E}/\partial a)\delta w/w\right]$$

where DET is the determinant of the transformation matrix.
These expressions of the relative uncertainties in the Watt fission spectrum parameters, *a* and *b*, in terms of the uncertainties in the relative variations in $\overline{E}$ and in *w*, can be substituted into the equation of the relative variation in $f(E \to E')$, in terms of the variations in the parameters *a* and *b*, which is given by:

$$\delta f/f = \left[(a/f)\partial f/\partial a\right](\delta a/a) + \left[(b/f)\partial f/\partial b\right](\delta b/b).$$

The sensitivities in these square brackets can be analytically calculated and averaged over the respective energy groups with the fission spectrum as the weighting function, ultimately leading to the multi group fission spectrum covariance matrix as a function of the relative variances of $\overline{E}$ and w and their covariance:

$$
\begin{aligned}
&\left\langle (\partial f/f)_i (\partial f/f)_j \right\rangle = \\
&1/DET\left\{\overline{(a/f)\partial f/\partial a}^{\,i}(b/w)\partial w/\partial b - \overline{(b/f)\partial f/\partial b}^{\,i}(a/w)\partial w/\partial a\right\}\bullet \\
&1/DET\left\{\overline{(a/f)\partial f/\partial a}^{\,j}(b/w)\partial w/\partial b - \overline{(b/f)\partial f/\partial b}^{\,j}(a/w)\partial w/\partial a\right\}\bullet \\
&\left\langle (\delta\overline{E}/\overline{E})(\delta\overline{E}/\overline{E})\right\rangle + \\
&1/DET\left\{\overline{(b/f)\partial f/\partial b}^{\,i}(a/\overline{E})\partial\overline{E}/\partial a - \overline{(a/f)\partial f/\partial a}^{\,i}(b/\overline{E})\partial\overline{E}/\partial b\right\}\bullet \\
&1/DET\left\{\overline{(b/f)\partial f/\partial b}^{\,j}(a/\overline{E})\partial\overline{E}/\partial a - \overline{(a/f)\partial f/\partial a}^{\,j}(b/\overline{E})\partial\overline{E}/\partial b\right\}\bullet \\
&\left\langle (\delta w/w)(\delta w/w)\right\rangle + \\
&\begin{pmatrix}
1/DET\left\{\overline{(a/f)\partial f/\partial a}^{\,i}(b/w)\partial w/\partial b - \overline{(b/f)\partial f/\partial b}^{\,i}(a/w)\partial w/\partial a\right\}\bullet \\
1/DET\left\{\overline{(b/f)\partial f/\partial b}^{\,j}(a/\overline{E})\partial\overline{E}/\partial a - \overline{(a/f)\partial f/\partial a}^{\,j}(b/\overline{E})\partial\overline{E}/\partial b\right\}+ \\
1/DET\left\{\overline{(a/f)\partial f/\partial a}^{\,j}(b/w)\partial w/\partial b - \overline{(b/f)\partial f/\partial b}^{\,j}(a/w)\partial w/\partial a\right\}\bullet \\
1/DET\left\{\overline{(b/f)\partial f/\partial b}^{\,i}(a/\overline{E})\partial\overline{E}/\partial a - \overline{(a/f)\partial f/\partial a}^{\,i}(b/\overline{E})\partial\overline{E}/\partial b\right\}
\end{pmatrix}\bullet \\
&\left\langle (\delta\overline{E}/\overline{E})(\delta w/w)\right\rangle
\end{aligned}
$$

The relative covariance matrix associated with the fission spectrum was expressed in terms of the relative uncertainties in the mean fission energy $\overline{E}$ and in the fission width w. This expression is valid even when the relative uncertainties in the Watt fission spectrum parameters are not known. The Watt fission spectrum was used only to calculate the sensitivities of the fission spectrum to the uncertainties in the mean fission energy $\overline{E}$ and in the fission width w.

References

1. J.L. Lucius and J.H. Marable, "Influence of Fission Spectra Uncertainties on Calculated Fast Reactor Parameters," *Trans. Am. Nucl. Soc.*, **32**, 731 (1979).

2. R.E. Maerker, J.H. Marable and J.J. Wagschal, "Estimation of the Uncertainties in the ENDF/B-V ^{235}U Fission Spectrum," *Trans. Am. Nucl. Soc.*, **35**, 555 (1980).

3. P. Staples, J.J. Egan, G.H.R. Kegel, A. Mittler, M.L. Woodring, "Prompt fission neutron energy induced by fast neutrons," *Nuclear Physics A*, **591**, 41 (1995).

4. V. McLane, C.L. Dunford, P.F. Rose, *ENDF-102 Data Formats and Procedures for the Evaluated Nuclear Data File ENDF-6,* BNL-NCS-44945 (1997).