Ciemat 687
Sp ISSN 614-087-X

ELECTRON - CYCLOTRON RESONANCE HEATING AND CURRENT DRIVE

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CENTRO DE INVESTIGACIONES
ENERGETICAS, MEDIOAMBIENTALES Y TECNOLOGICAS

MADRID, 1992

CLASIFICACION DOE Y DESCRIPTORES:

700101
ECR HEATING
NON-INDUCTIVE CURRENT DRIVE
ELECTRON CYCLOTRON-RESONANCE
WAVE-PROPAGATION
LENEAR ABSORPTION MODELS
RELATIVISTIC PLASMA

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Este trabajo se ha recibido para su impresión en Enero de 1992.

Depósito Legal nº M-2798-1992
ISBN 84-7834-132-3
ISSN 614-087-X
NIPO 238-92-004-9

IMPRIME CIEMAT

ELECTRON-CYCLOTRON RESONANCE HEATING AND CURRENT DRIVE

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ABSTRACT

A brief summary of the theory and experiments on electron-cyclotron heating and current drive is presented. The general relativistic formulation of wave propagation and linear absorption is considered in some detail. The O-mode and the X-mode for normal and oblique propagation are investigated and illustrated by several examples. The experimental verification of the theory in T-10 and D-III-D is briefly discused. Quasilinear evolution of the momentum distribution and related applications as, for instance, non linear wave damping and current drive, are also considered for special cases of wave frequencies, polarization and propagation. In the concluding section we present the general formulation of the wave damping and current drive in the absence of electron trapping for arbitrary values of the wave frequency.

I. GENERALITIES

Electron-cyclotron resonance occurs when the following conditions are fulfilled

$$\omega \approx \frac{\omega_c}{\gamma} \qquad v_{\parallel} \approx \left(\omega - \frac{\omega_c}{\gamma} \right) / k_{\parallel}$$

where ω is the wave frequency, $k_{||}$ is the parallel wave momentum, $\gamma=(1+p^2/m^2c^2)^{1/2}$ is the relativistic factor, $\mathbf{v}= \mathbf{p}/mc$, and $\omega_c=eB_0/mc$. High harmonics resonance can also occur for $\omega=n\omega_c/\gamma$, n=1, 2, 3,... Letting $N_{||}=(c/\omega)k_{||}$, the resonance relation reads

$$\gamma = \left(\frac{n\omega_c}{\omega} \right)^2 + N_{\parallel} \left(\frac{p_{\parallel}}{mc} \right) \tag{1}$$

We only consider the case of $|N_{\parallel}|<1$. From equation (1) we obtain

$$\left(\frac{p_\perp}{mc}\right)^2=\left(\frac{N_{\parallel}p_{\parallel}}{mc}-\frac{n\omega_c}{\omega}\right)^2-1-\left(\frac{p_{\parallel}}{mc}\right)^2$$

Since finite absorption occurs for $p_\perp>0$, we obtain that for the resonant electron $p_-<p_{\parallel}<p_+$, where

$$\left(\frac{p_{\parallel}}{mc}\right)_\pm=\frac{N_{\parallel}\left(\frac{n\omega_c}{\omega}\right)\pm\left(N_{\parallel}^2-1+\frac{n^2\omega_c^2}{\omega^2}\right)^{1/2}}{1-N_{\parallel}^2} \qquad (2)$$

Real values of $p_\pm$ are obtained for $n^2\omega_c^2/\omega^2-1+N_{\parallel}^2\geq 0$; thus nonzero electron-cyclotron absorption can only occur for

$$\left(\frac{n\omega_c}{\omega}\right)^2>1-N_{\parallel}^2,$$

which implies that the absorption spectrum is always asymmetric around $\omega=\omega_c$. This result is independent of the electron temperature and, in fact, is independent of the electron distribution. This relativistic asymmetry disappears in the so called non-relativistic approximation, obtained by letting $\gamma=1$. For $N_{\parallel}=0$ and $n=1$, $\omega_c>\omega$. Furthermore

$$\frac{\omega_c}{\omega}=\gamma>1$$

This relation shows that resonance absorption can occur even when $\omega_c=\omega$ outside the plasma region.

II. WAVE PROPAGATION

An electromagnetic wave can propagate in the vacuum, where $\mathbf{k}=(\omega/c)\mathbf{N}$, $|N|=1$. The effect of a plasma is due to the induced current

$$\vec{J}=\overline{\overline{\sigma}}\vec{E}$$

In first approximation σ_{ij} is obtained from the fluid equation

$$\frac{\partial \vec{J}}{\partial t}=\frac{\omega_p^2}{4\pi}\vec{E}(\vec{r})\exp(-i\omega t)-\frac{e}{mc}\vec{J}\times\vec{B}_0 \tag{3}$$

where $\omega_p^2=4\pi e^2 n_e/m$ and

$$\vec{E}(\vec{r},t)=\vec{E}_0\exp\left(i\int_x dx\frac{\omega}{c}N_\perp+i\frac{\omega}{c}zN_\parallel-i\omega t\right)$$

thus

$$\sigma_{ij}=\frac{\omega}{4\pi i}\left(\varepsilon_{ij}-\delta_{ij}\right)$$

$$\varepsilon_{xx}=\varepsilon_{yy}=1-\frac{\omega_p^2}{\omega^2-\omega_c^2}$$

$$\varepsilon_{xy}=\varepsilon_{yx}=i\frac{\omega_c}{\omega}\frac{\omega_p^2}{\omega^2-\omega_c^2}$$

$$\varepsilon_{zz}=1-\frac{\omega_p^2}{\omega^2}\qquad \varepsilon_{xz}=\varepsilon_{zx}=\varepsilon_{yz}=\varepsilon_{zy}=0$$

and

$$D(\vec{N},\omega)=\varepsilon_{xx}N_\perp^4-N_\perp^2\left[\varepsilon_{xy}^2+\left(\varepsilon_{xx}-N_\parallel^2\right)\left(\varepsilon_{xx}+\varepsilon_{zz}\right)\right]$$
$$+\varepsilon_{zz}\left[\left(\varepsilon_{xx}-N_\parallel^2\right)^2+\varepsilon_{xy}^2\right]=0 \tag{4}$$

The solutions of equation (4) can be written as

$$N_o^2=1-N_\parallel^2-X+\frac{XY}{2}\frac{\Delta-Y\left(1+N_\parallel^2\right)}{1-X-Y^2} \tag{5}$$

$$N_x^2=1-N_\parallel^2-X-\frac{XY}{2}\frac{\Delta+Y\left(1+N_\parallel^2\right)}{1-X-Y^2} \tag{6}$$

where

$$X=\frac{\omega_p^2}{\omega^2}\ ;\ Y=\frac{\omega_c}{\omega}\ ;\ \Delta=\left[\left(1-N_\parallel^2\right)^2Y^2+4N_\parallel^2(1-X)\right]^{1/2}$$

For $N_{||}=0$

$$N_o^2=1-\frac{\omega_p^2}{\omega^2} \quad ; \; E_z\neq 0, \; E_x=E_y=0$$
$$N_x^2=1-\frac{\omega_p^2(\omega^2-\omega_p^2)}{\omega^2-\omega_p^2-\omega_c^2} \quad ; \; E_z=0, \; E_y\neq 0, \; E_x=-\frac{\varepsilon_{xy}}{\varepsilon_{xx}}E_y$$

For $N_{||}\neq 0$, in general $E_x\neq 0$, $E_y\neq 0$ and $E_z\neq 0$ for both modes. Equations (5) and (6) are valid as long as

$$N_o\neq N_x \;, \; N_o\neq 0 \text{ and } N_x\neq 0 \text{ and } N_x\neq\infty$$

For instance, for $N_{||}=0$, $N_o=0$ for $\omega_p=\omega$ (O-mode cut-off) and $N_x=0$ for $\omega_p^2/\omega^2=1\pm\,\omega_c/\omega$ (X-mode cut-off) , and $N_x\to\infty$ for $\omega^2=\omega_p^2+\omega_c^2$ (Upper-hybrid resonance). In the region where $N_o=0$ and $N_x=0$, the wave is reflected back [1]. In the region where $N_x\to\infty$, the X-mode is strongly absorbed (collisional absorption). In general, in an inhomogeneous plasma, a ray launched from outside propagates into the plasma following a trajectory determined by the condition of stationary phase. Letting [2]:

$$\vec{E}(\vec{r},t)\propto\exp\left[i\left(\int\vec{k}\cdot d\vec{r}-\int\omega dt\right)\right]=\exp(i\Psi)$$

i.e. $\mathbf{k}=\nabla\Psi$ and $\omega=-\partial\Psi/\partial t$. Thus the dispersion relation (4) is a first-order partial differential equation for Ψ and can be solved by the method of characteristics and we obtain

$$\vec{\nabla}\omega+\frac{\partial\vec{k}}{\partial t}=0$$

Now $\omega=\Omega(\mathbf{k},\mathbf{r},t)$, thus

$$\left(\frac{\partial\Omega}{\partial\vec{r}}\right)_{\vec{k}}+\left(\frac{\partial\Omega}{\partial\vec{k}}\right)_{\vec{r}}\cdot\frac{\partial\vec{k}}{\partial\vec{r}}+\frac{\partial\vec{k}}{\partial t}=0$$

