OPTIMIZED MODELLING OF THE TORE SUPRA TOKAMAK FOR PLASMA EQUILIBRIUM CALCULATIONS WITH THE PROTEUS CODE

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Abstract

Plasma equilibrium codes solve numerically Grad-Shafranov and Maxwell equations on a two-dimensional finite element meshing of a tokamak, assuming a global axial symmetry to compute the poloidal magnetic field everywhere in a meridian plane of the machine. To provide relevant results, this modelling must take into account accurately the plasma facing components, and the iron circuit, with a consistent magnetic permeability, for machines with iron core.

For a better prediction of optimized pre-magnetization configurations and long discharge analysis with the PROTEUS code [1], the modelling of the Tore Supra tokamak iron circuit and new plasma facing components has been refined: a new meshing has been built, respecting the limiter positions and the inner surfaces of the iron return arms. The magnetic permeability has been adjusted to minimize errors between the code results and magnetic measurements during special calibration shots, at different levels of iron saturation.

With this new modelling, magnetic configurations with field lower than 1 mT over the whole vacuum vessel have been predicted, and plasma slow derive during long pulses in preparation to the gigajoule discharges has been confirmed by the PROTEUS code.

1. Introduction

Magnetic configuration calculations with plasma equilibrium codes are based upon numerical solving of Grad-Shafranov and Maxwell equations, respectively inside and outside the plasma. These finite element codes compute the poloidal flux and magnetic field everywhere on a two-dimensional (2D) modelling of a tokamak meridian plane, assuming a global axial symmetry of the machine. This 2D modelling must be carried out very carefully in the case of machines with iron core, where the return circuits violate the axial symmetry: for example the vertical arms are assumed to be equivalent to a vertical axis cylinder with the same total horizontal cross section, and modelled by a thin vertical rectangle in the meridian plane meshing. The iron magnetic permeability must be also accurately determined, and the plasma facing components must be carefully modelled, to provide consistent results in direct or reverse plasma equilibrium computations, respectively plasma boundary determination with a given current configuration in the poloidal field coils, and calculation of the current configuration for a given plasma boundary: in JET plasma equilibrium calculations with the PROTEUS code, this allows a determination of the X point position with a centimetre accuracy.

Intending to use the PROTEUS code to predict optimized pre-magnetization configurations and to perform accurate analysis of the plasma slow derive during long discharges, a new modelling of the Tore Supra tokamak has been designed: the meshing takes into account the new limiter positions in the CIEL ("Composants Internes Et Limiteurs") configuration, the inner surfaces of the iron circuit horizontal and vertical arms, and it includes all the inter-iron gaps. The magnetic permeability has been adjusted using an optimization method to minimize errors between the code results and magnetic measurements which have been recorded during calibration shots with significant horizontal or vertical field, at low, intermediate and high level of iron saturation.

2. New meshing of the Tore Supra tokamak

The PROTEUS code performs plasma equilibrium calculations using second order finite elements (triangles with 6 nodes and basis functions with both linear and quadratic terms). There existed a preliminary 2D meshing of a Tore Supra meridian plane, which had been designed for another equilibrium code working only with linear basis functions and triangles with 3 nodes. A special processor has been developed to adapt this meshing to the required format for PROTEUS, introducing additional nodes at the middle of the triangle sides and re-numbering all the nodes. As this meshing was not consistent with the CIEL plasma facing component configuration, the first plasma equilibrium calculations had relatively poor accuracy and very often did not converge at all (the PROTEUS code was trying to build a plasma boundary passing close to one of the given limiter points, all of them being distant from the actual limiters of several centimetres).

A new meshing of Tore Supra has been designed to cope with these difficulties. The CIEL project plasma facing components are exactly taken into account: the inner protect rings and the outer limiters (LPA, “Limiteur de Protection d’Antenne”) are modelled by circle arcs of respective radius $\rho_{AG} = 0.84$ m and $\rho_{LPA} = 0.80$ m, centred in the equatorial plane ($z = 0$) at respective radial positions (distances from machine vertical axis) $R_{AG} = 2.42$ m and $R_{LPA} = 2.32$ m. The bottom toroidal pumped limiter (LPT, “Limiteur Pompé Toroïdal”) is modelled by a straight segment (vertical position $Z_{LPT} = -0.72$ m, radial positions $R_{iLPT} = 2.235$ m and $R_{eLPT} = 2.715$ m). The thick actively cooled stainless steel ring of the LPT, which experiences eddy currents greater than 10 kA during plasma start-up, is modelled by a rectangle of sides $h_{LPT} = 0.155$ m (vertically) x $d_{LPT} = 0.174$ m (horizontally), centred vertically at $Z_{cLPT} = -0.823$ m and horizontally at $R_{cLPT} = 2.50$ m.

