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Abstract:

When a photon impinges upon a photo-multiplier tube, an electron is emitted with certain probability and, after several amplification stages, an electron shower is collected at the anode. However, when the first electron is emitted from one of the amplification dynodes or the photo-multiplier is operated under untoward conditions (external magnetic fields...) smaller showers are collected. In this paper, we present a bi-parametric model which describes the response of a photo-multiplier tube over a wide range of circumstances.

Modelo Paramétrico para la Respuesta de un Fotomultiplicador

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Resumen:

Cuando un fotón incide en un tubo fotomultiplicador se emite un electrón con cierta probabilidad y, tras sucesivas etapas de amplificación, una avalancha de electrones llega al ánodo. Sin embargo, si el primer electrón se emite por uno de los dinodos de amplificación o el fotomultiplicador opera bajo condiciones poco favorables (campo magnético externo...) se recogen avalanchas más pequeñas. En este artículo presentamos un modelo biparamétrico que describe la respuesta del fotomultiplicador en un amplio rango de condiciones de operación.

CLASIFICACIÓN DOE Y DESCRIPTORES

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Parametric model for the response of a photo-multiplier tube

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Abstract

Cuando un fotón incide en un tubo fotomultiplicador se emite un electrón con cierta probabilidad y, tras sucesivas etapas de amplificación, una avalancha de electrones llega al ánodo. Sin embargo, si el primer electrón se emite por uno de los dinodos de amplificación o el fotomultiplicador opera bajo condiciones poco favorables (campo magnético externo,...) se recogen avalanchas más pequeñas. En este artículo presentamos un modelo biparamétrico que describe la respuesta del fotomultiplicador en un amplio rango de condiciones de operación.

1 Introduction

Photo-multiplier tubes are widely used devices to detect electromagnetic radiation by means of the external photo-electric effect. A typical photo-multiplier tube consists of an input window, a photo-emissive cathode (photo-cathode), a focusing and a series of amplifying electrodes (dynodes) and an electron collector (anode). Several materials are used for the input window (borosilicate glass, synthetic silica,...) which transmit radiation in different wave-length ranges. Due to absorptions (in particular in the UV range) and external reflexions, the transmittance of the window is never 100 %. Most photo-cathodes are compound semiconductors consisting of alkali metals with a low work function. When the photons strike the photo-cathode and the excited electrons in the valence band get enough energy to overcome the vacuum level barrier, then they are emitted into the vacuum as photo-electrons. The electron trajectory inside the photo-multiplier tube is determined basically by the applied voltage and the geometry of the focusing electrode and the first dynode. Usually, the photo-electron is driven towards the first dynode and originates an electron shower which is amplified in the following dynodes and collected at the anode. However, a fraction of the incoming photons pass through the photo-cathode and originates a smaller electron shower when it strikes the first dynode of the amplification chain.

Different models are described in the literature [1,2,3] to analyse the response of a photo-multiplier tube. However, as explained in [4], they perform poorly if the number of photo-electrons collected at the anode is small or if the signals which correspond to a different number of incoming photons are not well differentiated. In this paper, we present a simple parametric model which describes satisfactorily the response of a photo-multiplier tube under different conditions and show its performance with a Monte Carlo simulation of a Hamamatsu H6568 R7600-00-M16 head-on photo-multiplier [5]. The good behaviour of the model under more extreme conditions (incoming light intensity, noise, magnetic fields,...) has been analised with an experimental set-up and the results are left for a future paper.

2 Model for the electron shower

We shall assume that the response of each dynode to the number of incoming electrons is well described by a Poisson model. Hence, when one electron hits the m^{th} dynode, n secondary electrons will be emitted with probability

$$P_m(n|\mu_m) = e^{-\mu_m} \frac{\mu_m^n}{\Gamma(n+1)} \quad ; \qquad n = 0,1,2,\dots \tag{1}$$

where $\Gamma(z)$ stands for the gamma function and the parameter μ_m accounts for the collection and emission efficiencies of the m^{th} dynode. When the Poisson model is applied to the $M > 1$ dynodes of the photo-multiplier, the probability to get n electrons from the last one (and hence collected at the anode) is given by:

$$\begin{aligned} P_M(n|\mu_M,\dots,\mu_1) &= \sum_{k=1}^{\infty} e^{-\mu_M k} \frac{(\mu_M k)^n}{\Gamma(n+1)} P_{M-1}(k|\mu_{M-1},\dots,\mu_1) + \\ &+ P_{M-1}(0|\mu_{M-1},\dots,\mu_1)\,\delta(n,0) \quad ; \qquad n = 0,1,2,\dots \end{aligned} \tag{2}$$