Letting $\mathbf{v_g}=(\partial\Omega/\partial\mathbf{k})_\mathbf{r}=$ group velocity, we obtain

$$\frac{\partial\vec{k}}{\partial t}+\left(\vec{v}_g\cdot\frac{\partial}{\partial\vec{r}}\right)\vec{k}=\frac{\partial\vec{k}}{\partial t}=-\left(\frac{\partial\Omega}{\partial\vec{r}}\right)_{\vec{k}}$$
$$\frac{d\vec{r}}{dt}=\vec{v}_g=\left(\frac{\partial\Omega}{\partial\vec{k}}\right)_{\vec{r}}$$

Furthermore

$$\left(\frac{\partial D}{\partial \vec{r}}\right)_{\omega}+\left(\frac{\partial \Omega}{\partial \vec{r}}\right)_{\vec{k}}\left(\frac{\partial D}{\partial \omega}\right)_{\vec{r}}=0$$

and

$$\frac{\partial \vec{k}}{\partial t}=\left(\frac{\partial D}{\partial \vec{r}}\right)\left(\frac{\partial D}{\partial \omega}\right)^{-1} \quad ; \quad \frac{d\vec{r}}{dt}=-\left(\frac{\partial \Omega}{\partial \vec{k}}\right)\left(\frac{\partial D}{\partial \omega}\right)^{-1} \tag{7}$$

Equations (7) are the ray-tracing equations.

III. WAVE DAMPING

In addition to to refractive effects, the plasma absorbs the wave energy. In order to compute wave damping **J** must be deuced from Boltzman equation

$$\vec{J}=-en_e\int d\vec{p}\,\vec{v}\,f$$

$$\frac{\partial f}{\partial t}+\vec{v}\cdot\frac{\partial f}{\partial \vec{r}}-\frac{e}{c}\vec{v}\times\vec{B}_0\frac{\partial f}{\partial \vec{p}}=e\left(\vec{E}+\frac{\vec{v}}{c}\times\vec{B}\right)\frac{\partial f_M}{\partial \vec{p}} \tag{8}$$

where f_M is the relativistic maxwellian

$$f_M=\frac{\mu}{4\pi(mc)^3K_2(\mu)}\exp(-\mu\gamma) \quad ; \quad \mu=\frac{mc^2}{T_e}$$

Equation (8) is the linearized Boltzman equation, i.e. we neglect for the moment non-linear effects. The solution of equation (8) yields [3]

$$\varepsilon_{ij}=\delta_{ij}+\frac{\omega_p^2}{\omega^2}\sum_{s=-\infty}^{\infty}\int d\vec{p}\,\frac{p_\perp\Pi_{is}^{*}\Pi_{js}Lf_M}{\gamma+\frac{s\omega_c}{\omega}-\frac{k_\parallel p_\parallel}{m\,\omega}}+\frac{\omega_p^2}{\omega^2}\delta_{i3}\,\delta_{j3}\int d\vec{p}\,\frac{p_\parallel}{\gamma}\left(\frac{\partial}{\partial p_\parallel}-\frac{p_\parallel}{p_\perp}\frac{\partial}{\partial p_\perp}\right)f_M \tag{9}$$

where

$$L=\left(1-\frac{k_\parallel v_\parallel}{\omega}\right)\frac{\partial}{\partial p_\perp}+\frac{k_\parallel v_\perp}{\omega}\frac{\partial}{\partial p_\parallel} \quad ; \quad \Pi_{xs}=\frac{sJ_s(\rho)}{\rho} \quad ; \quad \Pi_{ys}=-iJ'_s(\rho) \quad ; \quad \Pi_{zs}=\frac{p_\parallel}{p_\perp}J_s(\rho)$$

and

$$\rho=-\frac{k_{\perp}p_{\perp}}{m\,\omega_c}$$

The dispersion relation can be written as

$$D=\det(D_{ij})=0 \tag{10}$$

where

$$D_{ij}=\left(N_i N_j-N^2\delta_{ij}\right)+\varepsilon_{ij}$$

Note that equations (8), (9) and (10) are valid for arbitrary distribution f_0.

At the lowest order in $|\rho|$ for n=1, equation (10) becomes [4]

$$\begin{aligned}
&N_{\perp}^{4}\left[\varepsilon_{xx}\left(1-\chi_{zz}\right)+\chi_{xz}\left(\chi_{xz}+2N_{\parallel}\right)+N_{\parallel}^{2}\chi_{zz}\right]-N_{\perp}^{2}\Big\{\varepsilon_{xy}^{2}\left(1-\chi_{zz}\right)+\\
&\quad+\left(\varepsilon_{xx}-N_{\parallel}^{2}\right)\left[\left(\varepsilon_{xx}-N_{\parallel}^{2}\right)\left(1-\chi_{zz}\right)+\varepsilon_{zz}^{0}+\chi_{xz}^{2}+\left(N_{\parallel}+\chi_{xz}\right)^{2}\right]-2i\varepsilon_{xy}\chi_{xz}\left(N_{\parallel}+\chi_{xz}\right)\Big\}+\\
&\quad+\varepsilon_{zz}^{0}\left[\left(\varepsilon_{xx}-N_{\parallel}^{2}\right)^{2}+\varepsilon_{xy}^{2}\right]=0
\end{aligned} \tag{11}$$

And equations (9)

$$\varepsilon_{xx}=1+\frac{\omega_p^2}{4\omega^2}\sum_{+,-}\int d\vec{p}\,p_{\perp}L^{(\pm)}f_0 \quad ; \quad \varepsilon_{xy}=i\frac{\omega_p^2}{4\omega^2}\sum_{+,-}(\pm)\int d\vec{p}\,p_{\perp}L^{(\pm)}f_0 \quad ,$$

$$\varepsilon_{zz}^{0}=1-\frac{\omega_p^2}{\omega^2} \quad , \quad \chi_{xz}=-\left(\frac{\omega_p^2}{4mc\;\omega\,\omega_c}\right)\sum_{+,-}(\pm)\int d\vec{p}\,p_{\parallel}p_{\perp}L^{(\pm)}f_0=\frac{\varepsilon_{xz}}{N_{\perp}} \quad ,$$

$$\chi_{zz}=\left(\frac{\omega_p^2}{4m^2c^2\,\omega_c^2}\right)\sum_{+,-}\int d\vec{p}\,p_{\parallel}^{2}p_{\perp}L^{(\pm)}f_0=\frac{\varepsilon_{zz}-\varepsilon_{zz}^{0}}{N_{\perp}^{2}}$$

where

$$L^{(\pm)}=\left(\gamma\pm\frac{\omega_c}{\omega}-\frac{N_{\parallel}p_{\parallel}}{mc}\right)^{-1}\left[\left(1-\frac{N_{\parallel}v_{\parallel}}{c}\right)\frac{\partial}{\partial p_{\perp}}+\frac{N_{\parallel}v_{\perp}}{c}\frac{\partial}{\partial p_{\parallel}}\right]$$

We split the component of the dielectric tensor into two parts

$\varepsilon_{ij} = \varepsilon'_{ij} + i\varepsilon''_{ij}$

$\varepsilon'_{ij} = \text{Hermitian part} = \frac{1}{2}\left(\varepsilon_{ij} + \varepsilon^*_{ji}\right)$

$\varepsilon''_{ij} = \text{anti-Hermitian part} = \frac{1}{2i}\left(\varepsilon_{ij} - \varepsilon^*_{ji}\right)$

For a maxwellian distribution we obtain (x,y,z=1,2,3)

$$\varepsilon'_{11} = 1 - \frac{\omega_p^2/2\omega^2}{1+\omega_c/\omega} - \frac{\omega_p^2}{2\omega^2}\mu\left(1 + \int_{-\infty}^{\infty} du\, \Lambda(u)\right)$$

$$\varepsilon'_{12} = -i\frac{\omega_p^2/2\omega^2}{1+\omega_c/\omega} + i\frac{\omega_p^2}{2\omega^2}\mu\left(1 + \int_{-\infty}^{\infty} du\, \Lambda(u)\right)$$

$$\varepsilon'_{13} = -\frac{\omega_p^2}{2\omega_c\omega} N_\perp \mu \int_{-\infty}^{\infty} du\, u\,\Lambda(u)$$

$$\varepsilon'_{33} = \varepsilon^0_{33} - \frac{\omega_p^2}{2\omega_c^2} N_\perp^2 \mu\left(\mu^{-1} + \int_{-\infty}^{\infty} du\, u^2 \Lambda(u)\right) \tag{12}$$

where

$$\Lambda(u) = \frac{a}{(2\pi)^{1/2}}\mu^{3/2}\,\mathrm{Ei}(-\mu a)\exp\left(-\mu a - \frac{\mu u^2}{2}\right)$$

$$a = 1 - \frac{\omega_c}{\omega} - N_\parallel u + \frac{u^2}{2}$$

and Ei(x) is the exponential integral.

