

## Development of Direct Transmission Probability Method for Criticality Safety Analysis

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We have developed new deterministic Sn-type two dimensional transport calculation method using direct transmission probabilities. The present method has advantage on calculation accuracy for geometries with much void or neutron absorption regions, because paths of neutron are calculated from generation to reaction without approximation. Checking calculations are carried out for a criticality safety problem of fuel assemblies in a spent fuel storage pool with neutron absorption materials, which show difference between the present method and the conventional Sn methods of DOT3.5 on eigenvalues and flux distributions. The other checking calculations for a neutron shielding problem show advantage of the present method comparing with the conventional Sn methods from the viewpoint of ray effects.

**KEYWORDS:** transport calculation, direct transmission probability, two dimension, criticality safety, radiation shielding

### 1. Introduction

Deterministic transport calculations are effective for criticality safety and radiation shielding problems. However conventional two and three dimension Sn codes still have ray effect problems and less reliability in accuracy for geometries with much void or neutron absorption regions. We have developed new deterministic Sn-type of two dimensional transport calculation method using direct transmission probabilities. The present method has advantage on calculation accuracy for geometries with much void or neutron absorption regions, and it has similarity to Exponential Characteristics (EC) method <sup>1)</sup>, Method of Characteristics (MOC) <sup>2,3)</sup> or Method of Transmission Probabilities <sup>4)</sup>. These methods solve transport equation along straight lines for discrete ordinates in basic. While the MOC and other methods mainly deal with characteristics within each calculation cell or mesh, the present method of "Direct Transmission Probability Method (DTPM)" solves transport equation along paths of neutron from generation to reaction for the whole geometry of calculation without approximation. Therefore the DTPM has advantage on calculation accuracy for geometries with much void or neutron absorption regions. Additionally the DTPM uses FDM-like 5-points type differential equation to accelerate transport calculation <sup>5,6)</sup>. In this paper, we describe DTPM, and show the checking calculation results comparing with the conventional Sn method of the DOT3.5 code <sup>7)</sup>, for a criticality safety and a radiation shielding problems.

### 2. Direct Transmission Probability Method

#### 2.1 Transport Equation

A transport calculation considers angular neutron flux ( $\psi$ ). Boundary outgoing angular flux of a mesh ( $\psi_{1,m}$ ) is calculated with boundary income angular neutron flux ( $\psi_{0,m}$ ), transmission probability ( $T_m$ ), neutron source ( $S_m$ ) and escape probability ( $TS_m$ ) of the mesh ( $i,j$ ) for angle ( $m$ ) as:

$$\psi_{1,m} = \psi_{0,m} \cdot T_m + S_m \cdot TS_m \quad (1)$$

$$T_m = \exp(-\Sigma t \cdot l)$$

$$TS_m = \{1 - \exp(-\Sigma t \cdot l)\} / \Sigma t$$

Where,  $\Sigma t$ : Total cross section,  $l$ : Length of neutron path within mesh ( $i,j$ ).

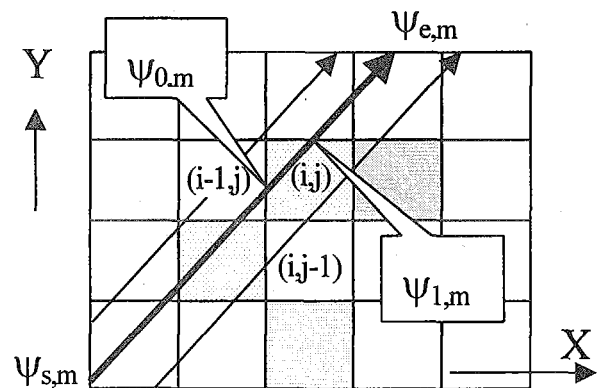


Fig.1 Basic Concept of the Present Transport Method.

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In the DTPM, transport calculation starts from boundary angular neutron flux ( $\psi_{s,m}$ ) and terminates at boundary angular neutron flux of other side ( $\psi_{e,m}$ ) with consideration of transmission probability of each mesh (Fig. 1). This transport calculation process is repeated for all of proper starting points at geometry boundaries and all of proper angles. Because the DTPM uses no approximation in calculation, correct values of neutron transport equation can be expected without approximation, under condition of enough starting points and enough neutron angles.

### 2.2 Acceleration Method

To accelerate calculation, the DTPM uses finite differential method (FDM)-like 5-points type differential equation.

Income neutron current ( $J^+_{i,j}$ ) of mesh ( $i,j$ ) can be calculated with angular neutron flux ( $\psi_{0,m}$ ) by summing for all of the angles and the starting points at geometry boundaries as:

$$J^+_{i,j} = \sum_{(m, \text{ starting points})} \psi_{0,m} \cdot \mu_m \quad (2)$$

Where,  $\mu_m$  is cosine of neutron angle ( $m$ ) to X (or Y) direction.