The Tore Supra tokamak iron magnetic circuit consists in a central yoke made up of 5 co-axial cylinders, plus 6 vertical return arms and 2 x 6 horizontal return arms with a rectangular cross section of poloidal width $d_{pol} = 1.20$ m and toroidal width $d_{tor} = 0.80$ m. The meshing respects accurately the geometry of the sides, which are facing the plasma and the PF (Poloidal Field) coils. The vertical arms (total height $H_{vert} = 4.8048$ m, inner radial position $R_{ivert} = 2.4965$ m) are assumed to be equivalent to a cylinder of height H_{vert}, inner radius R_{ivert}, with the same total cross section: the outer radius R_{evert} is such that $\pi(R_{evert}^2 - R_{ivert}^2) = 6d_{pol}d_{tor}$. The 6 upper and 6 lower horizontal arms are assumed to be equivalent to circular plates of outer radius R_{evert}, with an inner horizontal face (respective vertical positions $+H_{vert}/2$ and $-H_{vert}/2$), and a vertical thickness $d_{hor}(r) = 6d_{pol}d_{tor}/(2\pi r)$ decreasing with the radial position r to keep the total cross section constant.

The inter-iron gaps (4 gaps of thickness $e_1 = 2.7$ mm between the central yoke cylinders, and 2 gaps of thickness $e_2 = 2.1$ mm between the vertical and horizontal arms) have been modelled in the meshing, because most of the plasma equilibriums that will be computed correspond to low iron saturation and high relative permeability, and in these conditions, the inter-iron gaps may have a noticeable influence on the results. The total thickness of the gaps is $e_{tot} = 6e_1 + 2e_2 = 15$ mm and the total length of the magnetic circuit is $L_{tot} = 2(R_{ivert} + H_{vert}) - e_{tot} \approx 19.2$ m, but if the relative permeability μ_r ranges between 1000 and 1500, their reluctances are equivalent: if I_{tot} is the total current in the machine (PF coil currents plus plasma current), B_{iron} the magnetic induction along the iron circuit, and $B_{air} = B_{iron}/\mu_r$ the induction in the air just near the iron, Ampere's law reads:

$$\mu_0 I_{tot} = L_{tot} (B_{iron}/\mu_r) + e_{tot} B_{iron} = (L_{tot} + \mu_r e_{tot})B_{air} \quad (1)$$

From (1), it is clear that neglecting the gaps could leads to a strong over estimation of B_{air} for high values of μ_r, which could propagate towards the plasma boundary and modify it. The final version of the new meshing (see Figure 1), including all these gaps, has 3090 triangles and 6263 nodes.

3. Optimization of the relative permeability function in the PROTEUS code

During each iteration of the plasma equilibrium calculations with the PROTEUS code, the relative permeability is computed in each mesh of the magnetic circuit from the induction modulus B_{iron} in this mesh. This computation is performed in a special sub-program of the code, where the function $\mu_r = f(B_{iron})$ is given as a table of couples (B_k, μ_{rk}). This function can be accurately fitted by a sum of 4 Gaussian functions $\alpha_k \exp[-(B_{iron}-\beta_k)^2/(2\sigma_k^2]$ and an asymptotical function $B_{iron}/(B_{iron}-B_{sat})$ for $B_{iron} > 2.35$ T (the saturation magnetization B_{sat} is typically 2.056 T).

