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Neutrino Dark Energy – Revisiting the Stability Issue

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Abstract. A coupling between a light scalar field and neutrinos has been widely discussed as a mechanism for linking (time varying) neutrino masses and the present energy density and equation of state of dark energy. However, it has been pointed out that the viability of this scenario in the non-relativistic neutrino regime is threatened by the strong growth of hydrodynamic perturbations associated with a negative adiabatic sound speed squared. In this paper we revisit the stability issue in the framework of linear perturbation theory in a model independent way. The criterion for the stability of a model is translated into a constraint on the scalar-neutrino coupling, which depends on the ratio of the energy densities in neutrinos and cold dark matter. We illustrate our results by providing meaningful examples both for stable and unstable models.

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1. Introduction

Precision observations of the cosmic microwave background [1–3], the large scale structure of galaxies [4], and distant type Ia supernovae [5–8] have led to a new standard model of cosmology in which the energy density is dominated by dark energy with negative pressure, leading to an accelerated expansion of the universe.

The simplest possible explanation for dark energy is the cosmological constant which has $P = w\rho$ with $w = -1$ at all times. However, since the cosmological constant has a magnitude completely different from theoretical expectations one is naturally led to consider other explanations for the dark energy. A light scalar field rolling in a very flat potential would for instance be a candidate better motivated from high energy physics [9–11]. In the limit of a completely flat potential it would have $w = -1$. Such models are generically known as quintessence models [12–17]. The scalar field is usually assumed to be minimally coupled to matter and to curvature, but very interesting effects can occur if this assumption is relaxed (see for instance [18–24]). In general such models alleviate the required fine tuning in order to achieve $\Omega_X \sim \Omega_m$, where Ω_X and Ω_m are the dark energy and matter densities at present. Also by properly choosing the quintessence potential it is possible to achieve tracking behaviour of the scalar field so that one also avoids the extreme fine tuning of the initial conditions for the field.

Many other possibilities have been considered, like k-essence, which is essentially a scalar field with a non-standard kinetic term [25–27]. It is also possible, although not without problems, to construct models which have $w < -1$, the so-called phantom energy models [28–30]. Finally, there are even more exotic models where the cosmological acceleration is not provided by dark energy, but rather by a modification of the Friedmann equation due to modifications of gravity on large scales [31, 32], or even due to higher order curvature terms in the gravity Lagrangian [33–35].

A very interesting proposal is the so-called mass varying neutrino (MaVaN) model [36–38] in which a light scalar field couples to neutrinos. Due to the coupling, the mass of the scalar field does not have to be as small as the Hubble scale but can be much larger, while the model still accomplishes late-time acceleration. This scenario also holds the interesting possibility of circumventing the well-known cosmological bound on the neutrino mass [3, 4, 40–51]. The scenario is a variant of the chameleon cosmology model [52–54] in which a light scalar field couples democratically to all non-relativistic matter.

The idea in the MaVaN model is to write down an effective potential for the scalar field which as a result of the coupling contains a term related to the neutrino energy density. If the pure scalar field potential is tuned appropriately the effective potential including the neutrino contribution will have a minimum with a steep second derivative for some finite scalar field VEV. The scalar field is therefore locked in the minimum and when the minimum evolves due to changing neutrino energy density the field tracks this evolution adiabatically. This naturally leads to a dynamical effective equation of state for the combined scalar - neutrino fluid close to $w = -1$ today, and to a neutrino mass which is related to the combined neutrino-scalar field fluid's energy density ρ_{DE}. Since ρ_{DE} decreases with time, also the neutrino mass varies in this kind of scenario, where its present value is explained in terms of $\rho_{\mathrm{DE}}^{1/4}(a = 1)$. Possible tests for the MaVaN scenario can be found in Ref. [55–62].

MaVaN models, however, suffer from the problem that for some choices of scalar-neutrino couplings and scalar field potentials the combined fluid is subject to an instability once the neutrinos become non–relativistic. Effectively the scalar field mediates an attractive force between neutrinos which can possibly lead to the formation of neutrino nuggets [63]. This in turn would make the combined fluid behave like cold dark matter and thus render it non-viable as a candidate for dark energy.

In perturbation theory the formation of these nuggets can be seen as a consequence of an imaginary speed of sound for the combined fluid, signaling fast growth of instabilities. However, an imaginary speed of sound cannot be generally used as a sufficient criterion for the instabilities, since it is also crucial that the scalar field-induced interaction between neutrinos dominates over the drag provided by cold dark matter.

The instability can occur in these models because the effective mass associated with the scalar field is much larger than H. Accordingly, its effective Compton wavelength sets the scales as of which perturbations are adiabatic to be much smaller than the Hubble radius . This is a consequence of the steepness of the effective potential and can be remedied by making the potential sufficiently flat. In this case the evolution of the field is highly non-adiabatic [64, 65]. However, this model has the disadvantage that the neutrino mass is no longer related naturally to the dark energy density and equation of state.

In this paper we study various choices of scalar-neutrino couplings and scalar field potentials with the aim of identifying the conditions for the instability to occur. We show that the condition of positive sound speed squared severely constrains the allowed neutrino mass variation. In the next section we review the formalism needed to study mass varying neutrinos and in section 3 we derive the equation of motion of the neutrino perturbations. Section 4 contains our results for various couplings and potentials, and finally section 5 contains a discussion and conclusion.

2. Formalism

The idea in the so-called Mass Varying Neutrino (MaVaN) scenario [36–38] is to introduce a coupling between (relic) neutrinos and a light scalar field and to identify this coupled fluid with dark energy. As a direct consequence of this new interaction, the neutrino mass m_ν is generated from the vacuum expectation value (VEV) of the scalar field and becomes linked to its dynamics. Thus the pressure $P_\nu(m_\nu(\phi), a)$ and energy density $\rho_\nu(m_\nu(\phi), a)$ of the uniform neutrino background contribute to the effective potential $V(\phi, a)$ of the scalar field. The effective potential is defined by

$$\frac{dV(\phi)}{d\phi} = \frac{dV_\phi(\phi)}{d\phi} + \beta(\rho_\nu - 3P_\nu) \tag{1}$$

where $V_\phi(\phi)$ denotes the fundamental scalar potential, a is the scale factor, and $\beta = \frac{d\log m_\nu}{d\phi}$ is the coupling between the scalar field and the neutrinos. Throughout the paper we assume a flat Friedman-Robertson-Walker cosmology and use the convention $a_0 = 1$, where we take the subscript 0 to denote present day values.

Assuming the neutrino distribution to be Fermi-Dirac and neglecting the chemical potential, the energy density and pressure of the neutrinos can be expressed in the following form [39]

$$\begin{aligned}\rho_\nu(a,\phi) &= \frac{T_\nu^4(a)}{\pi^2}\int_0^\infty \frac{dy\, y^2\sqrt{y^2+\frac{m_\nu^2(\phi)}{T_\nu^2(a)}}}{e^y+1},\\ P_\nu(a,\phi) &= \frac{T_\nu^4(a)}{3\pi^2}\int_0^\infty \frac{dy\, y^4}{\sqrt{y^2+\frac{m_\nu^2(\phi)}{T_\nu^2(a)}}(e^y+1)},\end{aligned} \tag{2}$$

where $T_\nu = T_{\nu_0}/a$ is the neutrino temperature and y corresponds to the ratio of the neutrino momentum and neutrino temperature, $y = p_\nu/T_\nu$.