The main features of the electron shower at the anode are easily obtained from the discrete Laplace Transform:

$$\Phi_M(s|\mu_M,\ldots,\mu_1) = \sum_{n=0}^{\infty} e^{-sn}\, P_M(n|\mu_M,\ldots,\mu_1) = \Phi_{M-1}(\mu_M(1-e^{-s})|\mu_{M-1},\ldots,\mu_1) \quad (3)$$

being $\Phi_1(s|\mu_1) = e^{-\mu_1(1-e^{-s})}$. In particular, if the random quantity ζ describes the number of electrons collected at the anode we have, from recurrent derivatives of the relation (3), that the expected value and the variance are given by:

$$\begin{aligned} E[\zeta] &= \prod_{i=1}^{M} \mu_i = \mu_1{\cdot}\mu_2{\cdots}\mu_M \\ V[\zeta] &= \left(1 + \sum_{k=2}^{M}\prod_{j=k}^{M}\mu_j\right)\prod_{i=1}^{M}\mu_i = \\ &= (\mu_1{\cdot}\mu_2{\cdots}\mu_M)^2\left(\frac{1}{\mu_1} + \frac{1}{\mu_1\mu_2} + \ldots + \frac{1}{\mu_1\mu_2\cdots\mu_M}\right) \end{aligned} \quad (4)$$

Therefore, it is clear that although the expected number of electrons collected at the anode is independent on how the dynode amplification factors are disposed, the variance will be smaller if the larger ones are set at the beginning of the dynode chain.

When the number M of amplification stages (dynodes) is large and/or the amplification factors μ_i are big, the recurrence relation for the Laplace Transform can be well approximated by:

$$\Phi_M(s|\mu_M,\ldots,\mu_1) \simeq \Phi_{M-1}(\mu_M s|\mu_{M-1},\ldots,\mu_1) \quad (5)$$

and, accordingly, the probability densities by:

$$P_{M-1}(x|\mu_{M-1},\ldots,\mu_1) \simeq P_M(\mu_M x|\mu_M,\ldots,\mu_1) \quad (6)$$

being $x > 0$ a continuous variable. Thus, for $n{\neq}0$, the expression (2) can be approximated by the integral equation

$$P_M(x|\theta) \simeq \int_0^{+\infty} \frac{e^{-z}\, z^x}{\Gamma(x+1)}\, P_M(z|\theta)\, dz \quad (7)$$

with the limiting conditions:

$$\lim_{x\to 0} P_M(x|\theta) = 0 \qquad \text{and} \qquad \lim_{x\to +\infty} P_M(x|\theta) = 0 \quad (8)$$

and where θ denotes the corresponding set of parameters. A continuous bi-parametric model which approximates the solution of this equation is given by:

$$P(x|\lambda,s) = \begin{cases} N(\lambda,s)\,\dfrac{\lambda^{x/s}\,(x/s)^{1/2}}{\Gamma(x/s+1)} & ; \quad x > 0 \\ 0 & ; \quad x \leq 0 \end{cases} \quad (9)$$

with real parameters $\lambda \geq 0$ and $s > 0$ (scale factor) and $N(\lambda, s)$ the corresponding normalisation factor. This density describes very well the final charge (or ADC counts) collected at the anode of a standard photo-multiplier tube and performs reasonably well even in the extreme cases of small number of dynodes ($M \simeq 2$) or low amplification factors ($\mu \simeq 1$).