Aiming to optimize this function, calibrations shots have been performed with the Tore Supra tokamak, with high vertical or radial magnetic field (with currents in the range of +/-1 kA in the 95 or 96 turns of the 6 larger PF coils, +/-4 kA in the 176 turns of the upper and lower solenoid, and a ramp between 0 and 25 kA in the 195 turns of the central solenoid). The plasma feedback loop [2] of Tore Supra is based upon real time determination of its position and current from magnetic measurements set on a circle $(C_{m)}$ of typical radius $a_m \approx 0.91$ m, at a radial position $R_m \approx 2.42$ m. During the calibration shots, the magnetic field components $B\theta_{ijm}$ and $B\rho_{ijm}$, respectively tangentially and orthogonally to (C_m), on the sensor number $j = 1,...,n_m$ $(n_m = 51)$, at magnetic configuration number $i = 1,...,n_t$ $(n_t = 10)$ have been stored for low, intermediate and high level of iron saturation (5 configurations with radial field, and 5 with vertical field). These n_t magnetic configurations have been modelled with the PROTEUS code, computing the n_t x n_m field components $B\theta_{ij0}$ and $B\rho_{ij0}$ on the sensor positions. The derivatives $\partial B\theta_{ij}/\partial X_k \approx (B\theta_{ijk} - B\theta_{ij0})/\Delta X_k$ and $\partial B\rho_{ij}/\partial X_k \approx (B\rho_{ijk} - B\rho_{ij0})/\Delta X_k$ of these numerical results with respect to the free parameters X_k defining the function $\mu_r = f(B_{iron})$ have been linearly approximated by computing again the field components $B\theta_{ijk}$, $B\rho_{ijk}$ after slight modifications ΔX_k of the parameters in the PROTEUS code sub-program (these free parameters X_k can be some of the μ_{rk} values or of the α_k, β_k, σ_k parameters).

The optimization consists in finding $(\Delta' X)$, the set of parameter modifications $\Delta' X_k$ ($k = 1,...,N$) which minimizes the cost function $F(\Delta' X)$ defined in (2):

$$F(\Delta' X) = \sum_i \sum_j [B\theta_{ij0} - B\theta_{ijm} + \sum_k (\partial B\theta_{ij}/\partial X_k)\, \Delta' X_k]^2 + \sum_i \sum_j [B\rho_{ij0} - B\rho_{ijm} + \sum_k (\partial B\rho_{ij}/\partial X_k)\, \Delta' X_k]^2 \tag{2}$$

The cost function $F(\Delta' X)$ is minimum when the N derivatives $\partial F/\partial(\Delta' X_k)$ are equal to 0, which gives a linear system $(A)(\Delta' X) = (Y)$ of order N, where the general term of the system matrix (A) and of the right hand side vector (Y) read respectively:

$$A_{kl} = \sum_i \sum_j (\partial B\theta_{ij}/\partial X_k)(\partial B\theta_{ij}/\partial X_l) + \sum_i \sum_j (\partial B\rho_{ij}/\partial X_k)(\partial B\rho_{ij}/\partial X_l) \tag{3}$$

$$Y_k = \sum_i \sum_j (\partial B\theta_{ij}/\partial X_k)(B\theta_{ijm} - B\theta_{ij0}) + \sum_i \sum_j (\partial B\rho_{ij}/\partial X_k)(B\rho_{ijm} - B\rho_{ij0}) \tag{4}$$

A regulation term $\Sigma_k C_k(\Delta' X_k)^2$ has been added to the cost function, which corresponds to add the C_k coefficient to the diagonal term A_{kk} of the system matrix. The most consistent results have been obtained for values of C_k in the same range of magnitude as the other matrix term, leading to the optimized permeability function, which is compared to the previous one in Figure 2. The different improvements, which have been obtained on the model accuracy, are illustrated in Figure 3 (taking into account the inter-iron gaps, the eddy currents in the LPT and optimizing the iron permeability).

4. Optimization of Tore Supra pre-magnetization configurations

After the end of the CIEL project, a new pre-magnetization configuration has been experimentally determined, with noticeable differences between the currents in the upper and lower PF coils (for example, typically 5 kA in the upper solenoid and 4 kA in the lower solenoid), to hold the plasma far from the eddy currents in the LPT conductive ring that could attract it very quickly after start-up.

