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FUEL CYCLE OPTIMIZATION USING THE NONLINEAR REACTIVITY MODEL

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ABSTRACT

Fuel cycle optimization is one of the key subjects of reactor operation. In this study, fuel cycles of Spectral Shift PWR and Pebble Bed HTGR are optimized by using nonlinear reactivity model.

The Spectral Shift concept is based on the adjustments of fuel to moderator ratio as a function of burnup. For n-batch fuel cycle, where n is equal to 3 and 4, the fuel to moderator ratio is determined as a function of burnup to maximize discharge burnup, B_d. Results show that it is possible to increase discharge burnup up to 25 percent compared to typical commercial PWR designs.

Another problem arises in the design of PB-HTGR's fuel pebbles and mixing ratio. The optimization of the composition of fuel pebbles and mixing ratio for direct and n-pass fuel cycles are analyzed to maximize discharge burnup. We compared our results with the current design parameters of HTR-10 and PBMR.

1 INTRODUCTION

One of the main goals of in core fuel management activities is to determine reactivity (enrichment) of a fixed number of fuel assemblies or fuel elements to achieve a prescribed cycle or discharge burnup depending on utilities demand. For this purpose, the linear reactivity model (LRM), based on the assumption that reactivity is a linear function of burnup, was successfully used for Pressurized Water Reactors PWRs [1].

The nonlinear reactivity model is the extension of the LRM and used for CANDU type of reactors. [2] The results of nonlinear reactivity models were in good agreement with the measured data of CANDU reactors [2].

The main goal of this study is to demonstrate the use of nonlinear reactivity model for in core fuel management.

First, the nonlinear reactivity model is used for PWRs to calculate equilibrium cycle discharge burnup. Then, the nonlinear reactivity model is used for Spectral Shift Controlled reactor to determine optimum fuel to moderator ratio and shift points to maximize discharge burnup.

The spectral shift concept was introduced in order to extend the cycle length as a result of more efficient utilization of the fuel, and controlling the parasitic losses as a result of the adjustment of fuel to moderator ratio [3].

Finally, the nonlinear reactivity model is used for high temperature gas cooled reactor to predict the equilibrium cycle discharge burnup.

For this purpose, PBMR and HTR type of gas cooled reactors equilibrium cycle discharge burnup is calculated using the design parameters of PBMR and HTR-10 utilizing KenoV.a and Origen-S modules of scale code system [4], [5], [6], [7].

The outline of the manuscript is as fallows:

In section 2, nonlinear reactivity model is introduced and used for optimization of fuel to moderator ratio to maximize discharge burnup of PWRs utilizing WIMSD5a code.

In section 3, the nonlinear reactivity model is used for spectral shift reactor to determine optimum shift points and fuel to moderator ratio utilizing lattice cell code WIMSD5a.

Section 4 is devoted for the equilibrium cycle discharge burnup calculations of the PBMR and HTR. In these calculations, KenoV.a and Origen-S codes are utilized to determine burnup dependent nonlinear reactivity models.

2 NONLINEAR REACTIVITY MODEL

The reactivity of the fuel elements, even the reactivity of PWR fuel elements or assemblies with burnable poison is non-linear function of burnup. As a result, for most of the reactor applications LRM is not acceptable and need to be improved.

Improvement was done by using high order polynomials to represent reactivity as a function of burnup. Hence, the reactivity is written as;

$$\rho(B) = a_0 + a_1 B + a_2 B^2 + \cdots + a_n B^n \tag{1}$$

where a_0, a_1, ..., a_n are the coefficients of n^{th} degree polynomial and obtained from linear regression using the reactivity values obtained from lattice cell code after saturating fission products as a function of burnup.

As an illustration of the nonlinear reactivity model, equilibrium cycle calculations of 3 and 4-batch PWR with equal power sharing constraint was performed to determine optimum fuel to moderator ratio to maximize discharge burnup.

The discharge burnup B_d is calculated by using the nonlinear reactivity model and equal power sharing constraint from the following equality;

$$\frac{1}{N}\sum_{i=1}^{N} \rho(B_i) - \rho_l = 0 \tag{2}$$

The equation is written for N batch core and assuming equal power sharing for each batch. The first part of the equality denotes the system reactivity and ρ_l denotes leakage reactivity of the system. The discharge burnup is defined as $B_d = B_N$ and, due to equal power sharing constraint $B_i = iB_c$ and cycle burnup is denoted as B_c which is equal to B_N/N.