$$\varepsilon''_{11} = \frac{\pi\omega_p^2}{2\omega^2}\left(\frac{R}{N_\parallel}\right)^{3/2}\frac{\mu A\, S}{\left(1-N_\parallel^2\right)^{1/2}} I_{3/2}(\xi)\exp\left[\mu\left(1 - \frac{\omega_c/\omega}{1-N_\parallel^2}\right)\right]$$

$$\varepsilon''_{13} = \frac{\pi\omega_p^2}{2\omega_c\omega}\frac{R^{5/2}}{N_\parallel^{3/2}}\frac{N_\perp \mu A\, S}{\left(1-N_\parallel^2\right)^{3/2}}\left[N_\parallel\frac{\omega_c}{\omega R} I_{3/2}(\xi) - I_{5/2}(\xi)\right]\exp\left[\mu\left(1 - \frac{\omega_c/\omega}{1-N_\parallel^2}\right)\right]$$

$$\varepsilon''_{33} = \frac{\pi\omega_p^2}{2\omega_c^2}\frac{R^{7/2}}{N_\parallel^{3/2}}\frac{N_\perp^2 \mu A\, S}{\left(1-N_\parallel^2\right)^{5/2}}\left[\left(1 + \frac{N_\parallel^2\omega_c^2}{\omega^2 R^2}\right) I_{3/2}(\xi) - 2 I_{5/2}(\xi)\left(\frac{2}{\xi} + \frac{N_\parallel\omega_c}{\omega R}\right)\right]\exp\left[\mu\left(1 - \frac{\omega_c/\omega}{1-N_\parallel^2}\right)\right] \tag{13}$$

where

$$R=\left(\frac{\omega_c^2}{\omega^2}-1+N_{\parallel}^2\right)^{1/2},$$

$$S=\frac{1}{2}\left[\frac{\left(\frac{\omega_c^2}{\omega^2}-1+N_{\parallel}^2\right)}{|R|^2}+1\right],$$

$$\xi=\frac{N_{\parallel}\mu R}{1-N_{\parallel}^2}$$

and

$$A=\frac{(\pi/2\mu)^{1/2}}{\exp(\mu)K_2(\mu)}\approx 1 \quad , \; N_{\parallel}^2\geq 1-\frac{\omega_c^2}{\omega^2}$$

From equations (12) and (13) we obtain the O-mode damping for $N_{||}=0$

$$N_o^2\approx 1-\frac{\omega_p^2}{\omega^2}$$

$$k_o''=\frac{\omega}{c}N_o\frac{2\pi^{1/2}}{15}\frac{\omega_p^2}{\omega^2}\left[\mu\left(\frac{\omega_c}{\omega}-1\right)\right]^{5/2}\exp\left[-\mu\left(\frac{\omega_c}{\omega}-1\right)\right] \qquad (14)$$

where $k_0''=\mathrm{Im}(k_0)$. In figure 1 we present the qualitative feature of equation (14). For propagation in the equatorial tokamak plane

$$B_0(x)=\frac{B_0(0)}{1+x/R_0}$$

and the energy flux through the resonance $\omega=\omega_c$ is given by

$$S(x)=S_0\exp(-\tau_o)$$

$$\tau_o=\pi^2\left(\frac{T_e}{mc^2}\right)\frac{R_0}{\lambda_0}\left(\frac{\omega_p}{\omega_c}\right)^2\left(1-\frac{\omega_p^2}{\omega^2}\right)^{1/2} \quad ; \; \lambda_0=\frac{2\pi c}{\omega}$$

In general $\tau_o>>1$ and the wave energy is deposited near $\omega\approx\omega_c$.

Electron heating using the O-mode for outside launch has been investigated in several devices and, in particular, in T-10 and DIII-D tokamaks. In the T-10 (a=40 cm, R_0=150 cm) at B_0=30 kG and f=84 GHz up to 1 Mw has been coupled to a moderate density (n_e=3 x 10^{13} cm^{-3}) plasma [5]. A final electron temperature of $T_e(0)$=4 keV is attained from the initial ohmic heating $T_e(0)$=1.2 keV. At lower densities (n_e=1.5 x 10^{13} cm^{-3}) and W≈2Mw, $T_e(0)$=8-9 keV. In the DIII-D [6] (a=52 cm, b/a=1.7, R_0=168 cm) at B_0=21.4 kG and f=60 GHz the O-mode is coupled at n_e=1.2 x 10^{13} cm^{-3}. For W≈1Mw a peak temperature of $T_e(0)$=4 keV is recorded (see figure 2).

For $N_{\|}\neq 0$, the O-mode damping is obtained numerically [4]. In figure 3(a,b), we present $k''_{\perp o}$ and

$$\eta(x)=1-\exp\left(-2\int_0^x dx' k''_{\perp o}(x')\right)$$

for $T_e(x)=T_e(0)(1-x^2/a^2)^2$, $n_e(x)=n_e(0)(1-x^2/a^2)$, a=40 cm, R_o=130 cm, $T_e(0)$=4 keV, $n_e(0)$=0.5 x 10^{14} cm^{-3}, $B_o(0)$=40 kG, $\omega=\omega_c$(x=-10 cm) at an angle with respect to the magnetic field $\theta=65^o$. The dashed curves are the obtained from the relativistic formulation given by eqs. (12) and (13). For comparison we also present the corresponding results in the non-relativistic approximation (full curves). It appears that the two results are significantly different in the region of absorption (x>-10 cm). For instance, at x≈0, the non-relativistic formulation gives near 50 % absorption to be compared with the correct result zero per cent absorption.

It is straightforward to include high Larmor radius effects and high harmonics. We obtain [7]

$$\begin{aligned}D \equiv\ & \varepsilon_{11}N_\perp^4+2N_\|\varepsilon_{13}N_\perp^3+\\ &+N_\perp^2\left(\varepsilon_{13}^2+N_\|^2\varepsilon_{11}-\varepsilon_{11}\varepsilon_{22}-\varepsilon_{12}^2+N_\|^2\varepsilon_{33}-\varepsilon_{11}\varepsilon_{33}\right)+\\ &+2N_\|N_\perp\left(N_\|^2\varepsilon_{13}-\varepsilon_{13}\varepsilon_{22}+\varepsilon_{12}\varepsilon_{23}\right)+\\ &+\varepsilon_{11}\varepsilon_{23}^2-\varepsilon_{22}\varepsilon_{13}^2+2\varepsilon_{12}\varepsilon_{13}\varepsilon_{23}+N_\|^2\left(\varepsilon_{13}^2-\varepsilon_{23}^2\right)+\\ &+N_\|^2\varepsilon_{33}\left(N_\|^2-\varepsilon_{11}-\varepsilon_{22}\right)+\varepsilon_{33}\left(\varepsilon_{12}^2+\varepsilon_{11}\varepsilon_{22}\right)=0\end{aligned} \qquad (15)$$

Now ε'_{ij} is still approximately given by eqs. (12). For ε''_{ij} we obtain

$$\varepsilon''_{11}=a_{11}+N_\perp^2\left(b_{11}+c_{11}\right)+N_\perp^4\left(d_{11}+f_{11}+g_{11}\right),$$

$$\varepsilon''_{12} = -i\left[a_{11} + N_\perp^2\left(2b_{11} + c_{11}\right) + N_\perp^4\left(3d_{11} + 3f_{11}/2 + g_{11}\right)\right],$$

$$\varepsilon''_{22} = a_{11} + N_\perp^2\left(3b_{11} + c_{11}\right) + N_\perp^4\left(37d_{11}/5 + 2f_{11} + g_{11}\right),$$

$$\varepsilon''_{13} = N_\perp\left[a_{13} + N_\perp^2\left(b_{13} + c_{13}\right) + N_\perp^4\left(d_{13} + f_{13} + g_{13}\right)\right],$$

$$\varepsilon''_{23} = iN_\perp\left[a_{13} + N_\perp^2\left(2b_{13} + c_{13}\right) + N_\perp^4\left(d_{13} + 3f_{13}/2 + g_{13}\right)\right],$$

$$\varepsilon''_{33} = N_\perp^2\left[a_{33} + N_\perp^2\left(b_{33} + c_{33}\right) + N_\perp^4\left(d_{33} + f_{33} + g_{33}\right)\right] \qquad (16)$$

where

$$a_{11} = aRS\sum_{s=\pm1}\left(1 + \frac{s}{\xi}\right)F\left(p_s\right) \quad , \quad a_{13} = aRS\frac{\omega}{\omega_c}\sum_{s=\pm1}\left(p_s + \frac{s\left(p_s + 2sq\right)}{\xi} + \frac{3sq}{\xi^2}\right)F\left(p_s\right) \quad ,$$