This magnetic configuration (just before plasma start-up) has been modelled with the PROTEUS code, using the new meshing, the optimized iron relative permeability function and taking into account the eddy currents in the LPT ring. The magnetic field is higher than 5 mT in almost all the vacuum vessel, and analyzing the very first plasma cross sections with interpretation codes such as MAESTRO [3] shows that they are very small. Reducing the current difference between upper and lower solenoid to less than 0.5 kA and adjusting the small currents in the other PF coils to minimize the central solenoid stray field, it has been possible to predict with the PROTEUS code an optimized magnetic configuration with less than 1 mT in the whole vacuum vessel (see Figure 4). Adding a slight current multi-pole in the larger PF coils leads to another optimized configuration with very low field in the upper part of the vacuum vessel, and a slight repulsive radial field between the plasma and the LPT (see Figure 4). If the eddy currents in the LPT ring during next Tore Supra experiments differ from the previous ones, it will be relatively simple to adjust the PF coil currents, taking advantage of the quasi-linearity at high iron saturation, which has been confirmed by the PROTEUS code.

5. Tore Supra long pulse analysis with the PROTEUS code

During the preparation of the gigajoule pulses, a plasma discharge stopped after 3 minutes because of a slight derive of the plasma toward the Lower Hybrid Current Drive Antennae. This derive, which has been confirmed by IR camera recording, was due to a very small thermal deformation of the ring containing the magnetic sensors: the magnetic measurements were slightly modified, as if the plasma was moving inward, and then through the feedback loop, the plasma actually moved outward. Nine plasma equilibriums (every 20 s of the pulse) have been calculated with the PROTEUS code, taking into account the experimental currents in the PF coils. The values of the plasma internal parameters during this pulse (poloidal Beta $\beta_p \approx 0.56$, internal inductance $l_i \approx 1.7$) corresponded to a peaked current density profile, which complicated the modelling procedure: for each plasma equilibrium, a preliminary static calculation is performed with the actual PF coil currents and current density parameters leading to $l_i \approx 0.95$ (no convergence was obtained for higher values of l_i). Then a semi-dynamic calculation is performed with evolution of the current density parameters toward the required values to obtain $\beta_p \approx 0.56$, $l_i \approx 1.7$, and controlled evolution of the currents in the 2 larger PF coils. A last static calculation is carried out with the final values of these parameters, and the different flux maps are compared, showing for example a 2 cm increase of major radius R between 100 s and 120 s (see Figure 5). At 140 s, only a reduced value of l_i could be reached with the semi-dynamic calculation ($l_i \approx 1.67$), and at 160 s and 180 s, the PROTEUS diverges when the plasma limiter point tries to jump from the LPT (where it converged in the initial static calculation) to the LPA during the l_i evolution, while the plasma cross section is growing to keep Shafranov equilibrium field B_{eq}, plasma current I_p and β_p constant (5):

$$B_{eq} = (\mu_0 I_p/4\pi R)[Log(8R/a) + \beta_p + l_i/2 - 3/2] \qquad (5)$$

6. Conclusion

The modelling of the Tore Supra tokamak for plasma equilibrium calculations has been improved. A new meshing has been designed, taking into account accurately all the features (plasma facing components, iron circuit with inter-iron gaps) that may have an influence on the results given by an equilibrium code. The iron magnetic permeability model in PROTEUS has been mathematically adjusted taking into account experimental results of Tore Supra. Optimized magnetic configurations for plasma start-up have been derived with PROTEUS which also confirmed partially the plasma derive during long duration pulses.