The energy density and pressure of the scalar field are given by the usual expressions,

$$\begin{aligned}\rho_\phi(a) &= \frac{1}{2a^2}\dot{\phi}^2 + V_\phi(\phi),\\ P_\phi(a) &= \frac{1}{2a^2}\dot{\phi}^2 - V_\phi(\phi).\end{aligned} \tag{3}$$

Defining $w = P_{\rm DE}/\rho_{\rm DE}$ to be the equation of state of the coupled dark energy fluid, where $P_{\rm DE} = P_\nu + P_\phi$ denotes its pressure and $\rho_{\rm DE} = \rho_\nu + \rho_\phi$ its energy density, and the requirement of energy conservation gives,

$$\dot{\rho}_{DE} + 3H\rho_{DE}(1+w) = 0. \tag{4}$$

Here $H \equiv \frac{\dot{a}}{a}$ and we use dots to refer to the derivative with respect to conformal time. Taking Eq. (4) into account, one arrives at a modified Klein-Gordon equation describing the evolution of ϕ,

$$\ddot{\phi} + 2H\dot{\phi} + a^2 V'_\phi = -a^2\beta(\rho_\nu - 3p_\nu). \tag{5}$$

Here and in the following primes denote derivatives with respect to ϕ ($' = \partial/\partial\phi$).

2.1. The fully adiabatic case

In the following let us consider the late time evolution of the coupled scalar-neutrino fluid in the limit $m_\nu \gg T_\nu$ where the neutrinos are non-relativistic. It is in this regime that MaVaN models can potentially become unstable for the following reason: the attractive force mediated by the scalar field (which can be much stronger than gravity) acts as a driving force for the instabilities. But as long as the neutrinos are still relativistic, the evolution of the density perturbations will be dominated by pressure which inhibits their growth, as the strength of the coupling is suppressed when $\rho_\nu = 3P_\nu$.

In the non-relativistic limit $m_\nu \gg T_\nu$, the expressions for the energy density and pressure in neutrinos in Eq. (2) reduce to

$$\begin{aligned} \rho_\nu &\simeq m_\nu n_\nu, \\ P_\nu &\simeq 0, \end{aligned} \tag{6}$$

such that Eq. (1) takes the form

$$V = \rho_\nu + V_\phi = m_\nu n_\nu + V_\phi. \tag{7}$$

Assuming the curvature scale of the potential and thus the mass of the scalar field m_ϕ to be much larger than the expansion rate of the Universe,

$$V'' = \rho_\nu \left(\beta' + \beta^2\right) + V_\phi'' \equiv m_\phi^2 \gg H^2, \tag{8}$$

the adiabatic solution to the equation of motion of the scalar field in Eq. (5) applies [38]‡. As a consequence, the scalar field instantaneously tracks the minimum of its effective potential V, solution to the condition

$$V' = \rho_\nu' + V_\phi' = m_\nu' \left(\frac{\partial \rho_\nu}{\partial m_\nu} + \frac{\partial V_\phi}{\partial m_\nu}\right) = m_\nu' \left(n_\nu + \frac{\partial V_\phi}{\partial m_\nu}\right) = 0, \tag{9}$$

As the universe expands the neutrino energy density gets diluted, thus naturally giving rise to a slow evolution of $V(\phi)$. Consequently, the value of the scalar field ϕ evolves on cosmological time scales. Note that as long as m_ν' does not vanish, this implies that also the neutrino mass $m_\nu(\phi)$ is promoted to a time dependent, dynamical quantity. Its late time evolution can be determined from the last equality in Eq. (9).

In order to specify good candidate potentials $V_\phi(\phi)$ for a viable MaVaN model of dark energy, we must demand that the equation of state parameter w of the coupled scalar-neutrino fluid today roughly satisfies $w \sim -1$ as suggested by observations [66]. By noting that for constant w at late times,

$$\rho_{\mathrm{DE}} \sim V \propto a^{-3(1+w)} \tag{10}$$

and by requiring energy conservation according to Eq. (4), one arrives at [38]

$$1 + w = -\frac{1}{3}\frac{\partial \log V}{\partial \log a}. \tag{11}$$

In the non-relativistic limit $m_\nu \gg T_\nu$ this is equivalent to

$$1 + w = -\frac{a}{3V}\left(m_\nu \frac{\partial n_\nu}{\partial a} + n_\nu \frac{\partial m_\nu}{\partial a} + \frac{V_\phi'}{a'}\right) = -\frac{m_\nu V_\phi'}{m_\nu' V}, \tag{12}$$

where in the last equality it has been used that $V' = 0$ according to Eq. (9). To allow for an equation of state close to $w \sim -1$ today one can conclude that either the scalar potential V_ϕ has to be fairly flat or the dependence of the neutrino mass on the scalar field has to be very steep.

‡ In this case for $|\phi| < M_{\mathrm{pl}} \simeq 3 \times 10^{18}$ GeV the effects of the kinetic energy terms can be safely ignored [38].

2.2. The general case

As it will turn out later, the influence of the cosmic expansion in combination with the gravitational drag exerted by CDM on the neutrinos can have a major effect on the stability of a MaVaN model. However, to begin we will neglect any growth-slowing effects on the perturbations and proceed with a more general analysis of this case. In this case, the dynamics of the perturbations are solely determined by the sound speed squared which for a general fluid component i takes the following form,

$$c_{si}^2 = \frac{\delta P_i}{\delta \rho_i}, \tag{13}$$

where P_i and ρ_i denote the fluid's pressure and energy density, respectively. The sound speed c_{si}^2 can be expressed in terms of the sound speed c_{ai}^2 arising from purely adiabatic perturbations as well as an additional entropy perturbation Γ_i and the density contrast $\delta_i = \delta\rho_i/\rho_i$ in the given frame [67, 68],

$$w_i \Gamma_i = (c_{si}^2 - c_{ai}^2)\, \delta_i, \tag{14}$$

$$= \frac{\dot{P}_i}{\rho_i}\left(\frac{\delta P_i}{\dot{P}_i} - \frac{\delta\rho_i}{\dot{\rho}_i}\right). \tag{15}$$

Here w_i denotes the equation of state parameter and Γ_i is a measure for the relative displacement between hypersurfaces of uniform pressure and uniform energy density. For most dark energy candidates (like quintessence or k-essence) dissipative processes evoke entropy perturbations and thus $\Gamma_i \neq 0$.