$$a_{33} = aRS\left(\frac{\omega}{\omega_c}\right)^2\sum_{s=\pm1}\left(p_s^2 + \frac{sp_s\left(p_s + 4sq\right)}{\xi} + \frac{6sq\left(p_s + sq\right)}{\xi^2} + \frac{12sq^2}{\xi^3}\right)F\left(p_s\right) \quad ,$$

$$b_{11} = aRS\left(\frac{R}{\mu N_{\parallel}}\right)\left(\frac{\omega}{\omega_c}\right)^2\sum_{s=\pm1}\left(1 + \frac{3s}{\xi} + \frac{3}{\xi^2}\right)F\left(p_s\right) \quad ,$$

$$b_{13} = aRS\left(\frac{R}{\mu N_{\parallel}}\right)\left(\frac{\omega}{\omega_c}\right)^3\sum_{s=\pm1}s\left(p_s + \frac{3s\left(p_s + sq\right)}{\xi} + \frac{3\left(p_s + 4sq\right)}{\xi^2} + \frac{15q}{\xi^3}\right)F\left(p_s\right) \quad ,$$

$$b_{33} = aRS\left(\frac{R}{\mu N_{\parallel}}\right)\left(\frac{\omega}{\omega_c}\right)^4 \times$$

$$\times\sum_{s=\pm1}s\left(p_s^2 + \frac{3sp_s\left(p_s + 2sq\right)}{\xi} + \frac{3\left(p_s^2 + 8sqp_s + 4q^2\right)}{\xi^2} + \frac{30q\left(p_s + 2sq\right)}{\xi^3} + \frac{90q^2}{\xi^4}\right)F\left(p_s\right) \quad ,$$

$$d_{11} = aRS\left(\frac{R}{\mu N_{\parallel}}\right)^2\frac{5}{8}\left(\frac{\omega}{\omega_c}\right)^4\sum_{s=\pm1}\left(1 + \frac{6s}{\xi} + \frac{15}{\xi^2} + \frac{15s}{\xi^3}\right)F\left(p_s\right) \quad ,$$

$$d_{13} = aRS\left(\frac{R}{\mu N_{\parallel}}\right)^2\frac{5}{8}\left(\frac{\omega}{\omega_c}\right)^5\sum_{s=\pm1}\left[p_s\left(1 + \frac{6s}{\xi} + \frac{15}{\xi^2} + \frac{15s}{\xi^3}\right) + 2\frac{q}{\xi}\left(2 + \frac{15s}{\xi} + \frac{45}{\xi^2} + \frac{105s}{\xi^3}\right)\right]F\left(p_s\right) \quad ,$$

$$d_{33} = aRS\left(\frac{R}{\mu N_{\parallel}}\right)^2 \frac{5}{8}\left(\frac{\omega}{\omega_c}\right)^6 \sum_{s=\pm 1}\left[p_s^2\left(1+\frac{6s}{\xi}+\frac{15}{\xi^2}+\frac{15s}{\xi^3}\right)+4\frac{p_s q}{\xi}\left(2+\frac{15s}{\xi}+\frac{45}{\xi^2}+\frac{105s}{\xi^3}\right)+\right.$$

$$\left.+10\left(\frac{q}{\xi}\right)^2\left(2+\frac{18s}{\xi}+\frac{63}{\xi^2}+\frac{84s}{\xi^3}\right)\right]F(p_s) \ ,$$

$$c_{ij}=-4b_{ij}(\omega_c \Rightarrow 2\omega_c)\ , \ f_{ij}=-\frac{128}{5}d_{ij}(\omega_c \Rightarrow 2\omega_c)\ , \ g_{ij}=\frac{243}{5}d_{ij}(\omega_c \Rightarrow 3\omega_c) \qquad (17)$$

where the symbol $b_{ij}(\omega_c \Rightarrow 2\omega_c)$ means that ω_c must be replaced by $2\omega_c$ in b_{ij}. The various quantities in these expresions are

$$a=\pi\frac{(\omega_p/\omega)^2}{4N_{\parallel}^2 K_2(\mu)\exp(\mu)} \ , \ \mu=\frac{mc^2}{T_e}$$

$K_2(\mu)$ is the McDonald function, $q=R/(1-N_{\parallel}^2)$, $\zeta=\mu N_{\parallel}R/(1-N_{\parallel}^2)$, $R=[(\omega_c/\omega)^2-1+N_{\parallel}^2]^{1/2}$, S is a step function, i. e., S=1 for $R^2\geq 0$ and S=0 for $R^2<0$, $F(p_s)=\exp[-\mu(\gamma_s-1)]$, $\gamma_s=(1+p_s^2)^{1/2}$, and p_s is a solution of

$$(1+p_s)^{1/2}-\omega_c/\omega=N_{\parallel}p_s \Leftrightarrow p_s=\left[N_{\parallel}(\omega_c/\omega)+sR\right]/(1-N_{\parallel}^2)$$

Note that eqs. (17) at the lowest order in $N_\perp$ coincide with eq (13). The damping of the X-mode for $N_{\parallel}=\cos\theta$ ($\theta=60°$), and propagation in the tokamak equatorial plane from the low magnetic field side is shown [8] in fig. 4 for the following parameters: a=120 cm, R_0=500 cm, B(0)=55 kG, f=115 GHz, $T_e(0)$=5 keV, $n_e(0)=10^{14}$ cm^{-3} and $n_e(x)=n_e(0)(1-x^2/a^2)$, $T_e(x)=T_e(0)(1-x^2/a^2)^{3/2}$.

Note that $f_c(0)$=154 GHz and therefore nowhere within the plasma region $\omega=\omega_c$.

Wave damping for n≥2 can also be investigated using eqs. (15), (16) and (17). For $N_{\parallel}=0$, for non-overlapping harmonics, we obtain for the absorption coefficient $\alpha=2k''$

$$\alpha_{Xn} \cong (\omega/cN_X)(1-i\varepsilon_{12}/\varepsilon_{11})^2(2\pi)^{1/2}(\omega_p/\omega)^2(N_X/Y_n)^{2(n-1)}\mu^{2-n}\frac{2^{n+1/2}n^{2n}}{(2n+1)!}Z^{n+1/2}\exp(-Z)$$

$$\alpha_{On} \cong (\omega/cN_O)(2\pi)^{1/2}(\omega_p/\omega)^2(N_O/Y_n)^{2n}\mu^{1-n}\frac{2^{n+5/2}(n+1)n^{2n}}{(2n+3)!}Z^{n+3/2}\exp(-Z)$$

(18)

where

$$Y_n = \frac{n\omega_c}{\omega}, n = 1,2,3,...; \ \mu = \frac{mc^2}{T_e} \ ; \ Z = \mu(Y_N - 1),$$

ω_p and ω_c are the plasma and electron-cyclotron frequencies respectively, c is the speed of light, m the rest mass,

$$\varepsilon_{11} = 1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2}, \ \varepsilon_{12} = i\left(\frac{\omega_c}{\omega}\right)\frac{\omega_p^2}{\omega^2 - \omega_c^2},$$

$$\varepsilon_{33} = 1 - \frac{\omega_p^2}{\omega^2}$$

and $N_{O(X)}$ is the cold plasma refractive index for the ordinary (extraordinary) mode. The absorption coefficients are maxima at Z=n+1/2 and Z=n+3/2 for the X and O modes, respectively.

For propagation in the tokamak equatorial plane, it is easy to obtain from eqs. (18) the expresion of $\tau_{X,O}$ for $n\geq 2$. Letting

$$\alpha_{Xn} = \alpha_{Xn}^0 Z^{1/2+n}\exp(-Z) \ ; \ \alpha_{On} = \alpha_{On}^0 Z^{3/2+n}\exp(-Z)$$

we obtain

$$\tau_{Xn} = \alpha_{Xn}^0 \int_0^\infty dx \, Z^{1/2+n}\exp(-Z) = \frac{R_0\alpha_{Xn}^0}{\mu}\Gamma(n+3/2)$$

$$\tau_{On} = \frac{R_0\alpha_{On}^0}{\mu}\Gamma(n+5/2)$$

IV. QUASILINEAR EVOLUTION OF THE WAVE DAMPING.

So far we have assumed that during wave absorption the background distribution remains Maxwellian. In the low density regime ($<n_e>$ a few 10^{13} cm $^{-3}$) and high wave power W>2-3 Mw, a deformation of the momentum distribution function may occur. The quasilinear evolution of the electron distribution is mainly a diffusion in the $p_\perp$- direction. Here we present the case of the O-mode at $N_{||}\approx 0$ in some detail and show the result for the X-mode at oblique propagation. Diffusion in the $p_\perp$- direction is described by the equation

$$\frac{\partial f_0}{\partial t}=\frac{1}{p_\perp}\frac{\partial}{\partial p_\perp}\left(p_\perp D_{cy}\right)\frac{\partial f_0}{\partial p_\perp}+\left(\frac{\partial f_0}{\partial t}\right)_{col} \qquad (19)$$

where

$$\left(\frac{\partial f_0}{\partial t}\right)_{col}$$

is the Boltzman collision operator and

$$D_{cy}=\frac{\left\langle\left(\Delta p_\perp\right)^2\right\rangle}{2\Delta t}$$

