THE SPATIAL DISTRIBUTION OF ^{222}RN "INDOOR" IN SLOVENIA AS A STOCHASTIC MULTIFRACTAL PROCESS

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INTRODUCTION

The problem of exposure to *radon indoor* is the subject of many scientific investigations, since the radiation absorbed by human bodies might be the cause of tumours or leukaemia. In almost all Countries, sampling campaigns of *^{222}Rn* have been started with the purpose of monitoring its distribution and evaluating exposure risk levels. The *radon* is a gas that diffuses in the soil and penetrates into the buildings following multiple pathways. Indeed, many factors influence its migration, both of geological and atmospheric nature, and even anthropic. Therefore, it is evident that the presence of radon in the environment belongs to that class of complex phenomena that has originated the research on fractals (see, e.g., Mandelbrot, 1975, 1983; Falconer, 1988, 1990; Feder, 1989; Edgar, 1990). The possibility of modelling natural phenomena by means of fractals has recently been improved introducing the concept of *multifractals*, studying the fractal properties shown by the different intensity levels of a phenomenon (e.g., analysing the different fractal features of increasing concentrations of radon). In our case, given the complexity of the radon diffusion process, we shall adopt the *stochastic* formalism of *Universal Multifractals*, as introduced by Schertzer and Lovejoy (see, e.g., Schertzer and Lovejoy, 1983, 1987, 1989, 1991, 1992, 1993).

THE DATA

The principal sources of radon are soil, rocks, construction materials, water and air. We shall analyse ^{222}Rn data collected in Slovenia by the "J. Stefan" Institute in Ljubljana (Kobal et al., 1988a) in buildings such as schools and nurseries (see Fig. 1).

The measurement techniques of ^{222}Rn air concentration are based on the detection of α-particles produced by the radioactive decay (Kobal et al., 1988b); measurements have been performed within buildings made of wood or concrete a few hours before their opening and after the rooms have been closed for 1-2 days.

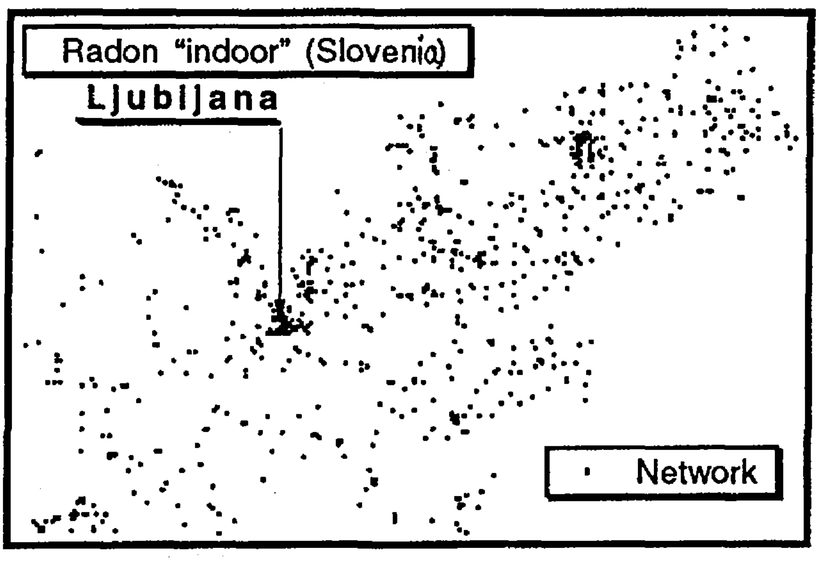

Fig. 1. The sampling network of all the available data.

The available samples are subdivided into three subsets depending on the construction material of the building where the measurements are collected, which is one of the variables influencing the radon emission: the label "T" identifies the *whole* data set, the label "M" identifies the data collected in buildings made of *wood*, and the label "Z" identifies the data collected in buildings made of *concrete*. In Tab. I we show the number of available data, the range of the values (in Bq/m^3), their average and standard deviation; in column "F_D" is reported the *fractal dimension* of the network, in column "h" the *hyperbolic exponent* of the data, and in columns "α" and "C_1" the *universal multifractal parameters* (see later).