5-points type differential coefficients for the mesh ( $i,j$ ) can be expressed with neutron current ( $J^+_{i,j}$ ) and total neutron flux ( $\phi_{i-1,j}$ ) of mesh ( $i-1,j$ ) as:

$$C_1(i,j) = J^+_{i,j} / \phi_{i-1,j} \quad (3)$$

and

$$C_0(i,j) = C_1(i+1,j) + C_2(i-1,j) + C_3(i,j+1) + C_4(i,j-1) + (\Sigma_{t,i,j} - \Sigma_{s,i,j}) \cdot \Delta X_{i,j} \cdot \Delta Y_{i,j} \quad (4)$$

Where,  $\Sigma_{s,i,j}$  is scattering cross sections of mesh ( $i,j$ ),  $\Delta X_{i,j}$  and  $\Delta Y_{i,j}$  are widths of mesh ( $i,j$ ) for X and Y direction respectively. Total neutron flux ( $\phi$ ) are calculated with the 5-points differential equation as:

$$C_0 \phi_{i,j} = C_1 \phi_{i-1,j} + C_2 \phi_{i+1,j} + C_3 \phi_{i,j-1} + C_4 \phi_{i,j+1} + S_{i,j} \quad (5)$$

The schemes of the DTPM are:

1. Calculation of angular neutron flux with transmission probabilities for all of the angles and the starting points at geometry boundaries.
2. Calculation of currents at each mesh boundaries with above calculated angular fluxes.
3. Calculation of differential coefficients such as  $C_1$ ,  $C_2$ , etc. of each meshes with the currents values.
4. Calculation of total fluxes with diffusion like differential calculation using the differential coefficients to accelerate flux distribution calculation (inner iterations).
5. Calculation of eigenvalues ( $k_{eff}$ ) by repeating the above 1-4 procedure (outer iterations).

### 3. Checking Calculation for a Criticality Safety Problem

#### 3.1 Calculation Geometry and Cross Sections

A checking calculation is performed with the DTMP in order to examine the applicability to the critical safety problem. Calculation geometry is array of fuel assemblies in spent fuel storage pool with neutron absorption materials (Fig. 2). The fuel assemblies and the moderator are included in the fuel region. Position of (c) is clipped corner of neutron absorbers and is in the region of moderator.

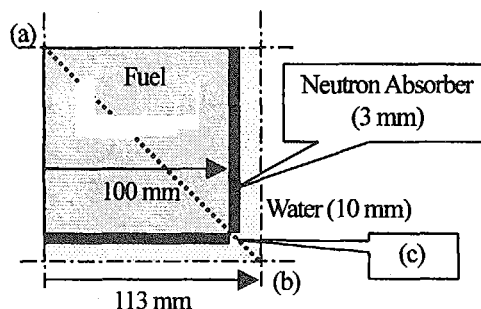


Fig.2 Calculation Geometry of a Criticality Safety Problem.

The cross sections of each region are listed in Table 1. The cross sections were calculated with 1-dimensional transport calculation code ANISN<sup>8)</sup> with the neutron library based on JENDL3.2<sup>9)</sup>.

Table 1 Cross Sections used for a Criticality Safety Problem in Two Energy Groups

Region	$\Sigma_1$	$\Sigma_2$	$\nu\Sigma f_1$	$\nu\Sigma f_2$	$\Sigma s_1$	$\Sigma s_2$	$\Sigma r$
Fuel	0.278	0.893	0.011	0.255	0.253	0.757	0.013
Water	0.730	2.199	0.0	0.0	0.721	2.186	0.007
Neutron Absorber	0.693	12.75	0.0	0.0	0.397	0.886	0.0001

(Unit:  $\text{cm}^{-1}$ )

#### 3.2 Calculated Results

Calculation was carried out by the DTPM and the conventional Sn method of DOT3.5 code. Although reactivity changes in mesh size, the present method and DOT3.5 show some difference (about 1%  $\delta k/k$ ) in reactivity (Fig.3).

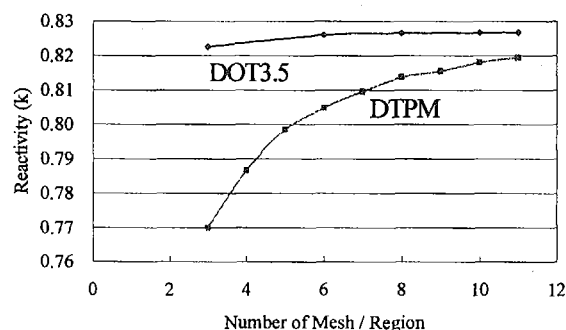


Fig.3 Reactivity's of the DTPM and the DOT3.5.