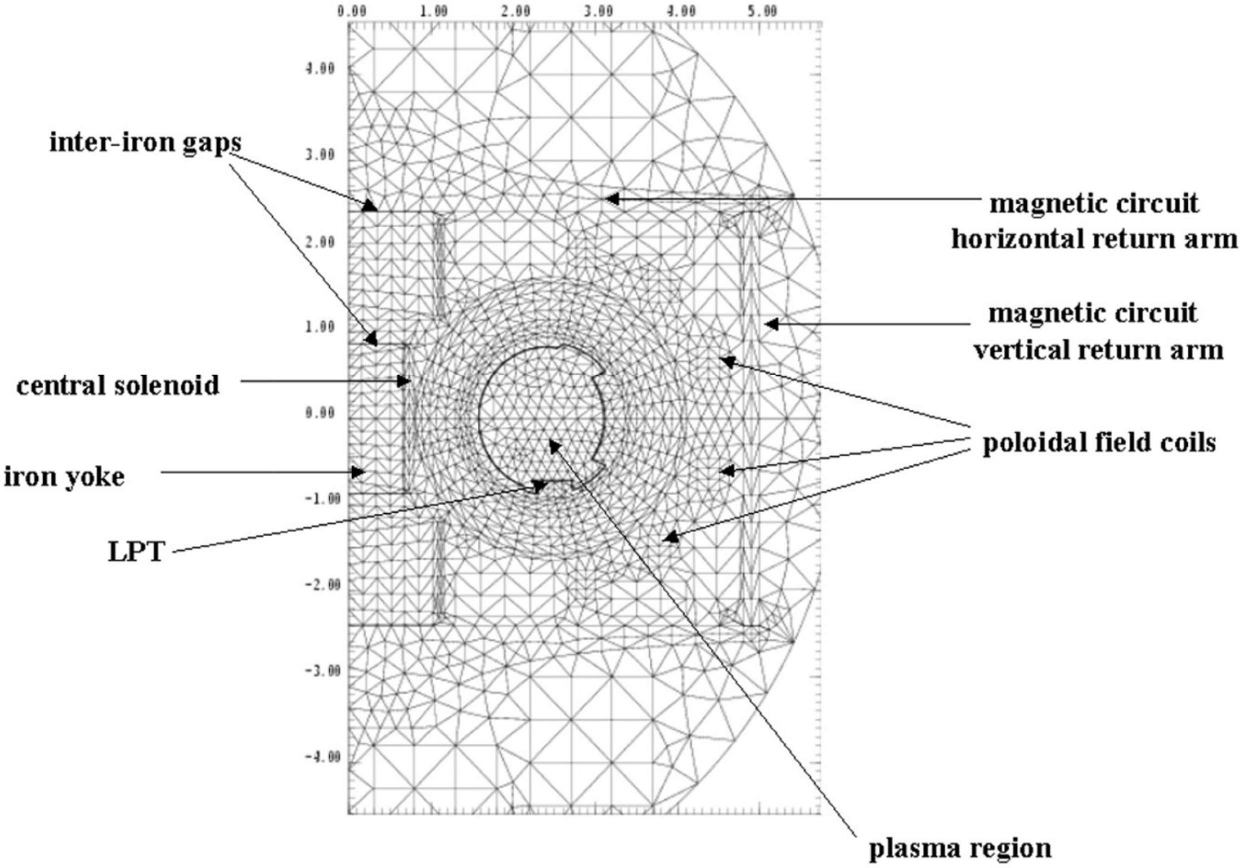


Figure 1: Two-dimensional meshing of the Tore Supra tokamak for plasma equilibrium calculations