Type	#	Min	Max	Av.	S.D.	F_D	h	α	C_1
"T"	1269	7	5750	135.5	362.9	≈1.62	≈1.3	0.58±0.02	0.95±0.01
"Z"	1015	7	5750	147.9	397.8	≈1.60	≈1.3	0.56±0.01	0.98±0.01
"M"	254	10	1306	85.9	150.7	≈1.55	≈1.1	0.38±0.01	1.17±0.01

Table I. Features of the available data (see text).

THE FRACTAL APPROACH

The radon emission features a *strong variability*, both in space and in time. We shall investigate the available data in an original way (Missineo, 1994) using both fractal and multifractal techniques, suitable to analyse phenomena originated by the non-linear interaction of many factors.

The dangerousness of radon may depend upon its *in situ* concentration; however, the presence of "hot spot" may not be detected due to averaging procedures or to insufficient *spatial* and/or *(fractal) dimensional resolution* of the sampling network (see Fig. 1). In our case (Missineo, 1994), the networks are fractal (see Tab. I and Fig. 2), indicating the presence of gaps at all scales: indeed, since the networks are not plane-filling (their fractal dimension is always less than *two*, the dimension of the plane), the sampling of the radon emission is necessarily inhomogenous and not-uniform.

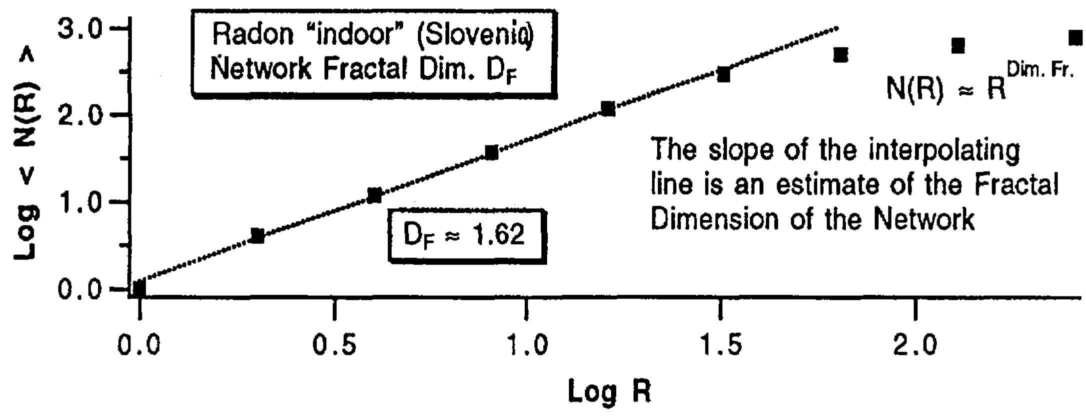

Fig. 2: Estimate of the Fractal dimension D_F of all the available data.

THE HYPERBOLIC ANALYSIS

The presence of "hot spots" can be regarded as the appearing of *anomalous* fluctuations in the dynamics of a phenomenon (the "Joseph Effect" and the "Noah Effect" described by Mandelbrot, 1975, 1983). Indeed, in our case the analysis of Tab. I (Missineo, 1994) shows that the radon concentration spans three orders of magnitude, and the variance is always large, indicating the presence of strong fluctuations. Such events may be interpreted by means of the *Self-Organised Criticality* theory (see, e.g., Schertzer et al., 1995, for a concrete case study) and suggest the use of specific (asymptotic) *hyperbolic* probability distributions (characteristic of fractal processes):

$$\Pr(V \geq v) \propto v^{-h} \; (v >> 1) \tag{1}$$

where h is a positive parameter called *hyperbolic exponent.* Such distributions assign a not-negligible probability even to very large fluctuations (i.e., asymptotically, for v>>1) which, on the contrary, would be impossible adopting other probabilistic models (e.g., Gaussian).