For the O-mode at $N_{||}\approx 0$ [9]

$$\frac{dp_\perp}{dt}=-\frac{e}{mc\,\gamma}p_\perp\times\left(B_0\vec{e}_z+B_y\vec{e}_y\right)$$

and therefore

$$D_{cy}=\pi\left(\frac{eN_o p_{||}}{2mc}\right)^2\left|E_z\right|^2\delta\left(\gamma-\frac{\omega_c}{\omega}-N_{||}\frac{p_{||}}{mc}\right)$$

Equation (19) is in general solved numerically using appropriate codes. The solution is used in equation (11) to obtain the self-consistent wave damping. For the O-mode, near perpendicular to B_0, we in general have

$$k"_o=-\frac{\pi}{8m^2c^2}\left(\frac{\omega N_o}{c}\right)\frac{\omega_p^2}{\omega_c^2}\int d\vec{p}\,p_{||}^2 p_\perp\frac{\partial f_0}{\partial p_\perp}\delta\left(\gamma-\frac{\omega_c}{\omega}-N_{||}\frac{p_{||}}{mc}\right) \qquad (20)$$

For $f_0=f_M$, eq. (20) becomes eq. (14). This is the initial value of $k"_o$. In figure 5, we present k_i at steady state for $P_0=1$ Mw and $n_e=3 \times 10^{13}$ cm^{-3}, $T_e=1$ keV, $B_0=30$ kG, f=84 GHz, a=40 cm and $R_0=150$ cm. The incident wave-packet has a half-width of a few degrees. It appears from fig. 5 that $k"_o$ decreases for increasing wave power.

The case of oblique propagation can be treated in a similar way. For $N_{||}\neq 0$, we have [10]

$$\frac{\partial f_0}{\partial \tau}=\frac{1}{u_\perp}\left(\frac{\omega_c}{\omega}\frac{\partial}{\partial u_\perp}+\frac{u_\perp}{\mu^{1/2}}\frac{\partial}{\partial u_{||}}n_{||}\right)u_\perp D_{cy}\left(\frac{\omega_c}{\omega}\frac{\partial}{\partial u_\perp}+\frac{u_\perp n_{||}}{\mu^{1/2}}\frac{\partial}{\partial u_{||}}\right)+\left(\frac{\partial f_0}{\partial \tau}\right)_{col} \qquad (21)$$

where $\tau=\nu_e t$, $\nu_e=2\pi e^4 n_e \Lambda/m^{1/2}T_e^{3/2}$, -e is the electron charge, n_e is the electron density, m is the electron rest mass, T_e is the body temperature, Λ is the Coulomb logarithm, $\mathbf{u}=\mathbf{p}/(mT_e)^{1/2}$, $\omega_c=eB/mc$, c is the speed of light, and

$$n_{\parallel}=\mu^{1/2}\left[\left(1+u^2/\mu\right)^{1/2}-\omega_c/\omega\right]/u_{\parallel}$$

The wave diffusion coefficient is given for a Gaussian wave packet, i. e.,

$$D_{cy}=\left(D_{\perp}\right)\exp\left(\frac{-\left(n_{\parallel}-\overline{N}_{\parallel}\right)^2}{\left(\Delta N_{\parallel}\right)^2}\right)\exp\left(-\int_r 2k''dr\right)$$

where $D_{\perp}$ is the appropriate diffusion coefficient for a given polarization and value of $N_{\parallel}$. The relativistic collision term is given in the case in which the velocity of the resonant electrons is greater than the thermal speed

$$\left(\frac{\partial f_0}{\partial \tau}\right)_{coll}\approx\frac{Z+1}{u^3\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial f_0}{\partial\theta}+\frac{2}{u^2}\frac{\partial}{\partial u}\left(\frac{1}{u}\frac{\partial f_0}{\partial u}+f_0\right) \qquad (22)$$

where Z is the effective ion charge, u and θ are polar coordinates, and equal electron and ion temperatures are assumed. Using a convenient variable time step $\Delta\tau$ and a 192x64 grid in the (u,θ) space, f is computed as a function of τ in the range $0\leq u\leq 13$. We keep the body temperature constant in time. This plasma model is legitimate if an unspecified thermal loss maintains the body temperature constant during the time interval in which the electron tail attains the steady-state. In figure 6 it is shown

$$\eta_x(x)=1-\exp\left(-2\int_0^x dx'k''_x\right)$$

versus x for the X-mode at an angle of $\theta=50^o$, $B_o=39$ kG, f=83 GHz, $n_e(x)=n_e(0)(1-x^2/a^2)$, $T_e(x)=T_e(0)(1-x^2/a^2)^2$, $T_e(0)=2.5$ keV, $n_e(0)=5\times 10^{13}$ cm^{-3}, Z=2, a=40 cm, $R_o=150$ cm and W=1 Mw.

V. ELECTRON-CYCLOTRON CURRENT DRIVE.

For finite values of $N_{\parallel}$, wave absorption in a maxwellian plasma is selective, i.e.

$$v_{\parallel}=\frac{\omega-\omega_c/\gamma}{k_{\parallel}}$$

This results in an asymmetric alteration of the distribution function [11] (see fig. 7) carrying an electron current in the parallel direction given by

$$\Delta J=-en_e\int d\vec{p}\, v_{\parallel}\Delta f$$

At steady-state, this current is sustained by the wave power

$$\Delta P=n_e\int d\vec{p}mc^2(\gamma-1)\left(\frac{\partial f_0}{\partial t}\right)_{cy}$$

where

$$\left(\frac{\partial f_0}{\partial t}\right)_{cy}\approx\frac{1}{p_\perp}\frac{\partial}{\partial p_\perp}\left(p_\perp D_{cy}\right)\frac{\partial f_M}{\partial p_\perp}$$

At steady-state

$$\left(\frac{\partial f_0}{\partial t}\right)_{cy}=\nu(p)\Delta f$$

thus

$$\frac{\Delta J}{\Delta P}=-\frac{e}{m^2c^2}\frac{\int d\vec{p}\,\frac{p_{\parallel}}{\gamma\nu(\gamma)}\left(\frac{\partial f_0}{\partial t}\right)}{\int d\vec{p}\,(\gamma-1)\left(\frac{\partial f_0}{\partial t}\right)} \qquad (23)$$

where

$$\nu(\gamma)=\frac{4\pi e^4 n_e\Lambda}{m^2c^3}\frac{(\gamma-1)^{3/2}}{(\gamma+1)^{1/2}(\gamma^2-2\gamma\ln\gamma-1)}$$

for Z=1.

Use of eq. (23) for the WENDELSTEIN VII-AS conditions predicts a value of the position of the maximum efficiency in good agreement with the experiment [12] (Fig. 8).

VI. GENERAL THEORY OF WAVE ABSORPTION AND CURRENT DRIVE.

A compact and general formulation of wave absorption (and emission) for arbitrary harmonics can be derived as follows.

The global results of emission and absorption are the cumulative contributions of electrons in a given velocity range, thus,

$$\alpha(\omega)=\int d\vec{p}\,\alpha(\vec{p})=\sum_{n=1}^{\infty}\int d\vec{p}\,\alpha_n(\vec{p}) \tag{24}$$

where **p** is the electron momentum. For cyclotron resonance processes

$$\alpha_n(\vec{p})\propto\delta\left(\gamma-Y_n-N_{\parallel}p_{\parallel}/mc\right)\ ;\ \text{with}\ \gamma^2=1+p^2/(mc)^2\ ,Y_n=\frac{n\omega_c}{\omega}$$

$N_{\parallel}$ is the parallel refractive index and m is the the electron rest mass. Using the δ-function, we obtain the one-particle resonance relation

$$\gamma=Y_n+N_{\parallel}p_{\parallel}/mc$$

from which

$$\left(\frac{p_{\perp}}{mc}\right)^2=\left(Y_n+N_{\parallel}p_{\parallel}/mc\right)^2-1-\left(p_{\parallel}/mc\right)^2=v_{\perp R}^2$$