The B_c value satisfying the Eq. (2) is the cycle burnup and B_N is the discharge burnup of the fuel assemblies used in the N-batch mode.

Assuming equal power sharing end equal batch sizes, as a function of initial enrichment and core leakage reactivity, resulting values of optimum fuel to moderator ratio and discharge burnup are given in Table 1. The maximizing discharge burnup is attained for fuel to moderator ratio is between 0.5 and 0.4.

Table 1. Discharge burnup and optimum fuel to moderator ratio with equal power sharing

		3-batch		4-batch	
Enrichment ($^{w}/_{o}$)	Leakage reactivity	V_F/V_M	B_d (MWd/kg)	V_F/V_M	B_d (MWd/kg)
2.6	0.0	0.50	35.7	0.50	38.4
	0.03	0.46	30.4	0.46	32.5
	0.06	0.42	25.5	0.42	27.2
3.1	0.03	0.44	37.8	0.45	40.4
	0.06	0.40	32.4	0.41	34.7
4.5	0.03	0.43	56.1	0.41	59.6
	0.06	0.39	50.1	0.39	53.3
2.6	0.0	0.5	35.7	0.5	38.4
	0.03	0.5	30.3	0.5	32.4
	0.06	0.5	24.8	0.5	26.5

The results show the dependence of the fuel to moderator ratio to the leakage reactivity of the system. Actually in this interval of fuel to moderator ratio, i.e., between 0.4 and 0.5 the increment in the discharge burnup is about 2-3%. Performing similar calculations to introduce low leakage core design concept, power sharing factors of 1.3, 1 and 0.7 for fresh, once burned and twice burned fuel assemblies are assigned in the case of 3 batch core; thus the equality is given as;

$$\frac{1.3\rho(1.3B_c)+\rho(2.3B_c)+0.7\rho(3B_c)}{3}-\rho_l=0 \tag{3}$$

The calculations are repeated for leakage reactivities of 0.0, 0.3 and 0.6. The results are given in Table 2 indicating that fuel to moderator ratio is in the range of 0.4-0.5 as expected. Moreover, unequal power sharing does not much effect the discharge burnup, even though the importance of each batch is increased, since the increase of $B_1 = 1.3B_c$ and $B_2 = 2.3\ B_c$ decreases the excess reactivity of the first and secand batch.

Table 2. Discharge burnup and optimum fuel to moderator ratio without equal power sharing

Enrichment ($^{w}/_{o}$)	Leakage reactivity	V_F/V_M	B_d (MWd/kg)
2.6	0.0	0.50	36.0
	0.03	0.45	30.4
	0.06	0.42	25.5

3 THE SPECTRAL SHIFT REACTOR

We utilized the nonlinear reactivity model for spectral shift reactors to determine optimum shift points and fuel to moderator ratios utilizing lattice cell code WIMSD5a.

In a spectral shift reactor, varying fuel to moderator ratio the parasitic absorptions are reduced and the utilization of fuel is increased. In this content, we used one, two and three shift for 3-batch and 4-batch PWR. The primary goal is optimization of shift points to maximize discharge burnup.

First, for one shift, we optimize the shift point and fuel to moderator ratio before and after the shift. For two and three shift, optimization calculations are repeated to determine discharge burnup.

As shown in Table 3, increasing the shift numbers increases discharge burnup, the improvement after one shift is about 24 percent and after two shift is about 5 percent more compared to one shift and if we introduce 3 and 4 shifts, the improvement is about 1-2 percent more. Total improvement is about 30 percent.