If we denote by $I(\lambda)$ the integral

$$I(\lambda) \;=\; \int_0^{+\infty} \frac{\lambda^u \, u^{1/2}}{\Gamma(u+1)}\, du \tag{10}$$

we have that $N(\lambda, s)^{-1} \;=\; s\, I(\lambda)$ and the corresponding moments of the distribution $P(x|\lambda, s)$ are easily expressed in terms of the derivatives of $I(\lambda)$; that is:

$$E[\zeta^n|\lambda, s] \;=\; \frac{s^n}{I(\lambda)} \sum_{p=1}^{n} S(n,p)\, \lambda^p \, \frac{\partial I^p(\lambda)}{\partial \lambda^p} \qquad ; \qquad n = 1, 2, \ldots \tag{11}$$

being $S(n,p)$ the Stirling numbers of second kind. In particular, the expected value and the variance are given by:

$$\begin{aligned} E[\zeta|\lambda, s] \;&=\; s\,\lambda \, \frac{\partial \ln I(\lambda)}{\partial \lambda} \\ V[\zeta|\lambda, s] \;&=\; s^2 \left(\lambda^2 \, \frac{\partial^2 \ln I(\lambda)}{\partial \lambda^2} + \lambda \, \frac{\partial \ln I(\lambda)}{\partial \lambda} \right) \;=\; s\,\lambda \, \frac{\partial E[\zeta|\lambda, s]}{\partial \lambda} \end{aligned} \tag{12}$$

It is customary to describe the performance of a photo-multiplier tube in terms of the *gain* $G = E[\zeta]$ and the *single photo-electron resolution* $R = \sqrt{V[\zeta]}/E[\zeta]$. From the previous expressions, it is clear that $R = R(\lambda)$ and $G = s\, G(\lambda)$. Thus, λ is the critical parameter of the model and s a scale factor.

In order to account for the small fraction (f_d) of incoming photons converted on the first dynode, we shall modelise the photo-multiplier response to one incoming photon by the density:

$$P_1(x|\theta_1) \;=\; f_d \, P(x|\lambda_d, s_d) \;+\; (1 - f_d)\, P(x|\lambda_p, s_p) \tag{13}$$

where $\theta_1 = \{\lambda_d, s_d, \lambda_p, s_p\}$ stands for the set of free parameters and the sub-indices d and p stand for *dynode* and *photo-cathode* respectively. Last, the electronic noise is always present so this expression has to be convoluted with the density function $P_{ped}(x|\theta_{ped})$ which accounts mainly for the pedestal. and whose explicit form depends on the experimental set-up. Therefore, the probability density function which describes the photo-multiplier response to one incoming photon is

$$P_{1\gamma}(x|\theta_1, \theta_{ped}) \;=\; P_1(x|\theta_1) \star P_{ped}(x|\theta_{ped}) \;=\; \int_0^x P_1(x - z|\theta_1)\, P_{ped}(z|\theta_{ped})\, dz \tag{14}$$

In an analogous manner, we have for n incoming photons that:

$$P_n(x|\theta_1) \;=\; P_1(x|\theta_1) \star P_{n-1}(x|\theta_1) \tag{15}$$

so the response of the photo-multiplier will be described by:

$$P_{n\gamma}(x|\theta_1, \theta_{ped}) \;=\; P_n(x|\theta_1) \star P_{ped}(x|\theta_{ped}) \tag{16}$$