Since $p_{\perp}^2>0$, we obtain $p_- < p_{\parallel} < p_+$, and

$$\alpha(\omega)=\sum_{n=1}^{\infty}\int_{p_-}^{p_+}dp_{\parallel}W_n(p_{\parallel})$$

where

$$p_{\pm}=mc\left[N_{\parallel}Y_n\pm\left(N_{\parallel}^2-1+Y_n^2\right)^{1/2}\right]/\left(1-N_{\parallel}^2\right)$$

and we consider for simplicity the case of most interest, i. e., $|N_{\parallel}|<1$. $W_n(p_{\parallel})$ is proportional to the power absorption per unit interval in momentum space. Now, from Poynting theorem $\alpha(\omega)=W/S$, where W is the power absorption per unit volume and S is the magnitude of the Poynting vector, and

$$W = n_e \int d\vec{p}\, mc^2 \gamma \left(\frac{\partial f}{\partial t}\right)_{cy} \tag{25}$$

where n_e is the electron density,

$$\left(\frac{\partial f}{\partial t}\right)_{cy} = 4\pi e^2 \sum_{n=1}^{\infty} \int_{-\infty}^{+\infty} dN_{\parallel} \frac{\gamma}{p_\perp} L_n D_n \,\delta\left(\gamma - Y_n - N_{\parallel} p_{\parallel}/mc\right) L_n f$$

$$\gamma L_n = Y_n \frac{\partial}{\partial p_\perp} + \frac{N_{\parallel} p_\perp}{mc} \frac{\partial}{\partial p_{\parallel}} \quad ; \; D_n = \frac{p_\perp}{8\omega} \left|\vec{E} \cdot \vec{\Pi}_n\right|^2$$

E is the wave electric field, $\Pi_{1n} = nJ_n(\rho)/\rho$, $\Pi_{2n} = -iJ_n'(\rho)$, $\Pi_{3n} = (p_{\parallel}/p_\perp)J_n(\rho)$, $\rho = k_\perp p_\perp / m\omega_c$, $-e$ is the electron charge, and **k** is the wave vector. Using eqs. (24) and (23), for a narrow wave-packet we obtain

$$\alpha(\omega) = \sum_{n=1}^{\infty} \int_{v_-}^{v_+} dv_{\parallel} W_n(v_{\parallel})$$

$$W_n(v_{\parallel}) = -2\pi (mc)^3 \omega_p^2 \left(\gamma \frac{D_n}{S} L_n f\right)_{v_\perp = v_{\perp R}} \tag{26}$$

$\mathbf{v} = \mathbf{p}/mc$, $(v_{\perp R})^2 = \gamma_n^2 - \gamma_{\parallel}^2$, $\gamma_n = Y_n + N_{\parallel} v_{\parallel}$, $\gamma_{\parallel} = (1 + v_{\parallel}^2)^{1/2}$ and ω_p is the plasma frequency. The emission coefficient $\beta(\omega)$ is obtained from the absorption coefficient $\alpha(\omega)$ using the transformation

$$L_n f \rightarrow \frac{\omega^2}{8\pi^3 c^2} p_\perp f / m\gamma$$

hence,

$$\beta(\omega) = \sum_{n=1}^{\infty} \int_{v_-}^{v_+} dv_{\parallel} G_n(v_{\parallel})$$

where

$$G_n(v_{\parallel}) = \frac{\omega^2}{8\pi^3 c^2} 2\pi (mc)^3 \omega_p^2 c \left(\frac{D_n}{S} v_\perp f\right)_{v_\perp = v_{\perp R}} \tag{27}$$

Equations (26) and (27) determine the momentum spectra of the absorption and emission coefficients. $W_n(G_n)$ describes the relative role of the electrons in a given velocity range on the global absorption (emission). In particular, the value of $v_{\parallel}$ for which

$W_n(G_n)$ is maximum characterizes the group of electrons which yields the predominant contribution to wave absorption (emission). In the case of a sharp maximum it is possible to define the resonant velocity for a system of electrons. Note that, for a maxwellian distribution, $G_n(v_{||})=B_o W_n(v_{||})$, $B_o=\omega^2 T_e/8\pi^3 c^2$, and we obtain that the electrons giving the predominant contribution to emission and absorption lie in the same range of velocities. For non-maxwellian momentum distributions, $G_n(v_{||})$ is not in general proportional to $W_n(v_{||})$ and emission and absorption depend on electrons in different regions of the momentum space.

The current drive efficiency is expressed in terms of $W_n(v_{||})$. Using the impulse-response method, the incremental efficiency is given by [13]

$$\frac{\delta J_{cy}}{\delta P_{cy}}=\frac{e}{mc}\left[N_{||}+v_{||}\gamma\nu(\gamma)\frac{\partial}{\partial\gamma}\left(\frac{1}{\gamma\nu(\gamma)}\right)\right]\left[\gamma\nu(\gamma)\right]^{-1}$$

where

$$\delta P_{cy}=\frac{4\pi e^2}{m}\sum_{n=1}^{\infty}D_n\delta\left(\gamma-n\omega_c/\omega-N_{||}v_{||}\right)L_n f$$

Combining the two equations we obtain

$$\frac{J_{cy}}{P_{cy}}=\left(\sum_{n=1}^{\infty}\int_{v-}^{v+}dv_{||}\,g(v_{||})W_n(v_{||})\right)\left(\sum_{n=1}^{\infty}\int_{v-}^{v+}dv_{||}W_n(v_{||})\right)^{-1} \qquad (28)$$

where

$$g(v_{||})=\frac{e}{mc}\left[N_{||}+v_{||}\Gamma(\gamma_n)\right]\left[\gamma_n\nu(\gamma_n)\right]^{-1}$$

and for the ion charge Z=1

$$\nu(\gamma)=\frac{4\pi e^4 n_e\Lambda}{m^2c^3}(\gamma-1)^{3/2}/(\gamma+1)^{1/2}(\gamma-2\gamma\ln\gamma-1),$$

$$\Gamma(\gamma)=\left[2\gamma^2(\gamma+2)\ln\gamma-(4\gamma-1)(\gamma^2-1)\right]\left[\gamma(\gamma^2-1)(\gamma-2\gamma\ln\gamma-1)\right]^{-1}.$$

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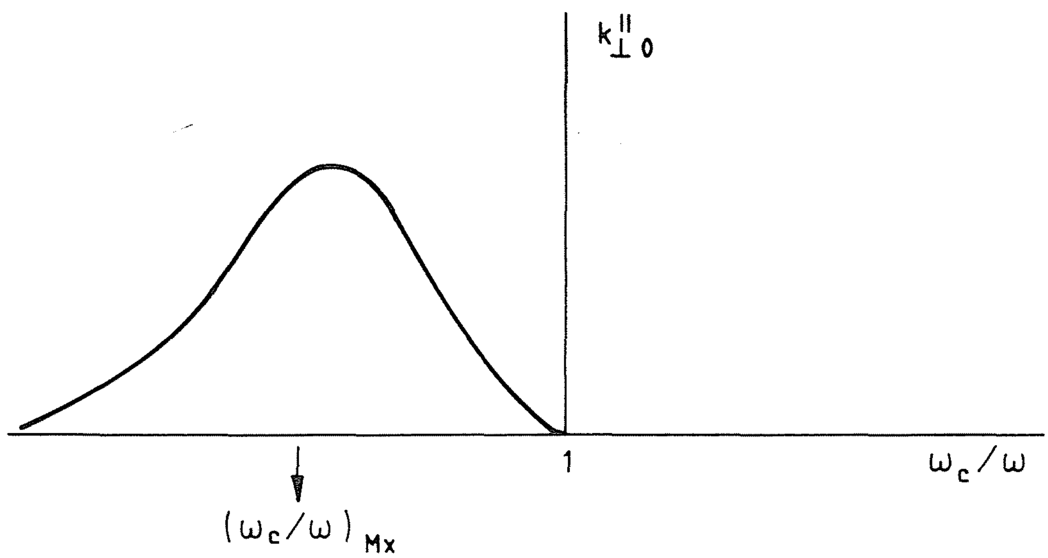
MAXIMUM AT $(\omega_c/\omega)_{Mx} = 1 + 5/2\ T_e/mc^2$