Table 3. Discharge burnup and optimum fuel to moderator ratios and shift points

			3-batch				4-batch			
1-shift step	Enrichment ($^w/_o$)	Leakage Reactivity	$(V_F/V_M)_1$	$(V_F/V_M)_2$	B_1 (MWd/kg)	B_d (MWd/kg)	$(V_F/V_M)_1$	$(V_F/V_M)_2$	B_1 (MWd/kg)	B_d (MWd/kg)
	2.6	0.0	1.80	0.39	9.8	44.4	1.80	0.41	8.5	46.0
		0.06	0.59	0.31	15.9	27.6	0.49	0.30	20.2	29.2
	3.1	0.06	0.90	0.40	7.4	35.1	1.10	0.39	5.3	37.0
	4.5	0.06	1.31	0.35	9.8	54.0	1.29	0.39	8.2	56.7
2-shift steps	3-batch									
	Enrichment ($^w/_o$)	Leakage reactivity	$(V_F/V_M)_1$	$(V_F/V_M)_2$	$(V_F/V_M)_3$		B_1(MWd/kg)	B_2 (MWd/kg)	B_d (MWd/kg)	
	2.6	0.0	1.80	0.80	0.40		10.0	13.3	46.5	

4 PBMR AND HTR REACTORS

The nonlinear reactivity model gives functional relationship between reactivity and burnup assuming that the reactivity depends on the value of the burnup and is insensitive to burnup history.

The main problem in the modeling is the construction of the functional relationship between reactivity and burnup for a given enrichment, fuel to moderator mixing ratio, packing factor and volumetric heat generation rate.

Once the reactivity as a function of burnup is determined, the nonlinear reactivity model can be used to calculate equilibrium cycle discharge burnup of the reactor using;

$$\int_0^{t_d} F(t)\rho(B(t))dt - \rho_l = 0 \tag{4}$$

where F(t) denotes the fraction of the power generated from fuel having burnup B(t), and it is acting like probability distribution function and it's integral is normalized to one. ρ(B(t)) is the reactivity of the fuel having burnup B(t) and B(t) depends on F(t). The integral term determines the system reactivity and ρ_l is defined as leakage reactivity. B(t) is function of time and $B(t_d)$ is the discharge burnup of the fuel. Assuming equal power sharing, i.e., using the linear relationship between burnup and time $B(t)=B_m t$, in our case time is used as day and B_m average burnup in one day and defined as; B_m=*Thermal Power (MW_{th})/Total-Heavy-Metal-Loaded,* assuming equal power sharing the equation is simplified to;

$$\int_0^{t_d} (\rho(t) - \rho_l)dt = 0 \tag{5}$$

Equilibrium cycle discharge burnup calculations for high temperature gas cooled reactors of PBMR and HTR-10 types are strongly dependent on the initial enrichment of fuel, fuel to moderator mixing ratio, packing factor and volumetric heat generation rate. Using the characteristic design data of PBMR and HTR-10 type of reactors given in Table 4 and

assuming 300 MW thermal power generation, t_d and $B(t_d)$ is calculated and given in Table 5 for PBMR and HTR type of reactors.

Table 4. PBMR and HTR characteristics

	PBMR	**HTR**
Power density (MW/m^3)	4.33	4.33
Number of fuel balls in core	~380000	~223000
Fuel ball diameter	6 cm	6 cm
Outer graphite shell thickness	0.5 cm	0.5 cm
Heavy metal loading per fuel ball	9 g	5 g
Average number of fuel kernels	15000	8335
Fuel enrichment	8%	17%
UO_2 kernel diameter	0.025 cm	0.025 cm
Density of UO_2	10.5 g/cm^3	10.4 g/cm^3
Coating layer materials	PyC/PyC/SiC/PyC	PyC/PyC/SiC/PyC
Coating layer thickness	95/40/35/40 μm	90/40/35/40 μm
Coating layer density	1.05/1.9/3.18/1.9 g/cm^3	1.1/1.9/3.18/1.9 g/cm^3
Fuel-graphite moderator ball ratio	-	0.57/0.43
Density of graphite moderator ball	-	1.73 g/cm^3

Table 5. Discharge burnup using Eq. (5)

	PBMR		HTR	
Leakage reactivity (ρ_l)	t_d (days)	$B(t_d)$ (MWd/kg)	t_d(days)	$B(t_d)$ (MWd/kg)
0.0	834	73.2	445	123.3
0.06	788	69.1	417	115.5
0.1	751	65.9	393	109.0

Since the burnup is not linearly dependent on time, the model requires the use of functional relationship between burnup and power sharing factors. In this study, using constant flux approximation, the power sharing factor is assumed to be linearly decreasing function of burnup.