P. Hertout Figure 2

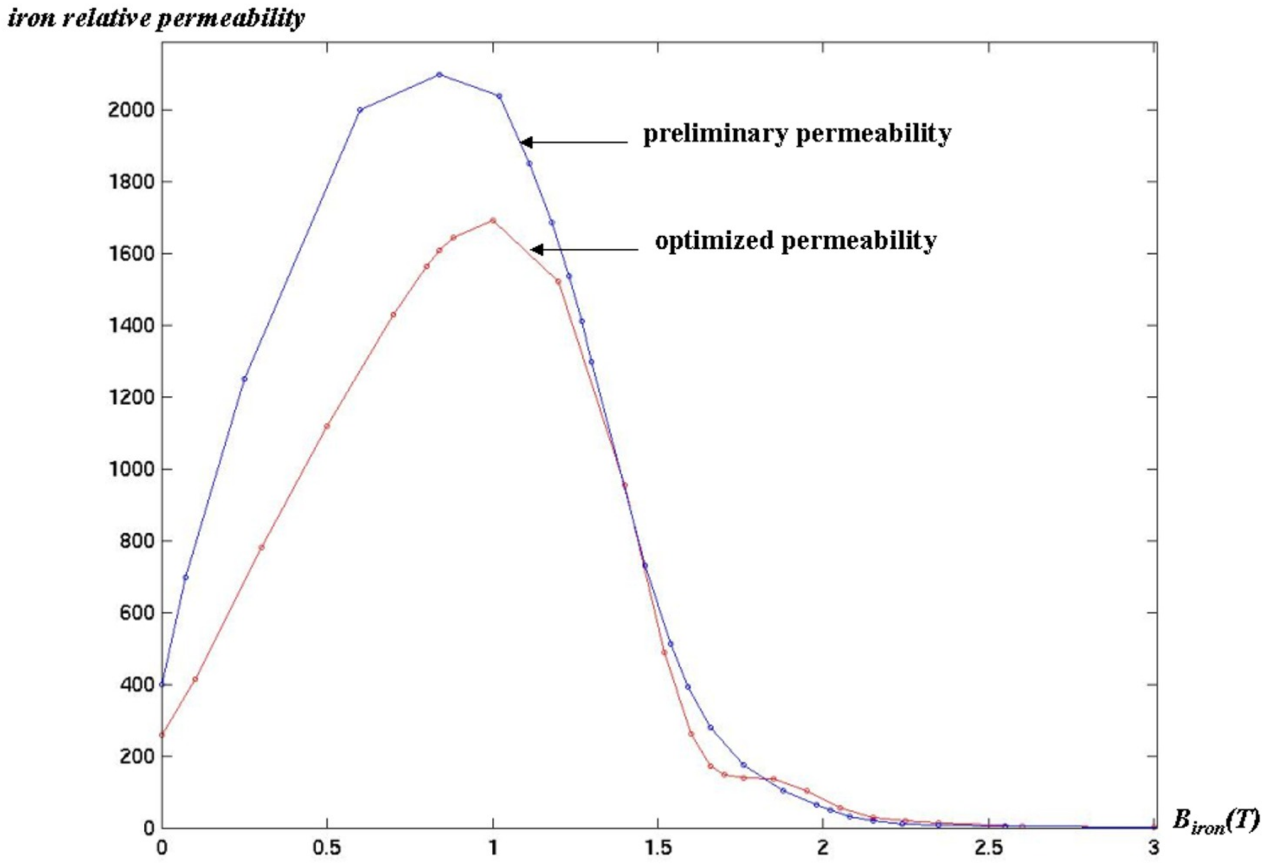


Figure 2: Preliminary and optimized iron relative permeability in the PROTEUS code