$\Delta\omega/\omega = 5/2\ T_e/mc^2$

FIG. 1

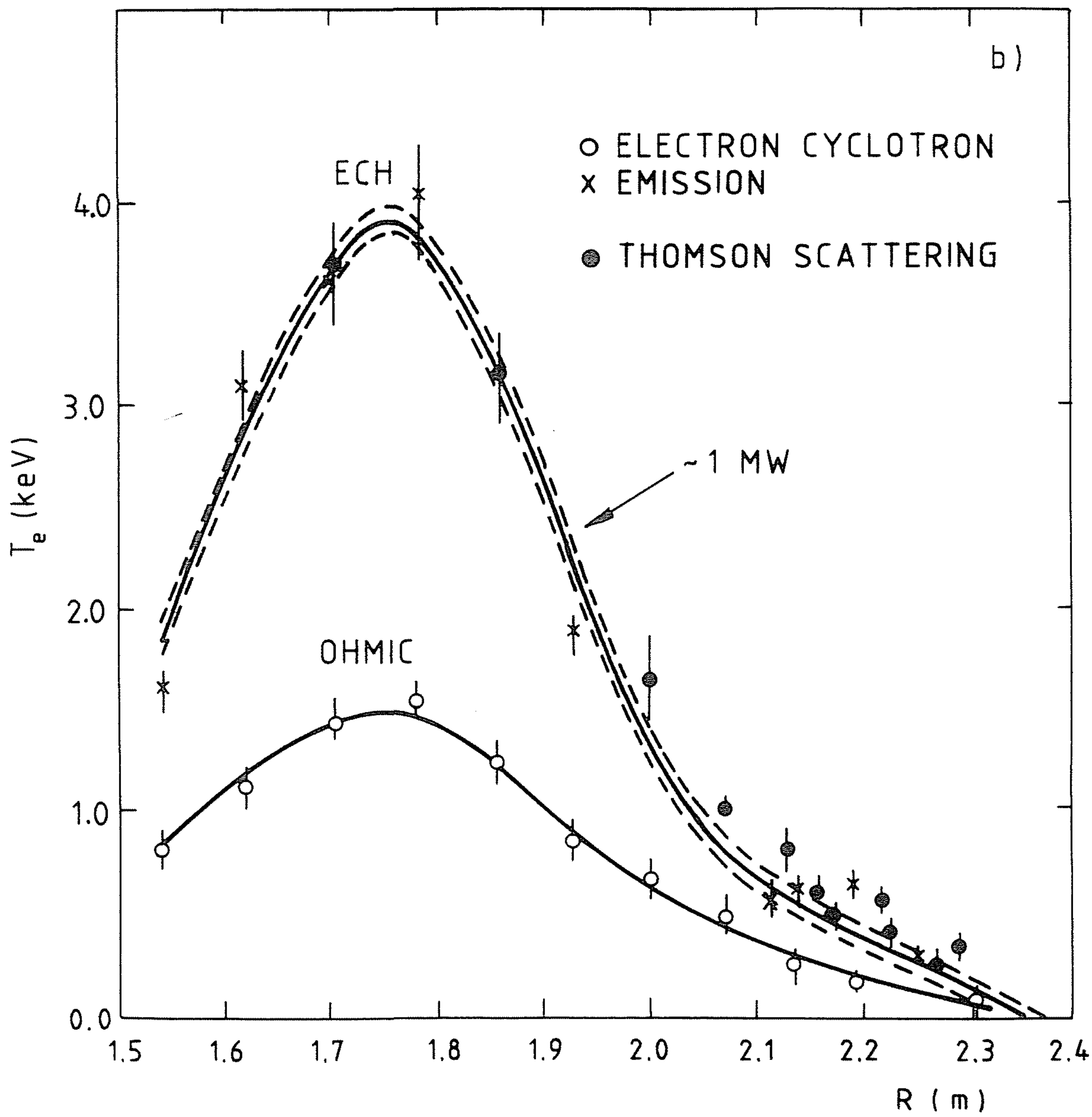

b)
○ ELECTRON CYCLOTRON
x EMISSION
● THOMSON SCATTERING
ECH
OHMIC
~1 MW
T_e (keV)
R (m)
4.0
3.0
2.0
1.0
0.0
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
2.4

FIG. 2

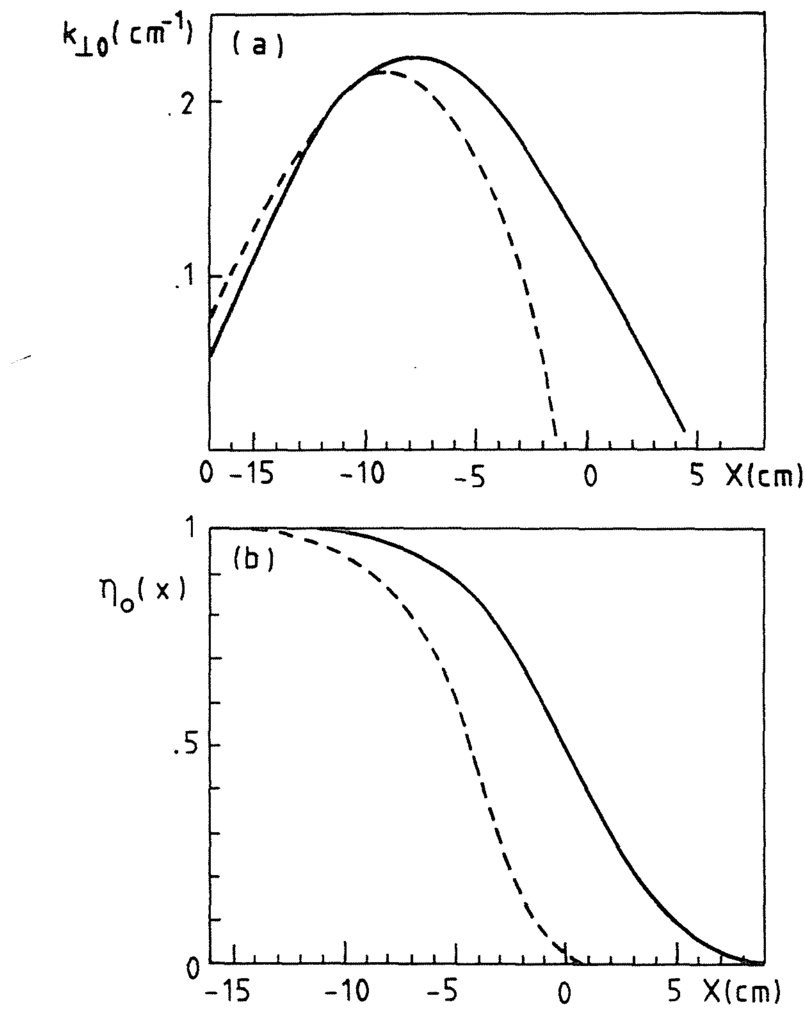


FIG. 3.- $k''_{\perp 0}$ and η_0 vs x for $T_e(0)=4keV$, $a=40cm$, $R=130cm$ for $\theta=25°$ ($x_c=-10cm$). Wave propagation is from the low magnetic field side.