From linearity between burnup and power peaking factors, f(t) is written as;

$$f(t) = \alpha_1 - \alpha_2 B(t) \tag{6}$$

Using the relationship,

$$B(t) = B_{mean} \int_0^t f(B(t'))dt' \tag{7}$$

one can easily obtain the resulting function that satisfies Eqs. (6) and (7) for power peaking factor and burnup as a function of time.

$$f(t) = \alpha_1 e^{-\alpha_2 B_{mean} t} \tag{8}$$

and

$$B(t)=\frac{\alpha_1}{\alpha_2}\left(1-e^{\alpha_2 B_{mean} t}\right) \tag{9}$$

Since, f(t) is fraction of the power generated at time t, for a given α_2, the integral of f(t) must be equal to t_d. As a result α_1 is determined from normalization condition as;

$$\alpha_1=\frac{t_d \alpha_2 B_m}{1-e^{-\alpha_2 B_m t_d}} \tag{10}$$

Hence, the power sharing factor F(t), acting like probability density function, is defined as $f(t)/t_d$ and used in the reactivity model as follows;

$$F(t)=\frac{\alpha_2 B_m}{1-e^{-\alpha_2 B_m t_d}} e^{-\alpha_2 B_m t} \tag{11}$$

Using Eq. (4) with the nonlinear reactivity model for PBMR and HTR type of fuels, the resulting t_d and $B(t_d)$ values as a function of α_2 and ρ_1 are given in Table 6.

Table 6. Discharge burnup using Eq.(4)

	PBMR		HTR	
Leakage reactivity (ρ_l)	t_d (days)	$B(t_d)$ (MWd/kg)	t_d(days)	$B(t_d)$ (MWd/kg)
0.0	834	73.2	445	123.3
0.06	788	69.1	416	115.3
0.1	751	65.9	392	108.7

As can be seen from the results given in Table 5 and Table 6 using equal power sharing constraint or Eq. (4) do not make any difference in the discharge burnup. Utilizing Eq. (4), the importance of fresh fuels and their burnup rates are increased. The increase of the importance of the fresh fuel increases core reactivity, however increase in the burnup rate decreases the core reactivity. Nonlinear model utilized for in core fuel management depends only on the burnup dependent reactivity model.

4.1 Reactivity Model

Burnup dependent reactivities are calculated by using KenoV.a and Origen-S codes. Due to double heterogeneity, dancoff factor used in the KenoV.a is adjusted until the reactivity values of a unit cell with uniformly distributed coated fuel particles with an assigned dancoff factor equals to the reactivity of unit cell with randomly distributed coated particles.

To perform these calculations 15000 fuel kernels randomly distributed into the fuel region of the pebble. These calculations for PBMR unit cell is repeated to improve reactivity model of the pebble as a function of burnup. The results are given in Figure 1.

Utilizing heterogeneity and randomness of the fuel kernels in the reactivity Model, similar calculations are repeated to calculate discharge burnup of PBMR. Results are given in Table 7.