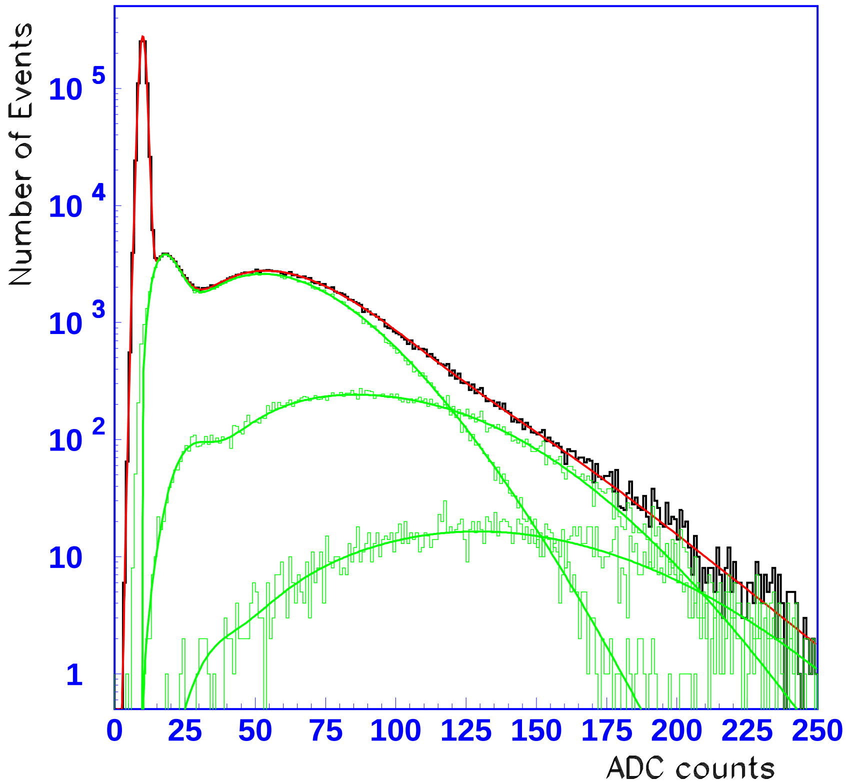

Figure 1.- Fits to the Monte Carlo simulated response of a photo-multiplier tube. The contribution of 1,2 and 3 incoming photons is shown as a histogram for the simulated data and as a solid line for the fitted distribution.

Usually, the noise distribution is very narrow compared with $P_n(x|\theta_1)$ and can be approximated under the previous integrals by a delta distribution simplifying the formulae.

Last, the fraction of incoming photons is well modelised by a Poisson distribution

$$P_{in}(n_\gamma|\mu) \;=\; e^{-\mu}\,\frac{\mu^{n_\gamma}}{\Gamma(n_\gamma+1)} \qquad ; \qquad n_\gamma = 0, 1, 2, \ldots \tag{17}$$

so the final response of the photo-multiplier will be described by:

$$P(x|\theta_1, \theta_{ped}, \mu) \;=\; P_{in}(0|\mu)\,P_{ped}(x|\theta_{ped}) \,+\, \sum_{k=1}^{+\infty} P_{in}(k|\mu)\,P_{k\gamma}(x|\theta_1, \theta_{ped}) \tag{18}$$

3 Performance of the model

In order to study the performance of the model, we have run a Monte Carlo simulation of a Hamamatsu H6568 head-on photo-multiplier with 12 dynodes and a 4x4 anode structure

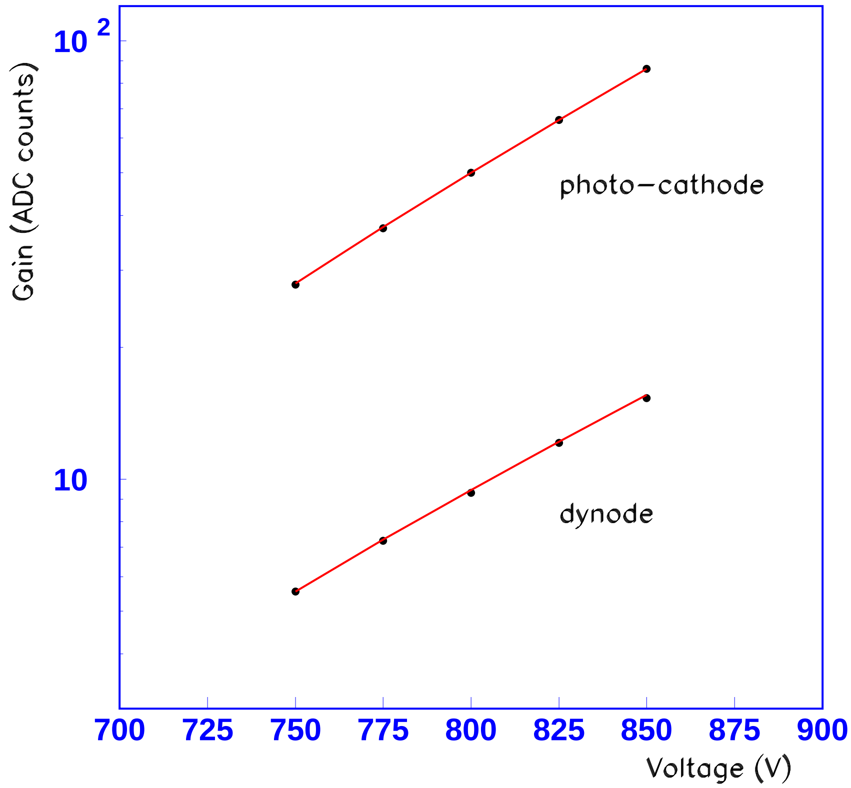

Figure 2.- Gains for the photo-cathode and the first dynode as function of the applied voltage. Dots correspond the values derived from the fit and solid lines to the expected behaviour as explained in [6].