However, in MaVaN models the effective mass of the scalar field $\sim m_\phi \gg H$ sets the scale, $\sim m_\phi^{-1}$, where these processes and the associated gradient terms become unimportant [63, 69], to be much smaller than the Hubble radius (in contrast to a quintessence field with finely-tuned mass $\lesssim H$ and long range $\gtrsim H^{-1}$). As a consequence, on sub-Hubble scales $> m_\phi^{-1}$ all dynamical properties of (non-relativistic) MaVaNs are set by the local neutrino energy density [63]. In particular, for small deviations away from the minimum of its effective potential, the scalar field re-adjusts to its new minimum on time scales $\sim m_\phi^{-1}$ small compared to the characteristic cosmological time scale H^{-1}. In this case the hydrodynamic perturbations in MaVaNs are adiabatic. This means the system of neutrinos and the scalar field can be treated as a unified fluid with pressure $P_{DE} = P_\nu + P_\phi$ and energy density $\rho_{DE} = \rho_\nu + \rho_\phi$ without intrinsic entropy, $\Gamma_{DE} = 0$.

If any growth-slowing effects can be neglected, the perturbations in a stable MaVaN model are driven by a positive effective sound speed squared,

$$c_a^2 = \frac{\dot{P}_{\rm DE}}{\dot{\rho}_{DE}} = \frac{\dot{w}\rho_{DE} + w\dot{\rho}_{DE}}{\dot{\rho}_{DE}} = w - \frac{\dot{w}}{3H(1+w)} > 0, \tag{16}$$

where Eq. (4) and Eq. (15) have been used. In the case $c_a^2 > 0$ the attractive scalar force is offset by pressure forces and the fluctuations oscillate as sound waves. However, for $c_a^2 < 0$ perturbations become unstable and tend to blow up.

After generalising the above treatment to include three neutrino generations, it can be shown [70] that the requirement of positive sound speed squared in Eq. (16)

leads to the following stability condition on the mass evolution for $m_{\nu_i}(a) \gg T_\nu(a)$ with $i = 1, 2, 3$,

$$\sum_{i=1}^{3} \frac{\partial m_{\nu_i}(a)}{\partial a} a^2 \left(\frac{5\alpha T_{\nu_0}^2(a)}{3m_{\nu_i}^2(a)} - 1 \right) + \sum_{i=1}^{3} \frac{25\alpha T_{\nu_0}^2(a)}{3am_{\nu_i}(a)} > 0, \text{ with}$$

$$\alpha \equiv \frac{\int_0^\infty \frac{dy\, y^4}{e^y+1}}{2\int_0^\infty \frac{dy\, y^2}{e^y+1}} \simeq 6.47. \tag{17}$$

Assuming a degenerate neutrino mass spectrum with $m_{\nu_i}(z) \sim m_\nu(z)$, the resulting first order differential equation in $m_\nu(z)$ can be solved for the maximally allowed mass evolution in a stable adiabatic MaVaN model. In fig. 1 the solution $m_\nu(z)$ is plotted for $m_{\nu_i}(0) \sim m_\nu(0) = 0.312$ eV. Apparently, if the perturbations are not stabilised by growth-slowing effects to be discussed later on, the condition of positive sound speed squared severely constrains the allowed mass variation at late times.

In the following we will argue that this result remains valid in the case of a hierarchical neutrino mass spectrum independent of the absolute neutrino mass scale. Therefore, let us consider two different scenarios possibly realised in nature. Either all neutrinos are (highly) non-relativistic today such that Eq. (17) is applicable which requires that all the neutrino masses are essentially constant at late times. Or, as allowed by neutrino oscillation experiments, one neutrino mass eigenstate is still relativistic today. It has been shown for a large class of MaVaN models that this case can only be realised if the heaviest, non-relativistic neutrino is stable [71]. Otherwise, the scalar field VEV and thus the neutrino masses are driven to a new scale such that none of the neutrinos remains relativistic until today. This results in a cascaded instability of the system because all components become unstable at nearly the same time§. Consequently, at least in this class of models the stability condition requires the mass of the heaviest, non-relativistic neutrino to be essentially constant at late times. Therefore, in the case that all neutrino masses have the same dependence on the scalar field value, they are forced to behave accordingly.

For simplicity, in the following stability analysis we will assume a degenerate neutrino mass spectrum corresponding to three highly non-relativistic neutrinos today. However, largely independent of the absolute scale and spectrum of neutrino masses our results can be generalised to apply for all standard MaVaN models possibly subject to a cascaded instability.

§ Possible alternative scenarios are so-called hybrid MaVaN models which involve a second light scalar field [71, 72]. Since they allow for a steeper scalar potential, while accomplishing late-time acceleration, they can be stable even in the presence of an unstable component until the present time.

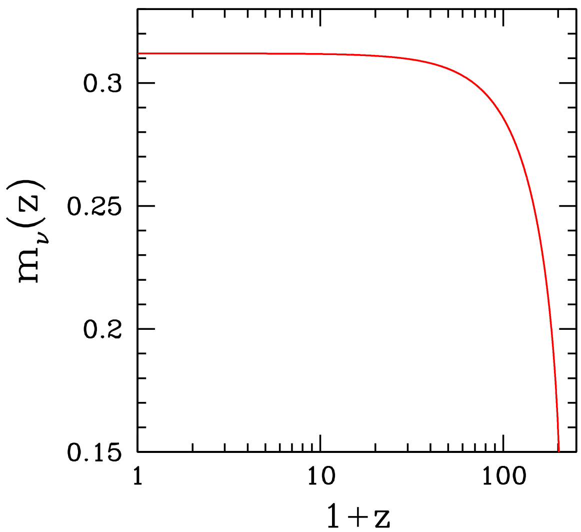

Figure 1. Maximally allowed mass evolution in a model with positive sound speed squared for a degenerate neutrino mass spectrum with $m_{\nu_i}(0) \sim m_\nu(0) = 0.312$ eV, where $i = 1, 2, 3$.

3. Evolution of the Perturbations

In this section we will analyse the linear MaVaN perturbations in the synchronous gauge, which is characterised by a perturbed line element of the form

$$ds^2 = a(\tau)^2(-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j), \tag{18}$$

where τ denotes conformal time and h_{ij} is the metric perturbation. Here and in the following dots represent derivatives with respect to τ. Most of our other notations and conventions comply with those in Ma and Bertschinger [73]. Consequently, the Friedmann equation takes the form

$$3H^2 = \frac{a^2}{M_{\rm pl}^2}\left(\frac{\dot{\phi}^2}{2a^2} + V_\phi(\phi) + \rho_m\right), \tag{19}$$

with $M_{\rm pl} \equiv (\sqrt{8\pi G})^{-1}$ denoting the reduced Planck mass and the subscript m comprising all matter species.

Since the following perturbation equations have been widely discussed in the literature (e.g. [23, 53, 65, 75, 76] and references therein), we will simply state them here for neutrinos coupled to a scalar field.