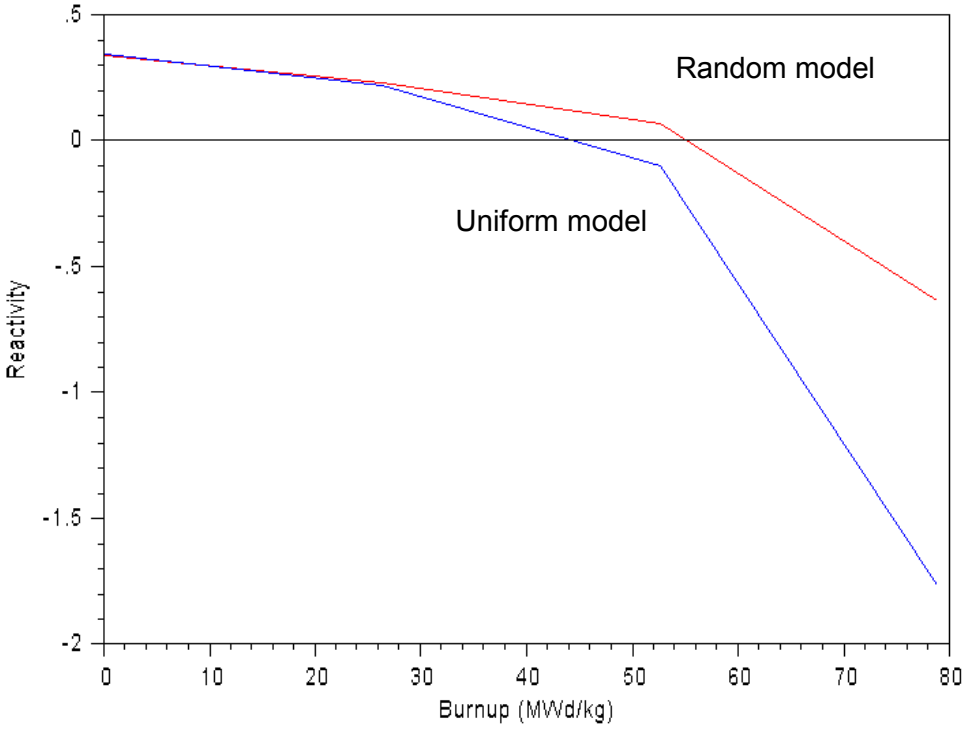

Figure 1: Reactivity model of the pebble as a function of burnup with adjusted dancoff factor

Table 7. Discharge burnup calculations using improved reactivity model for PBMR

Leakage reactivity (ρ_l)	t_d (days)	$B(t_d)$ (MWd/kg)
0.0	1275	111.8
0.06	1118	98.1
0.1	1005	88.1

5 CONCLUSION

In this study, utilization of nonlinear reactivity model for in core fuel management of PWRs and spectral shift reactors are discussed. The results show that it is possible to extend the cycle length up to 30 % utilizing spectral shift concept for an infinite reactor. If the core leakage reactivity of 0.06 is included in the model, the improvement is more than 8 % for one shift.

One of the main contribution of this study is the use of nonlinear reactivity model for PBMR and HTR type of reactors to determine discharge burnup.

For PBMR type of fuels, burnup dependent reactivities calculated including double heterogeneity and randomness of the locations of fuel kernels within the fuel region of the pebble.

Using the improved reactivity model utilizing randomness, the discharge burnup obtained from nonlinear reactivity model is more than 20 MWday/kg greater compared to the discharge burnup calculated by using reactivity model obtained from uniform lattice model with burnup independent dancoff factor.

Our modeling results show that, the dancoff factor used in unit cell calculations depends on burnup and due to double heterogeneity, at high burnup, unit cell calculations differ from calculations performed utilizing randomness of fuel kernel distributions.

REFERENCES

[1] M. J. Driscoll, T. J. Downar, E. E. Pilat, *The Linear Reactivity Model for Nuclear Fuel Management*, ANS, Illinois, 1990

[2] C. H. Millar, "Fuel Management in CANDU Reactors", Ann. Nucl. Energy, Volume 3, 359, 1976

[3] G. Abu-Zaied, "Control of PWRs Using Moderator Displacement and Fuel Assembly Reconstitution", Sc.D. Thesis, Department of Nuclear Engineering, MIT, Dec. 1985

[4] "Review of the Pebble Bed Modular Reactor (PBMR) Plant" Current Status and Future Development of Modular High Temperature Gas Cooled Reactor Technology, p.21, IAEA-TECDOC-1198, 2000.

[5] International Atomic Energy Agency, "Evaluation of High Temperature Gas Cooled Reactor Performance", Interim Draft of First CRP-5 TECDOC, 2001

[6] L. M. Petrie, N. F. Landers "KENOV.a: An Improved Monte Carlo Criticality Program With Supergrouping", NUREG/CR-0200, Revision 6, Volume 2, Section F11, ORNL/NUREG/CSD-2/R6, September 1998.

[7] O. W. Hermann, R. M. Westfall, "ORIGEN-S: Scale System Module to Calculate Fuel Depletion, Actinide Transmutation, Fission Product Buildup and Decay, and Associated Radiation Source Terms", NUREG/CR-0200, Revision 6, Volume 2, Section F7, ORNL/NUREG/CSD-2/V2/R6, September 1998.