[5]. Because of their efficient response to single-photon signals and their good spatial resolution, these photo-multipliers are the the ones chosen for the Cerenkov Radiation Detector of the AMS experiment [6] and the parameters of the simulation where set close to those of the final detector [7]. In particular, the ratios of the 13 resistances from the photo-cathode to the anode were set to 3x2.4, 8x1.0, 1.2 and 2.4 respectively. Thus, when a voltage V is applied, the amplification factors for each dynode are given by:

$$\mu_i \simeq a \left(\frac{V \cdot R_i}{\sum_j R_j} \right)^k \quad ; \qquad i = 1, \ldots, 12 \; ; \; j = 1, \ldots, 13 \tag{19}$$

The parameters a and k were set to 0.165 and 0.75 respectively so the simulated response resembles the real performance of a photo-multiplier for applied voltages between 750 and 850 V. Finally, the number of electrons collected at the anode was converted into ADC counts as:

$$n_{ADC\ counts} = n_{collected\ electrons} \cdot (e \cdot f_{ADC}) \tag{20}$$

being e the electron charge and $f_{ADC} = 2.1 \cdot 10^{14}$ the number of ADC counts per Coulomb of collected charge. According to the experimental observations [7], the pedestal is properly modelised by:

$$P_{ped}(x|\theta_{ped}) \; = \; \alpha \, N(x|\mu, \sigma_1) \; + \; (1-\alpha) \, N(x|\mu, \sigma_2) \tag{21}$$

where $N(x|\mu, \sigma)$ stands for a Normal density and the parameters set to $\alpha = 0.8$, $\sigma_1 = 1.$, $\sigma_2 = 1.5$ and μ (irrelevant) to 10 ADC counts. Other effects, like like dark current and cross-talk among pixels, have a negligible effect on the response and were not included in the simulation. The fraction of incoming photons was modelised by a Poisson distribution whose parameter μ accounted for the intensity of the LED source and the window set for data collection. Last, the fraction (f_d) of photons converted on the first dynode has been varied between 0 and 1. Since the model did perform quite well in all the cases we have analised, we present only the results obtained for $f_d = 0.2$ and $\mu = 0.25$ for a qualitative understanding.

Parameters	**MC input**	**Fitted values**
Incoming photons: μ	0.25	0.25
Pedestal: μ	10.00	9.99
σ_1	1.00	1.02
σ_2	1.50	1.51
α	0.20	0.21
Photo-cathode: Gain	49.80	50.20
r.m.s.	24.09	23.90
First dynode: Gain	8.96	9.18
r.m.s.	4.90	5.01
Relative factor (f_d)	0.20	0.21

Table 1.- Comparison between the values of the parameters used in the Monte Carlo simulation and those obtained after the fit.

The sample distribution of ADC counts for the Monte Carlo simulation is shown in Fig. 1 together with the corresponding contributions from 1,2,3 and 4 incoming photons (in decreasing order of importance) for an applied voltage of 800 V. The solid line corresponds to the fitted density as given by (18) and the fitted contributions of 1,2 and 3 incoming photons. The comparison between the input values of the parameters in the Monte Carlo simulation and those obtained from the fit are shown in table 1. The gains and r.m.s.'s shown for the photo-cathode and the first dynode are derived from the fitted parameters λ and s using the equations explicited in (12).

Last, the corresponding gains for the photo-cathode and the first dynode are shown in Fig. 2 versus the applied voltage. Dots correspond to the values derived from the fitted parameters (λ and s) and solid lines to what is expected from the characteristics of the simulated photo-multiplier as explained in [5].

4 Conclusions

We have presented a simple bi-parametric model to describe the response of a photo-multiplier tube. Under different conditions of light intensity and applied voltages, the model has performed well in describing the response of the photo-multiplier. As compared to other models described in the literature, the present one performs reasonably well even in the case of low applied voltages or few amplification stages.

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