The evolution equation for the MaVaN density contrast $\delta_\nu = \delta\rho_\nu/\rho_\nu$ is given by [65],

$$\dot{\delta}_\nu = 3\left(H + \beta\dot{\phi}\right)\left(w_\nu - c_\nu^2\right)\delta_\nu - (1 + w_\nu)\left(\theta_\nu + \frac{\dot{h}}{2}\right)$$
$$+ \beta\,(1 - 3w_\nu)\,\delta\dot{\phi} + \beta'\dot{\phi}\delta\phi\,(1 - 3w_\nu)\,, \tag{20}$$

where $\beta = \frac{d\log m_\nu}{d\phi}$ and c_ν is the neutrino sound speed (cf. Eq. 13). The quantity c_ν^2 can be calculated directly from the sound speed of the combined fluid and the scalar field perturbations. We note that in general $c_\nu^2 \neq \dot{P}_\nu/\dot{\rho}_\nu$ (i.e. the neutrino part of the fluid is not adiabatic in itself), and that c_ν^2 can in some cases become negative because of the interaction.

Furthermore, the trace of the metric perturbation, $h \equiv \delta^{ij} h_{ij}$, according to the linearised Einstein equations satisfies,

$$\ddot{h} + H\dot{h} = \frac{a^2}{M_{\rm pl}^2}[\delta T_0^0 - \delta T_i^i], \text{ where} \tag{21}$$

$$\delta T_0^0 \quad = -\frac{1}{a^2}\dot{\phi}\delta\dot{\phi} - V_\phi'(\phi)\delta\phi - \sum_m \rho_m \delta_m, \tag{22}$$

$$\delta T_i^i \quad = \frac{3}{a^2}\dot{\phi}\delta\dot{\phi} - 3V_\phi'(\phi)\delta\phi + \sum_r \rho_r\delta_r + 3c_b^2\rho_b\delta_b + 3c_\nu^2\rho_\nu\delta_\nu. \tag{23}$$

Here δT^μ_ν denotes the perturbed stress energy tensor and the subscripts m and r collect neutrinos, radiation, CDM and baryons (with sound speed c_b) as well as (relativistic) neutrinos and radiation, respectively.

The evolution equation for the neutrino velocity perturbation $\theta_\nu \equiv ik_i v_\nu^i$ with $v_\nu^i \equiv dx^i/d\tau$ reads [65],

$$\begin{aligned}\dot{\theta}_\nu = & -H(1-3w_\nu)\theta_\nu - \frac{\dot{w}_\nu}{1+w_\nu}\theta_\nu + \frac{c_\nu^2}{1+w_\nu}k^2\delta_\nu \\ & + \beta\frac{1-3w_\nu}{1+w_\nu}k^2\delta\phi - \beta(1-3w_\nu)\,\dot{\phi}\,\theta_\nu - k^2\sigma_\nu,\end{aligned} \tag{24}$$

where σ_ν denotes the neutrino shear as defined in [73].

Finally, the perturbed Klein-Gordon equation for the coupled scalar field is given by [65]

$$\begin{aligned}& \delta\ddot{\phi} + 2H\delta\dot{\phi} + \left[k^2 + a^2\left\{V_\phi'' + \beta'(\rho_\nu - 3P_\nu)\right\}\right]\delta\phi + \frac{1}{2}\dot{h}\dot{\phi} = \\ & -a^2\beta\delta_\nu\rho_\nu(1-3c_\nu^2).\end{aligned} \tag{25}$$

We note that instead of proceeding via the fluid equations, Eqs. (20) and (24), the evolution of the neutrino density contrast can be calculated from the Boltzmann equation [73]. We have verified analytically and numerically that the two methods yield identical results provided that the scalar-neutrino coupling is appropriately taken account of in the Boltzmann hierarchy [74].

As discussed in sec. 2 MaVaNs models can only possibly become unstable on sub-Hubble scales $m_\phi^{-1} < a/k < H^{-1}$ in the non-relativistic regime of the neutrinos, where the perturbations evolve adiabatically. For our numerical results in the next section we solve the coupled Eqs. (20-25) in the (quasi-)adiabatic regime by neglecting the neutrino shear σ_ν. This approximation is justified, since the scalar-neutrino coupling becomes important in this regime and m_ν is much larger than the mean momentum of the neutrino distribution.

For the purpose of gaining further analytical insight into the evolution of the neutrino density contrast, it is instructive to apply additional approximations to Eqs. (20-25) to be justified in the following.

Since the minimum of the effective potential tracked by the scalar field evolves only slowly due to changes in the neutrino energy density, we can safely ignore terms proportional to $\dot{\phi}$. Moreover, in the non-relativistic regime of the neutrinos on scales $m_\phi^{-1} < a/k < H^{-1}$, as a consequence of $P_\nu \sim 0$ it follows that $\sigma_\nu \sim 0$ and $w_\nu \sim 0$ as well as $\rho_r \sim c_b^2 \sim 0$. In addition, in the following we substitute $\delta\phi$ by its average value corresponding to the forcing term on the right hand side of Eq. (25) in the above limits,

$$\delta\bar{\phi} = -\frac{\beta \rho_\nu \delta_\nu (1 - 3c_\nu^2)}{(V_\phi'' + \rho_\nu \beta') + \frac{k^2}{a^2}}, \tag{26}$$

which solves the perturbed Klein-Gordon equation reasonably well on all scales [23, 75]. Finally, by combining the derivative of Eq. (20) with Eq. (21) – Eq. (24) and Eq. (26) in the non-relativistic limit, we arrive at the equation of motion for the neutrino density contrast valid at late times on length scales $m_\phi^{-1} < a/k < H^{-1}$,

$$\ddot{\delta}_\nu + H(1 + 3c_\nu^2)\dot{\delta}_\nu + \left(c_\nu^2 k^2 - \frac{3}{2}H^2\left[\Omega_\nu(\frac{G_{\rm eff}}{G} + 3c_\nu^2) - c_\nu^2\right]\right)\delta_\nu = \frac{3}{2}H^2\left[\ \Omega_{\rm CDM}\delta_{\rm CDM} + \Omega_b\delta_b\ \right] \tag{27}$$

where

$$G_{\rm eff} = G\left(1 + \frac{2\beta^2 M_{\rm pl}^2(1 - 3c_\nu^2)}{1 + a^2\{V_\phi'' + \rho_\nu\beta'\}/k^2}\right) \text{ and} \tag{28}$$

$$\Omega_i = \frac{a^2\rho_i}{3H^2 M_{\rm pl}^2}. \tag{29}$$

Since neutrinos not only interact through gravity, but also through the force mediated by the scalar field, they feel an effective Newton's constant $G_{\rm eff}$ as defined in Eq. (28). The force depends upon the MaVaN model specific functions β and V_ϕ and takes values between G and $G(1+2\beta^2 M_{\rm pl}^2(1-3c_\nu^2))$ on very large and small length scales, respectively. The scale dependence of $G_{\rm eff}$ is due to the finite range of the scalar field $(V_\phi'' + \rho_\nu\beta')^{-\frac{1}{2}}$, which according to Eq. (8) is equal to $(m_\phi^2 - \beta^2\rho_\nu)^{-\frac{1}{2}}$. For moderate coupling strength it is essentially given by the inverse scalar field mass, whereas for $\beta \gg 1/M_{\rm pl}$ it can take larger values. Accordingly, in a MaVaN model both the scalar potential V_ϕ and the coupling β influence the range of the scalar field force felt by neutrinos, whereas its strength is determined by the coupling β.