P. Hertout Figure 3

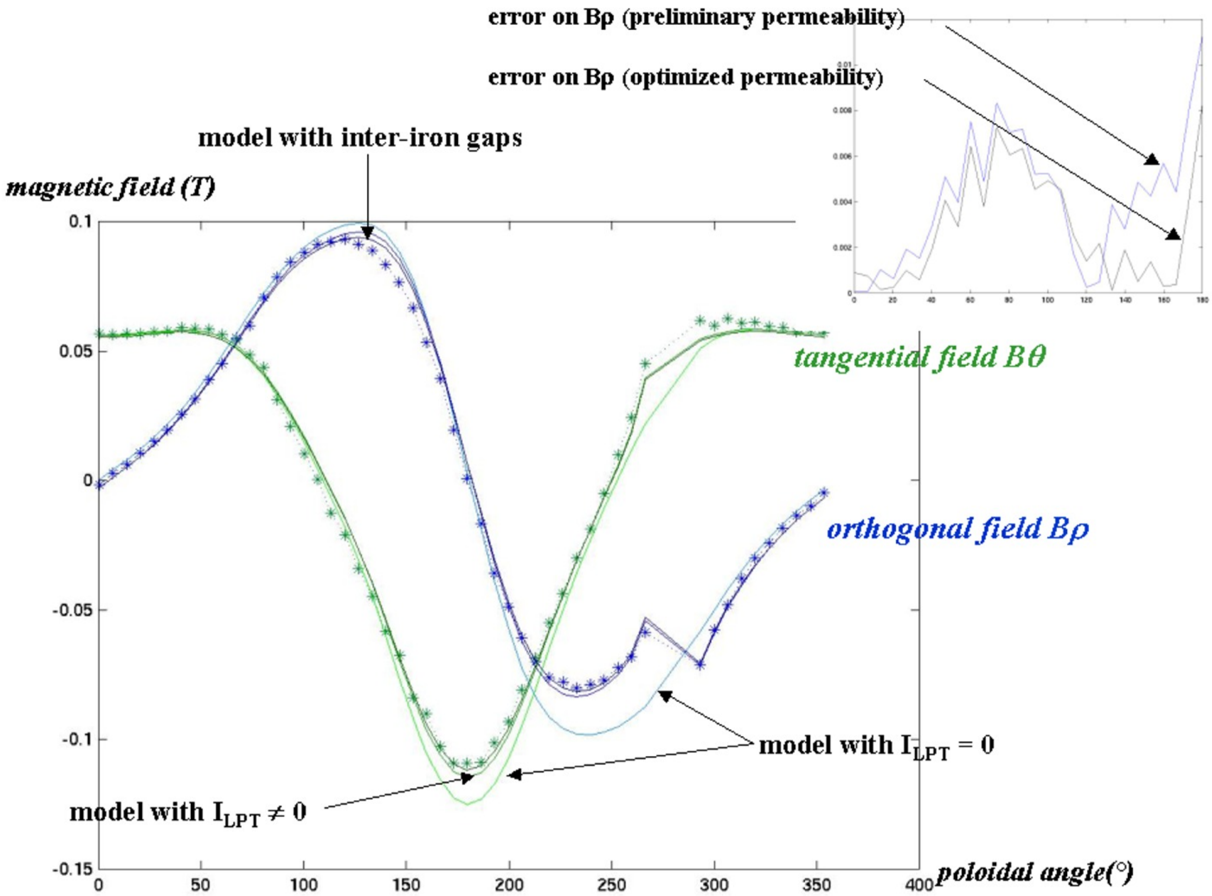
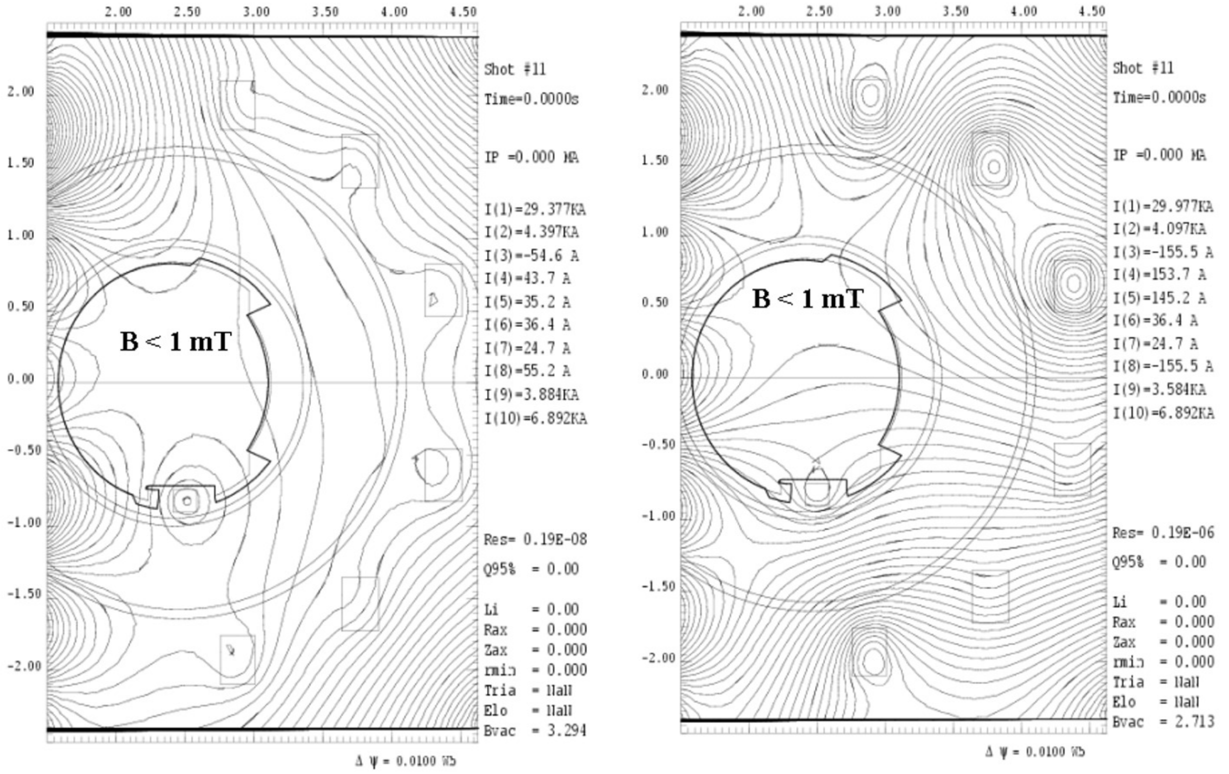