In Fig. 3 we show the (asymptotic) estimate of the hyperbolic exponent h of all the available data; the results are reported in Tab. I. The results show that the radon concentration follows, at least asymptotically, a hyperbolic behaviour, which is a fingerprint of *multifractality.*

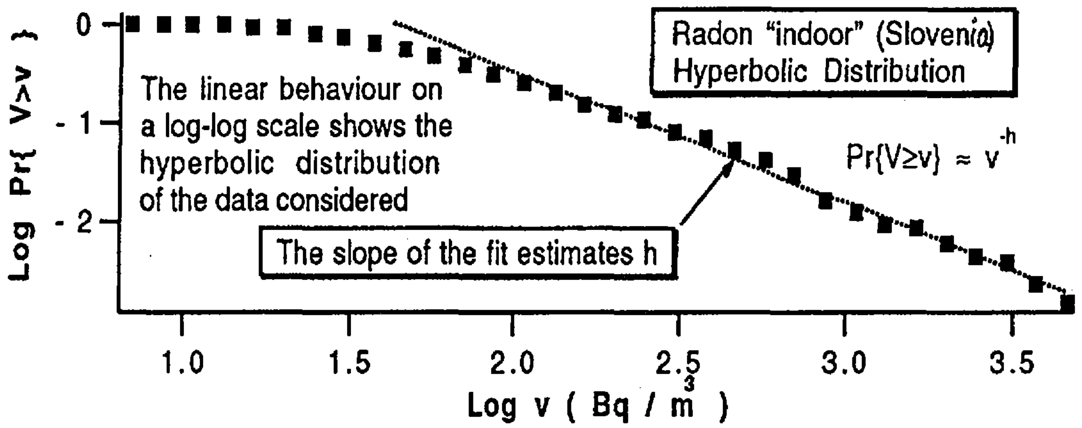

Fig. 3. Estimate of the hyperbolic exponent h of all the available data.

UNIVERSAL MULTIFRACTALS

As already mentioned, *multifractals* are more powerful than monofractals, since they are able to investigate the full spectrum of fractal properties shown by a system. Multifractals are characterised by the presence of a *hierarchy* of structures; they feature a strong space-time *variability* and show the *multiscaling* property, i.e. a precise *scaling* (power law) relationship between the intensity of a given event and its probability of occurrence (see, e.g., Schertzer and Lovejoy, 1987, 1993).

For a (stochastic) multifractal field at resolution λ the following (multiscaling) relation holds (Schertzer and Lovejoy, 1983, 1987):

$$\Pr\{\epsilon_\lambda \geq \lambda^{\gamma}\} \propto \lambda^{-c(\gamma)} \tag{2}$$

where ϵ_λ is the field intensity (in our case, the measured radon concentration), γ is the order of singularity and $c(\gamma)$ is the *codimension function* describing both the "sparseness" of the field intensities and the probability of given events. Thus, eq. (2) relates the intensity of the field ϵ_λ to its probability of occurrence through the function $c(\gamma)$: since $c(\gamma)$ is a convex increasing function, the strongest events are also the rarest.

In the limit of large λ, the corresponding law for the statistical moments $\langle \epsilon^q_\lambda \rangle$ is obtained via a *Laplace transform* and a *saddle point* approximation:

$$\langle \epsilon^q_\lambda \rangle \approx \lambda^{\max_\gamma \{q\gamma - c(\gamma)\}} = \lambda^{K(q)} \tag{3}$$

where $K(q)$ is the *moment scaling function* and q is the order of moment. Formula (3) also shows that $c(\gamma)$ and $K(q)$ form a *Legendre transformation pair* (Parisi and Frisch, 1985).