The evolution of perturbations in cold dark matter (CDM) coupled to a light scalar field in coupled quintessence [23] and chameleon cosmologies [53] is governed by an equation similar to Eq. (27). However, we would like to point out that for the same coupling functions the dynamics of the perturbations in neutrinos can be quite different from those in coupled CDM. This is a consequence of the relative smallness of the neutrino masses, due to which neutrinos provide a much smaller fraction to the total energy density than CDM and baryons, $\Omega_\nu \ll (\Omega_{\rm CDM} + \Omega_{\rm b})$. Whereas $\Omega_{\rm CDM} \sim 0.2$ and $\Omega_{\rm b} \sim 0.05$ [4] at present, Ω_ν depends on the so far not known absolute neutrino mass scale realised in nature. Taking as a lower bound the mass splitting deduced from atmospheric neutrino flavour oscillation experiments and the upper bound derived from the Mainz tritium beta-decay experiments [80], we get $10^{-4} \lesssim \Omega_\nu \lesssim 0.15$ today ‖. It is important to note that since in the standard MaVaN scenario the neutrino mass is an increasing function of time, at earlier times the ratio $\Omega_\nu/(\Omega_{\rm CDM} + \Omega_{\rm b})$ was even more suppressed than today. In general it follows that the smaller this ratio is, the larger the relative influence of the forcing term on the RHS of Eq. (27) becomes. The forcing term describes the effect of the perturbations in other cosmic components on the dynamics of the neutrino density contrast and competes with the scalar field dependent term $\propto \frac{G_{\rm eff}}{G}\Omega_\nu\delta_\nu$ on the LHS. Correspondingly, apart from the scalar field mediated force the neutrinos feel the gravitational drag exerted by the potential wells formed by CDM. Consequently, as long as the coupling function β does not compensate for the relative smallness of Ω_ν and thus enlarge the influence of the term $\propto \frac{G_{\rm eff}}{G}\Omega_\nu\delta_\nu$, the neutrinos will follow CDM (like baryons) just as in the Standard Model.

In the following we classify the behaviour of the neutrino density contrast in models of neutrino dark energy subject to all relevant kinds of coupling functions β. In the small-scale limit we distinguish the following three cases:

a) For $\beta < \sqrt{\frac{\Omega_{\rm CDM}-\Omega_\nu}{2M_{\rm pl}^2\Omega_\nu}}$ until the present time, $G_{\rm eff}\Omega_\nu\delta_\nu < G\Omega_{\rm CDM}\delta_{\rm CDM}$, the neutrino density contrast is stabilised by the CDM source term which dominates its dynamics. In this case the influence of the scalar field on the perturbations is subdominant and the density contrast in MaVaNs grows moderately just like gravitational instabilities in uncoupled neutrinos.

b) For $\beta \sim$ const. and much larger than all other parameters at late times, $G_{\rm eff} \gg G$, the damping term $H\dot{\delta}_\nu$ in Eq. (27) as well as the the terms proportional to $\delta_{\rm CDM}$ and δ_b can be neglected, leading to exponentially growing solutions.

c) For $\beta \neq$ const. and growing faster than all other parameters at late times, $G_{\rm eff} \gg G$, δ_ν is growing faster than exponentially¶.

In contrast, on scales $(V''_\phi + \rho_\nu\beta')^{-1/2} \ll a/k < H^{-1}$ much larger than the range of the ϕ-mediated force,

d) For $\beta \sim$ const. and of moderate strength, $G_{\rm eff} \sim G$ and the perturbations behave effectively like perturbations for uncoupled fluids in General Relativity.

e) For β growing faster than all other quantities at late times, $G_{\rm eff} \gg G$, instabilities develop on all sub-Hubble scales $a/k > (V''_\phi + \rho_\nu\beta')^{-1/2}$ according to *c*). However, on large length scales their growth rate is suppressed due to the corresponding small wave number k.

‖ Note that if the upper limit from the Mainz experiment is saturated the requirement $\Omega_\nu \ll \Omega_m$ is formally not satisfied. However, this case should be viewed as very extreme and is most likely excluded based on structure formation arguments

¶ In the limit $\beta(\tau) \to \infty$ for $\tau \to \infty$, Eq. 27 takes the form $\ddot{\delta}_\nu - 3H^2\Omega_\nu \frac{\beta^2(\tau)M_{\rm pl}^2}{1+a^2(V''_\phi+\rho_\nu\beta')/k^2}\delta_\nu = 0$, and it can be shown that $|\frac{\dot{\delta}_\nu}{\delta_\nu}| \to \infty$ for $\tau \to \infty$ [78]. Since this ratio is constant and thus not large enough for an exponentially growing δ_ν, the solution is required to grow faster than exponentially.

We note that for $|c_\nu^2| \ll 1$ Eq. 27 can be recast into the simple form,

$$\ddot{\delta}_\nu + H\dot{\delta}_\nu + \left[\frac{c_a^2}{c_a^2+1}k^2 - \frac{3}{2}H^2\Omega_\nu\right]\delta_\nu = \frac{3}{2}H^2\left[\ \Omega_{\rm CDM}\delta_{\rm CDM} + \Omega_b\delta_b\ \right]. \tag{30}$$

Apparently, as a consequence, all the effects of the scalar-neutrino coupling on the evolution of δ_ν are encoded in the term governed by the total sound speed squared c_a^2. Clearly, as soon as c_a^2 turns negative (but > -1), this term will change its sign and thus also its nature. Namely, it will amplify the effect of the gravitational term for neutrinos, which tends to drive instabilities to grow. Accordingly, in case the stabilising effect of CDM on the neutrino perturbations becomes negligible (cf. *b*), *c*)), it will cause the neutrino density contrast to strongly grow.

3.1. Potentials and Couplings

In the following, we consider two combinations of scalar potentials $V_\phi(\phi)$ and of scalar-neutrino couplings β which define our MaVaN models. The potentials are chosen (and fine-tuned) to accomplish the required cosmic late-time acceleration and for the couplings we take meaningful limiting cases.

Firstly, we consider a MaVaN model suggested by [38] which we will refer to as the *log-linear model*. The scalar field has a Coleman-Weinberg type [77] logarithmic potential,

$$V_\phi(\phi) = V_0\,\log(1+\kappa\phi), \tag{31}$$

where the constants V_0 and κ are chosen appropriately to yield $\Omega_{\rm DE} \sim 0.7$ and $m_\phi \gg H$ today. The choice of V_ϕ determines the evolution of ϕ according to Eq. (7) as plotted in fig. 2. Apparently, the neutrino background has a stabilising effect on ϕ. It drives the scalar field to larger values and stops it from rolling down its potential V_ϕ. This competition of the two terms in Eq. (7) results in a minimum at an intermediate value of ϕ (cf. Eq. 9), which slowly evolves due to changes in the neutrino energy density. As the universe expands and ρ_ν dilutes, both the minimum and the scalar field are driven to smaller values towards zero.