Figure 3: Comparison between measured and predicted magnetic field on Tore Supra sensors during a calibration shot with high vertical field

P. Hertout Figure 4

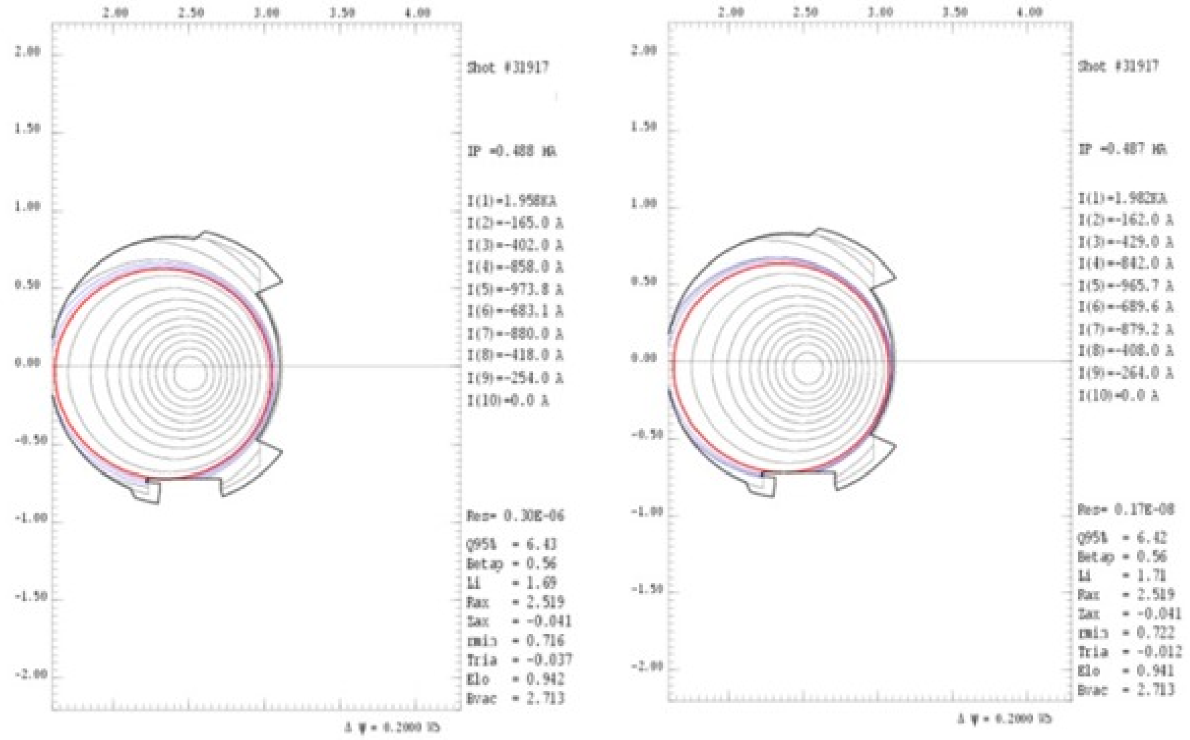


magnetic configuration with very low field in the whole vacuum vessel

magnetic configuration with slight radial field in the lower vacuum vessel

Figure 4: Optimized magnetic configurations for plasma start-up in Tore Supra

P. Hertout Figure 5



Plasma equilibrium at t = 100 s
(predicted major radius R ≈2.33 m)

Plasma equilibrium at t = 120 s
(predicted major radius R ≈2.35 m)

Figure 5: Plasma equilibrium at 100 s and 120 s during a long pulse with slow plasma derive in the Tore Supra tokamak

References

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