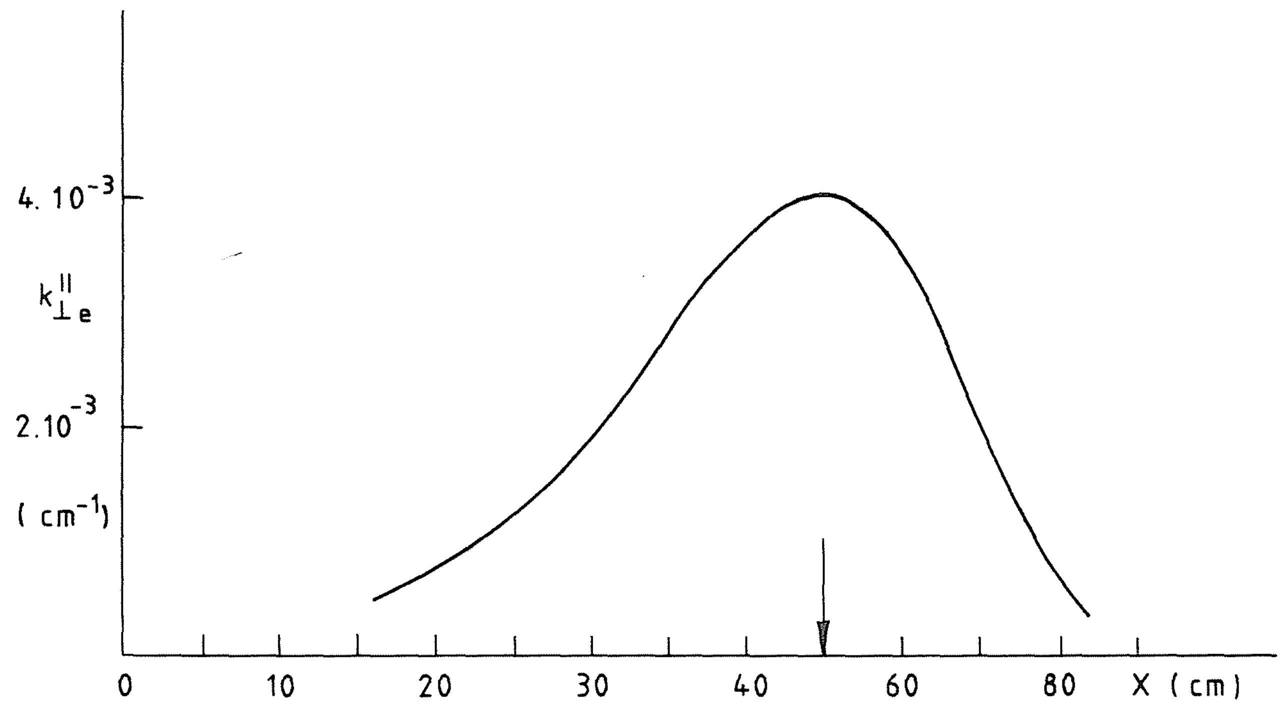


FIG. 4

$\eta_e = 1 - \exp(-\tau_e) \approx 90\ \%$

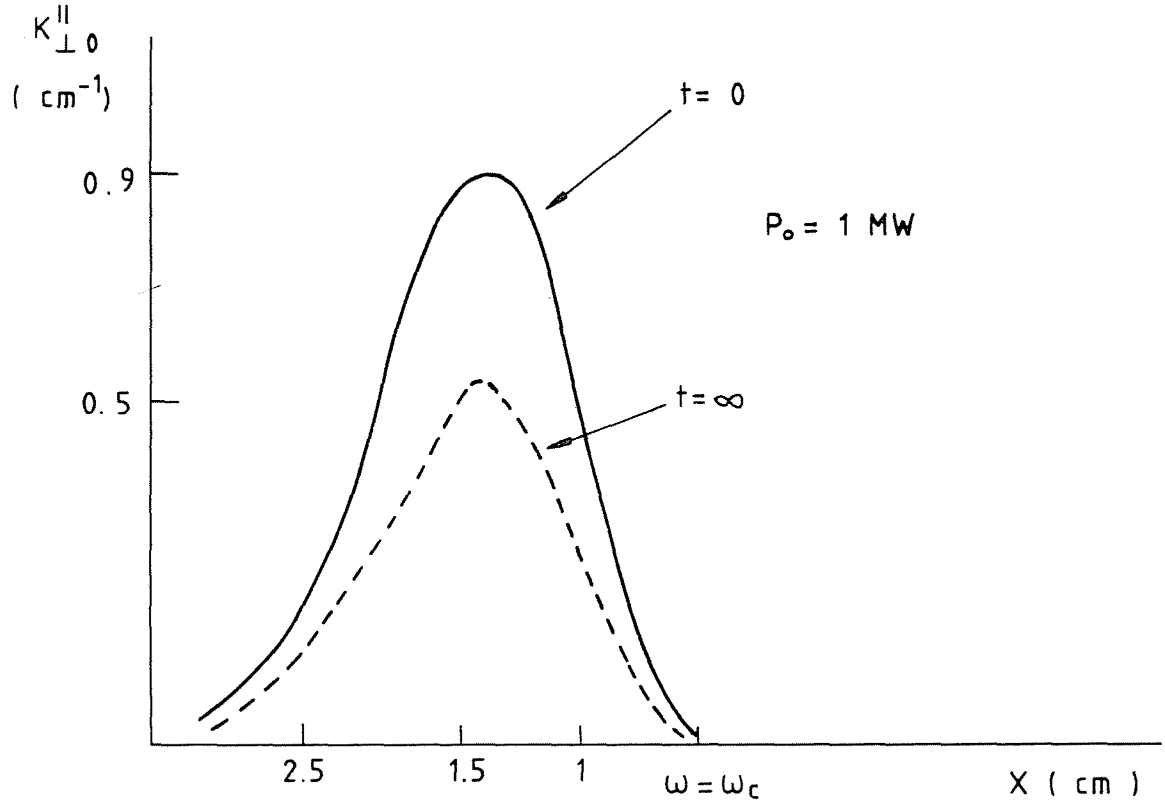


FIG. 5

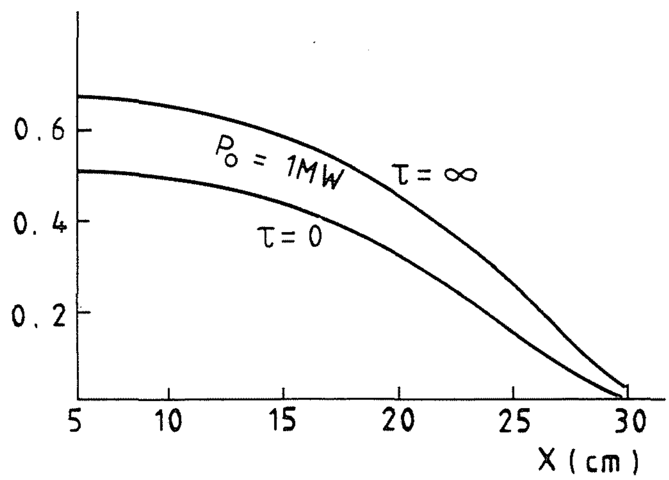


FIG. 6

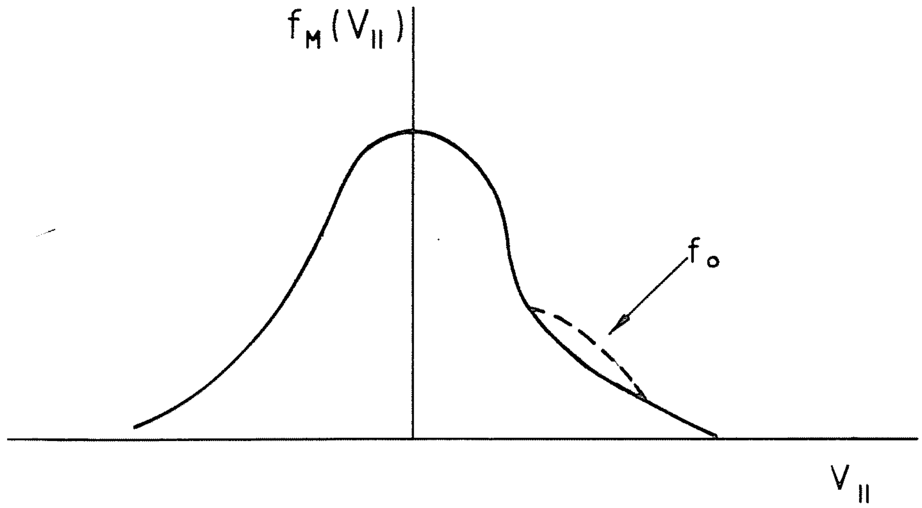


FIG. 7.- $\Delta f = f_o - f_M$

$|\Delta f| << f_M$

STELLARATOR : FOR PROFILE CONTROL OF I

WENDELSTEIN VII-AS

R_o= 200cm , a= 20cm

B_o= 12.5kG , f= 70 GH_z (SECOND HARM)

$n_e(0)$= $1.4\times10^{13}\,cm^{-3}$, $T_e(0)$= 1.2keV

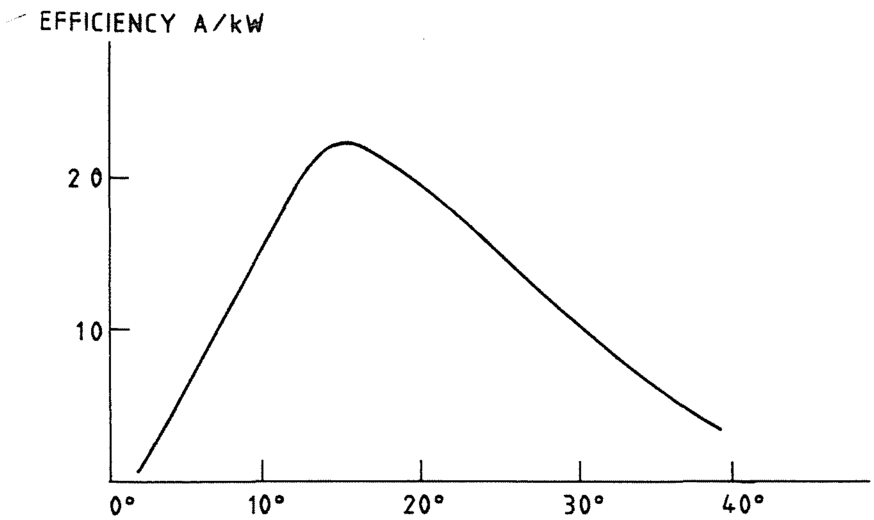


FIG. 8.- THEORY AND EXPERIMENTS YIELD MAXIMUM AT θ =15°

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"Calentamiento e inducción de corriente mediante la resonancia ciclotrónica electrónica".

FIDONE, I.; CASTEJON, F. (1992) 27 pp. 8 figs. 13 refs.

En este trabajo se presenta un sumario de la teoría y los experimentos de calentamiento e inducción de corrientes mediante la resonancia ciclotrónica electrónica. Se considera en detalle la formulación relativista general para la propagación de las ondas y la absorción en el régimen lineal. Se han investigado los modos O y X para propagación perpendicular y oblicua y se ilustran mediante varios ejemplos. Se discute brevemente la verificación experimental de la teoría en los tokamaks T-10 y D-III-D. También se considera la evolución cuasi-lineal de la distribución de momentos y sus aplicaciones, como por ejemplo, la inducción de corrientes y la absorción en régimen no lineal; este estudio se hace para algunos casos especiales de polarización, propagación y frecuencias. En la última sección presentamos la formulación general de inducción de corrientes en ausencia de electrones atrapados para valores arbitrarios de la frecuencia de la onda.

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CIEMAT-687

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