Let us now turn to the neutrino mass and its evolution. The dependence of m_ν on the scalar field is given by,

$$m_\nu(\phi) = \frac{m_0}{\phi}. \tag{32}$$

Such a dependence naturally emerges in the framework of the seesaw mechanism. In this case the light neutrino mass m_ν arises from integrating out a heavier sterile state, whose mass varies linearly with the value of the scalar field (as e.g. in Ref. [38, 63, 71]).

According to Eq. (32) this model is characterised by a field dependent coupling,

$$\beta(\phi) = \frac{1}{m}\frac{\partial m}{\partial\phi} = -\frac{1}{\phi}, \tag{33}$$

which corresponds to a time evolution as plotted in fig. 3.

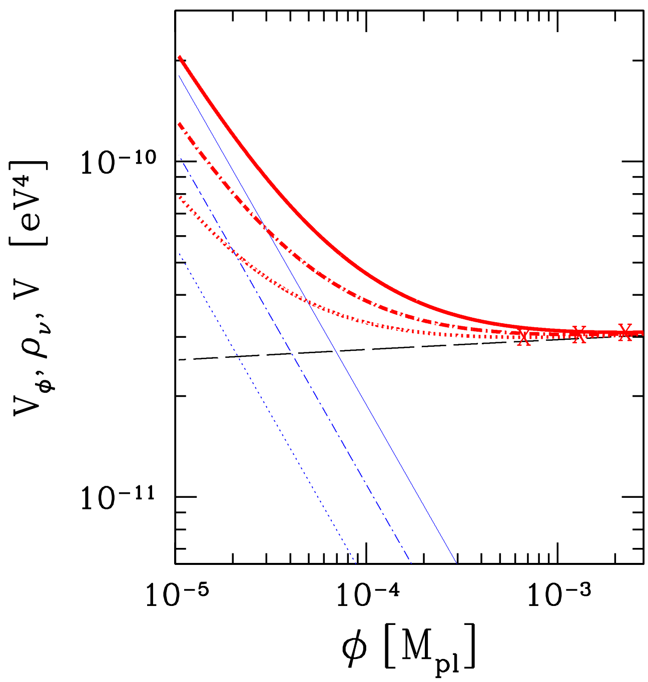

Figure 2. The effective potential V (thick lines), composed of the scalar potential V_ϕ (dashed) and the neutrino energy density ρ_ν, plotted for three different redshifts, $z = 5$ (solid), $z = 4$ (dashed-dotted), $z = 3$ (dotted). The VEV of ϕ tracks the minimum of V (marked by X) and evolves to smaller values for decreasing redshift. We have used $\kappa = 1 \times 10^{20} M_{\rm pl}^{-1}$ and $V_0 = 8.1 \times 10^{-13}{\rm eV}^4$.

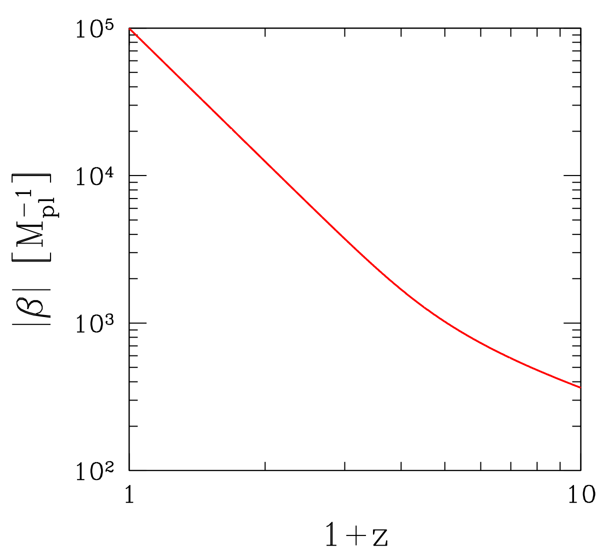

Figure 3. The evolution of the effective coupling, β (given by Eq. (33)), as a function of redshift for the potential Eq. (31). We have used $\kappa = 1 \times 10^{20} M_{\rm pl}^{-1}$ and $V_0 = 8.1 \times 10^{-13}{\rm eV}^4$.

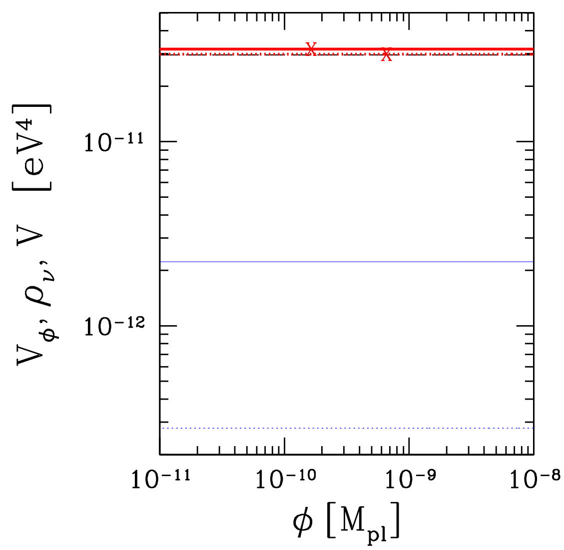

Figure 4. The effective potential V (thick lines), composed of the scalar potential V_ϕ (dashed) and the neutrino energy density ρ_ν, plotted for two different redshifts, $z = 1$ (solid), $z = 0$ (dotted). The VEV of ϕ tracks the minimum of V (marked by X) and evolves to larger values for decreasing redshift. We have used $M = 0.0023195$ eV.

Since the value of ϕ decreases with time (cf. fig. 2) this means the rate of energy transfer between the scalar field and the neutrinos and also the attraction felt between neutrinos increases with time. Consequently, both the neutrino mass m_ν in Eq. (32) and according to Eq. (2) also the neutrino energy density blow up when ϕ approaches zero. Thus, from these qualitative considerations it can already be expected that the model will run into stability problems in the non-relativistic neutrino regime.

Secondly, we consider a model proposed in the context of chameleon cosmologies [52, 53, 79], which we will refer to as the *power-model*. The scalar field has an exponential potential which at late times reduces to an inverse power-law potential,

$$V_\phi = M^4 \exp \frac{M^n}{\phi^n} \sim M^4 \left(1 + \frac{M^n}{\phi^n}\right), \tag{34}$$

since then $\phi \gg M$. Furthermore, the mass parameter M is fixed by the requirements $\Omega_{DE} \sim 0.7$ and $m_\phi \gg H$. In fig. 4 the evolution of ϕ is plotted according to Eq. (1). In contrast to the first model, the expectation value of ϕ is increasing with time. Note that the scalar potential V_ϕ thus at late times very weakly depends on changes in the VEV of ϕ, since $V_\phi \sim M^4 = \text{const.}$ for $\phi \gg M$.