Comparison of fast neutron flux distributions from the point (a) to the point (b) of Fig. 2 is shown in Fig. 4. The reason of the difference at point (c) is that the fast neutron passed through neutron absorbers is calculated from neutron source directly in the DTPM. When segment node was changed from S<sub>4</sub> to S<sub>16</sub>, reactivity and flux distributions differed little in the DTPM.

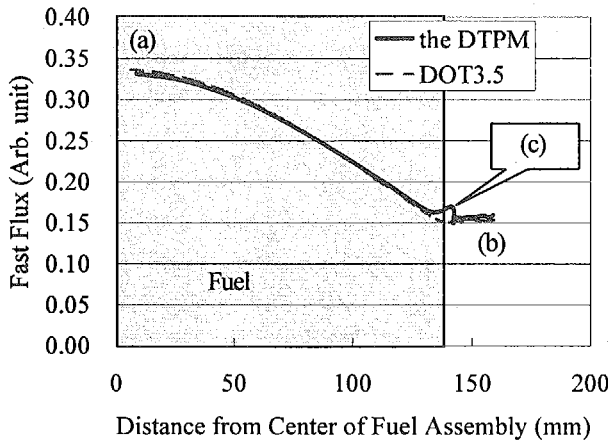


Fig.4 Fast Flux Distributions of the DTPM and the DOT3.5 Code on the Criticality Safety Problem.

4. Checking Calculation for a Radiation Shielding Problem

4.1 Calculation Geometry and Cross Sections

In order to examine the applicability of the DTPM to a radiation shielding problem, a neutron duct streaming problem was set up (Fig. 5), where a neutron source is located in void (no substance) region which is surrounded with the concrete walls. The external boundaries of the problem were on vacuum (no reflection) conditions.

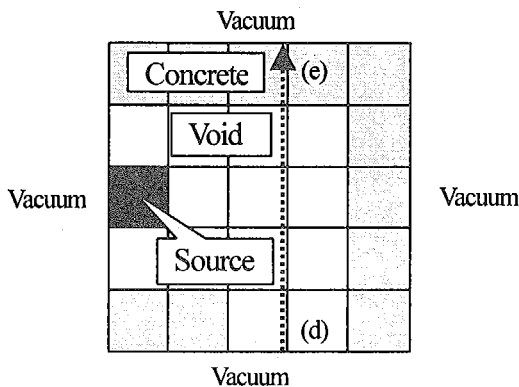


Fig.5 Calculation Geometry of a Neutron Duct Streaming for Radiation Shielding Problem.

Cross section of the concrete regions is listed in Table 2, which was also calculated with the ANISN code and the neutron library based on JENDL3.2. S<sub>12</sub> was used for both the DTMP and the DOT3.5 code in the neutron duct streaming problem.

Table 2 Cross Sections used for a Neutron Duct Streaming Problem in Two Energy Groups (Unit: cm<sup>-1</sup>)

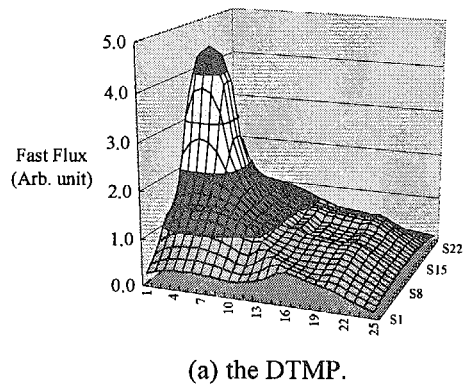
Region	$\Sigma_1$	$\Sigma_2$	$\Sigma_{s1}$	$\Sigma_{s2}$	$\Sigma_r$
Concrete	0.2706	0.3854	0.2678	0.3808	0.0016

4.2 Calculated Results

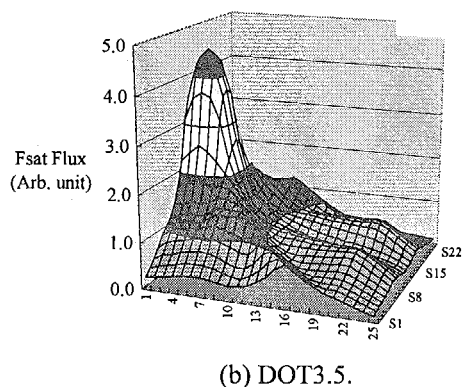
Figures 6 to 8 show fast and thermal neutron flux distributions calculated with the DTPM and the DOT3.5 code. Fig. 6 and 7 show the distributions in the whole geometry. The places of the peak in the Fig. 6 are the position of the neutron source in Fig. 5. Fig. 8 shows neutron flux distributions along the line of (d) to (e) in Fig. 5.