The statistical description of multifractal processes may be greatly simplified considering *universal multifractals* (Schertzer and Lovejoy, 1987). In this case, $c(\gamma)$ and $K(q)$ have the following parametric expressions:

$$\begin{cases} c(\gamma) = C_1\left(\dfrac{\gamma}{C_1\alpha'} + \dfrac{1}{\alpha}\right)^{\alpha'} & \alpha \neq 1 \\ c(\gamma) = C_1\, e^{\frac{\gamma}{C_1} - 1} & \alpha = 1 \end{cases} \tag{4}$$

$$\begin{cases} K(q) = \dfrac{C_1\alpha}{\alpha}\left(q^{\alpha} - q\right) & \alpha \neq 1 \\ K(q) = C_1\, q \log(q) & \alpha = 1 \end{cases} \tag{5}$$

where $\frac{1}{\alpha} + \frac{1}{\alpha'} = 1$, $0 \le a \le 2$ and $C_1 \ge 0$. The *parameter* α (called the *Lévy index*) represents the *degree of multifractality* of the process (e.g., the case $\alpha = 0$ corresponds to a *monofractal* process) and determines its probability class; the *parameter* C_1 is the *codimension of the average field* and measures its sparseness and inhomogeneity. Based on the value of α (Schertzer and Lovejoy, 1992), a *classification* of universal multifractals has been provided: for $\alpha \ge 1$ we have *unconditionally hard* multifractals (i.e. the process shows *divergence of moments* above a critical order q_D, where q_D is defined by $K(q_D) = D(q_D - 1)$ and D is the dimension of the embedding space); for $\alpha < 1$ we have *conditionally soft/hard* multifractals (i.e. for large enough but finite values of the dimension D all the moments converge).

The estimate of the multifractal parameters α and C_1 for all the available data can be carried out by means of the *Double Trace Moments* (DTM) technique (Lavallée, 1991; Schertzer and Lovejoy, 1993; see also Salvadori (1993) and Salvadori et al. (1994) for concrete case studies and the *Software Note*). Here we only discuss the results obtained (Missineo, 1994), summarised in Tab. I.

The multifractal parametrization of the data "T" and "Z" is almost identical, but differs from the one of the data "M": actually, the subset "M" represents only about 20% of the total data, and its extraction does not seem to have altered the global probabilistic structure. In all cases, the radon distribution shows evident features of multifractality: since $\alpha < 1$, we may classify the radon concentration as a "conditionally soft/hard" multifractal process. The value of $C_1 \approx 1.2$ for the subset "M" compared to the value $C_1 \approx 1$ for the data "T" and "Z" indicates a greater sparsity of the average level of pollution.

It is worth noting that the value of α characterises the "phenomenology" of type "M" and the different one of type "Z": in other words, it provides a probabilistic classification of the radon distribution in buildings having distinct features (wood or concrete), an experimental fact that may have relevant implications for epidemiological purposes and risk assessment.

CONCLUSIONS AND PERSPECTIVES

The present work shows that fractals and (stochastic) universal multifractals represent a proper mathematical framework for characterising the spatial distribution of radon indoor, at small as well as at large scales, without introducing *ad hoc* data smoothing and preserving the original intrinsic features of the phenomenon.

The fractal nature of the networks shows that the sampling is inhomogenous and indicates that some "information" is almost surely lost in the gaps of the network; in particular, it is possible that some "hot spot" regions are not (correctly) sampled, spoiling further operations of risk analysis and assessment. Indeed, the (asymptotic) hyperbolic distribution of the data indicates the probable presence of strong fluctuations (i.e. the "hot spot"). The estimate of the multifractal parameters α and C_1 leads to a classification of the radon concentration as a "conditionally soft/hard" multifractal process; the value of the Lévy index distinguishes between data of type "M" (wood) and "Z" (concrete). Such a parametrization may open new research perspectives, since the possibility of simulating and interpolating arbitrary multifractal processes is the object of recent studies and we foresee useful and interesting results in the near future.

SOFTWARE NOTE

A *free* software for the *multifractal analysis* of 1D and 2D data is available upon request. The program runs both on the standard 68K Macintosh and on the new PowerPC platform. For further information, please contact the author G. Salvadori (phone: +39 - 382 - 507438; fax: 526938; E-mail: salvadori@pavia.infn.it).

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