In this model the dependence of the neutrino mass on the scalar field is taken to be,

$$m_\nu = m_0 e^{\beta\phi}, \tag{35}$$

which is of a form expected in a special type of scalar-tensor theory, in which the scalar degree of freedom only couples to neutrinos in a conformal way (as eg. in [65]). It is important to note that for this model the coupling is constant,

$$\beta = \frac{1}{m_\nu}\frac{\partial m_\nu}{\partial \phi} = \text{const.} \tag{36}$$

Since according to fig. 4 the value of ϕ is very small until the present time, even for $\beta \gg \frac{1}{M_{\rm pl}}$ the exponential function in Eq. (36) takes values close to 1. Accordingly, the neutrino mass in Eq. (35) depends only weakly on changes in the scalar field VEV and thus hardly evolves with time. In contrast to the log-linear model the effective potential V in fig. 4 only evolves due to the dilution of the neutrino energy density and not additionally due to the mass variation. Furthermore, unlike the log-linear model, the attractive force between neutrinos is essentially time independent. This in turn makes the model much more stable.

4. Results

In this section we present the numerical results of our stability analysis for the two MaVaN models of the last section. They are obtained from modifying the CMBFAST code [81] to include a light scalar field coupled to neutrinos and were checked by altering the CAMB code [82] accordingly. We assume a neutrino energy density of $\Omega_\nu \sim 0.02$, which corresponds roughly to the current conservative upper limit on the sum of neutrino masses from CMB and LSS data [3, 4, 40] [+], where we take the present day normalised Hubble expansion rate to be $h = 0.7$. Ω_ν corresponds to the energy density of three neutrino species with degenerate mass $m_{\nu_i}(z = 0) \sim 0.312\,\text{eV} \gg T_{\nu_0}$, which are highly non-relativistic today.

4.1. Log-linear Model

The log-linear model is defined by Eq. (31) and Eq. (32). By fine-tuning the parameter V_0 for a fixed value of $\kappa = 10^{20} M_{\rm pl}^{-1}$ in Eq. (31), standard cosmology with $\Omega_{\rm DE} = 0.7$, $\Omega_{\rm CDM} = 0.25$, and $\Omega_{\rm b} = 0.05$ at present can be accomplished, where $\Omega_{\rm DE} = \Omega_\nu + \Omega_\phi$.

The mass of ϕ at present determined from Eq. (8) is $m_\phi = 5.74\ \text{Mpc}^{-1} \gg H$. Consequently, the Compton wavelength of the scalar field, m_ϕ^{-1}, sets the scales on which the perturbations in (non-relativistic) MaVaNs are adiabatic, $H \ll 0.1\,\text{Mpc}^{-1} \lesssim k$ (cf. the discussion in sec. 2.2). In fig. 5 we present our results for the evolution of the neutrino mass, the sound speed squared and the density contrast to be discussed in the following.

a) The evolution of the neutrino mass $m_\nu(z)$ and the neutrino temperature $T_\nu(z) = T_{\nu_0}(1 + z)$ is plotted as a function of redshift. As long as $m_\nu(z) \ll T_\nu(z)$, the neutrinos are relativistic, whereas for $m_\nu(z) \gg T_\nu(z)$ they have turned non-relativistic. The transition takes place at roughly $z + 1 \sim 7$, i.e. when

[+] Note those constraints were obtained assuming non-interacting neutrino models. Hence this assumption could be relaxed.

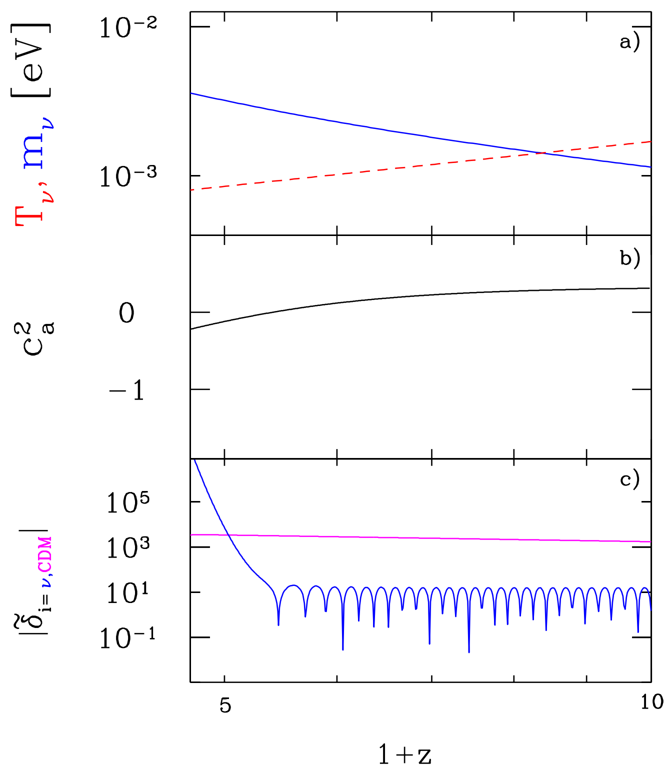

Figure 5. *a*) Neutrino mass m_ν (solid) and temperature T_ν (dotted) as a function of redshift. *b*) Total dark energy sound speed squared c_a^2 as a function of redshift. *c*) Density contrast in neutrinos (solid) $\tilde{\delta}_\nu$ and density contrast in CDM $\tilde{\delta}_{\mathrm{CDM}}$ (dash-dotted) as a function of redshift on a scale $k = 0.1\,\mathrm{Mpc}^{-1}$. We have used $\kappa = 1 \times 10^{20} M_{\mathrm{pl}}^{-1}$ and $V_0 = 8.1 \times 10^{-13}\mathrm{eV}^4$.