In the result of the DOT3.5 code shown in Fig. 6 (b) and Fig. 8 (a), wave phenomenon is clearer than that in the results of the DTPM shown in Fig.6 (a) and Fig. 8 (a). This is due to the ray effect of conventional Sn methods. Therefore the DTMP has advantage on ray effect in such duct streaming problems.

Differences of thermal neutron flux distribution are small between the DTPM and the DOT3.5 code, since thermal neutrons come from the whole concrete wall (see Fig. 7 and Fig. 8 (b)).



(a) the DTMP.



(b) DOT3.5.

Fig.6 Fast Flux Distributions of the DTPM and the DOT3.5 code in the Duct Streaming Problem.

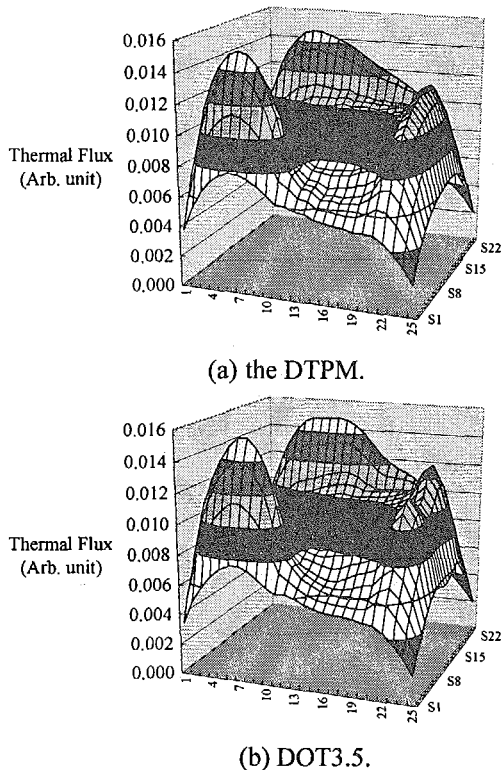


Fig.7 Thermal Flux Distributions of the DTPM and the DOT3.5 code in the Duct Streaming Problem.

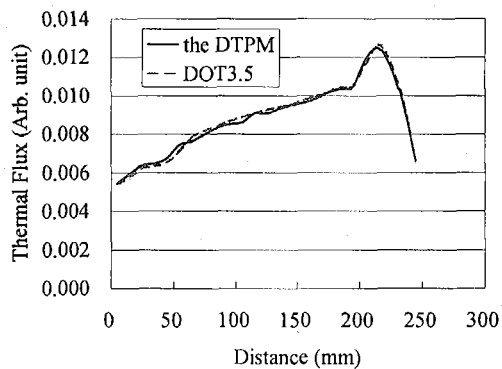
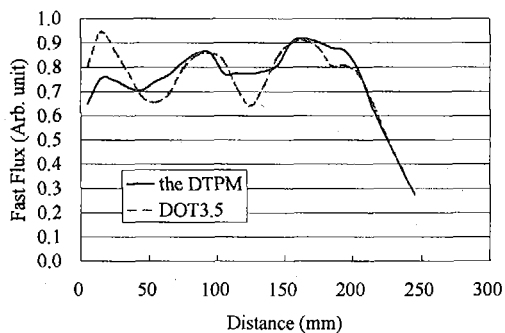


Fig.8 Fast and Thermal Flux Distributions along the line of (d) to (e) in Fig. 5 with the DTPM and DOT3.5 Code.

### 5. Conclusion

We have developed new deterministic Sn-type two dimensional transport calculation method using direct transmission probabilities. The present direct transmission probability method (DTPM) has advantage on accuracy for geometries with much void or neutron absorption regions, because paths of neutron are calculated from generation to reaction without approximation. The DTPM has possibility to solve correctly neutron transport equation, on condition that starting points and neutron angles are enough.

Checking calculations are carried out for array of fuel assemblies in spent fuel storage pool with neutron absorption materials, which show difference between the DTPM and the conventional Sn method of DOT3.5. Difference in eigenvalues is about 1%  $\delta k/k$ , and flux distribution in the crevice portion of neutron absorbers show remarkable difference between the DTPM and the DOT3.5 code.

The other checking calculations for neutron shielding problems show that the DTPM has caused smaller ray effect in duct streaming problems. In this point, the DTPM has advantage in radiation shielding problems.

### Acknowledgements

Mr. M. Sasaki (the Japan Research Institute, Limited) cooperated the present study with the authors.

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