$m_\nu(z) \simeq T_\nu(z)/3$. One interesting feature is that for $z \to 0$ the neutrino mass grows as $m_\nu(z) \propto a^3$ so that $\rho_\nu \to$ Constant.

b) A plot of the total adiabatic sound speed squared of the coupled fluid c_a^2. It decreases when the neutrinos approach the non-relativistic regime $m_\nu(z) \gg T_\nu(z)$(cf. *a*)). This is due to the drop in the neutrino pressure from initially $P_\nu \sim 1/3$ to $P_\nu \sim 0$ well after the transition of regimes.

c) A plot of the density contrast in neutrinos $\tilde{\delta}_\nu = \delta\rho_\nu/\rho_\nu$, and cold dark matter (CDM) $\tilde{\delta}_{\mathrm{CDM}} = \delta\rho_{\mathrm{CDM}}/\rho_{\mathrm{CDM}}$ on a scale of $k = 0.1$ Mpc^{-1}. As long as the neutrinos are still relativistic ($m_\nu(z) \ll T_\nu(z)$ cf. *a*)), the perturbations in the strongly coupled scalar-neutrino fluid oscillate like sound waves. However, after pressure cannot offset the attractive force anymore ($m_\nu(z) > T_\nu(z)/3$), the neutrino density contrast blows up and thus grows at a much faster rate than the density contrast in CDM (the fast growth sets in after the effective sound speed squared has turned negative). This can be understood by considering the evolution of the scalar neutrino coupling β (cf. fig. 3) for this model, since β^2 according to Eq. (27) governs the evolution of the density contrast in non-relativistic neutrinos. Since the choice of a large κ corresponds to $\phi \ll M_{\mathrm{pl}}$ at late times, β^2 is driven to larger and large values, while the VEV of ϕ approaches zero (cf. the discussion in the last section). Accordingly, $\tilde{\delta}_\nu$ is subject to an effective Newton's constant $G_{\rm eff} \gg G$ (cf. the discussion in sec. 3). However, $\tilde{\delta}_{\rm CDM}$ behaves essentially as in General Relativity, as long as the modification to the gravitational effect on CDM caused by the scalar-field induced change in the neutrino density contrast is not prominent. Since the coupling and thus $G_{\rm eff}$ rapidly increase with time, the scalar field transfers more and more energy to the neutrinos causing m_ν to increase (cf. *a*)). Therefore, both β as well the energy density in neutrinos increase such that the stabilising effect of the CDM becomes less and less important and finally becomes entirely negligible.

As a further consequence, the attraction between neutrinos also rises steadily, while the neutrino pressure drops and ceases to stabilise the perturbations. As demonstrated in *b*) the total sound speed squared is thus quickly driven to negative values, causing $\tilde{\delta}_\nu$ to grow faster than exponentially (cf. also the discussion in sec. 3). In the following, we will argue that as a result the neutrino density contrast has already turned non-linear in the past. Therefore, we take into account the normalisation of the CDM density contrast which gives us a rough estimate for the normalisation of $\tilde{\delta}_\nu$. As long as the dimensionless power spectrum $\Delta^2(k) = k^3 P(k)/(2\pi^2) \propto \delta^2_{\rm CDM} < 1$, CDM perturbations on a scale k are linear, where $P(k)$ denotes the power spectrum of CDM. Since on the considered scale of $k = 0.1\,{\rm Mpc}^{-1}$ we have $\Delta^2(k) \sim 0.3 - 0.4$ [83] for CDM, we can infer that for neutrinos $\Delta^2(k) \propto \delta^2_\nu > 1$, when $\tilde{\delta}_\nu$ exceeds $\tilde{\delta}_{\rm CDM}$ by more than a factor of $\sqrt{2}$. This is the case at roughly $1 + z \sim 5$, while afterwards linear perturbation theory breaks down. It is thus likely that neutrinos in this model are subject to the formation of non-linear structure in the neutrino energy density [63] before the present time.

Our numerical results presented in fig. 5 demonstrate that the total sound speed squared in the log-linear model is negative at late times, corresponding to a fast growth of perturbations. Thus, inevitably, the neutrino density contrast at some point in time will go non-linear and the model becomes unstable with the possible outcome of the formation of neutrino bound states [63]. Note that this result is not in strict accordance with the analytical considerations of Ref. [63], since the non-linear collapse does not happen *as soon as* the neutrinos become non-relativistic, as baryons and especially CDM, are able to attract the neutrinos in their potential wells formed through conventional gravitational collapse. It should be noted that in case the neutrino mass scale realised in nature is much lower than assumed in our analysis, the stabilising effect of CDM might prevent a clumping of neutrinos until the present time.

We thus add a remark previously unnoticed in the MaVaN literature: it is crucial to consider the magnitude and the growth rate of the scalar field-neutrino coupling and to compare its importance relative to other sources of gravitational attraction. As indicated in the previous section, the comparison can be made quantitatively through Eq. (27).

4.2. Power-law Potential

The power-law potential is defined by Eq. (34) and Eq. (35). We have chosen $n = 0.3$ in Eq. (34) and tuned the mass parameter M to accomplish the right cosmology at present. Furthermore, as suggested by naturalness we choose $\beta = 1/M_{\rm pl}$ in Eq. (35). With these choices of parameters the mass of the scalar field at present is $m_\phi \sim 0.1\,{\rm Mpc}^{-1} \gg H$ and accordingly the MaVaN perturbations are adiabatic on sub-Hubble scales $k \lesssim m_\phi$. We perform our perturbation analysis on a scale $k = 0.1\,{\rm Mpc}^{-1}$ and illustrate our results in fig. 6 to be described in the following:

a) The evolution of the neutrino mass $m_\nu(z)$ and the neutrino temperature $T_\nu(z)$ in the non-relativistic regime $m_\nu(z) \gg T_\nu(z)$ is plotted as a function of redshift. Since the neutrino mass depends only weakly on changes in the scalar field VEV, it hardly evolves with time (cf. sec. 3.1).

b) The evolution of the total sound speed squared c_a^2 of the coupled dark energy fluid is plotted as a function of redshift. We observe that, c_a^2 takes positive values even in the highly non-relativistic regime of the neutrinos.

c) The density contrast in neutrinos $\tilde{\delta}_\nu$, and cold dark matter $\tilde{\delta}_{\rm CDM}$ is plotted on a scale of $k = 0.1$ ${\rm Mpc}^{-1}$ for $\beta = 1/M_{\rm pl}$. It is found that the density contrast in MaVaNs grows just as in uncoupled neutrinos in General Relativity. The reason is that the effects of the scalar field on the neutrino perturbations are subdominant with respect to the gravitational influence of CDM and baryons. It follows that they considerably affect the growth of MaVaN perturbations. In contrast to the log-linear model, the coupling between the scalar field and the neutrinos is constant and the neutrino mass very weakly depends on changes in the scalar field VEV. Accordingly, both the energy transfer of the scalar field to the neutrinos as well as the attraction felt between neutrinos hardly increases with time but stays essentially constant. As a result, the growth of $\tilde{\delta}_\nu$ (as well as of $m_\nu(z)$) with time remains moderate and δ_ν turns out to be of comparable size as $\tilde{\delta}_{\rm CDM}$ today.

   As argued in sec. 4.1, the CDM perturbations are known to be linear at the scale considered and thus the neutrino perturbations also can be viewed as linear until the present time. This result is in accordance with our finding of positive total sound speed squared up to today (cf. fig. 6b).

   We have checked that the behaviour of the neutrino density contrast is retained on the same scale for an increased value of the coupling $\beta = 100/M_{\rm pl}$. It should be noted that in this case the range of the scalar field and thus the scales, where possible instabilities grow fastest, have dropped below the physical scales accessible with CMBFAST/CAMB. We thus ascribe the unaltered behavior of the perturbations to the suppression of the effective Newton's constant $G_{\rm eff}$ felt by neutrinos with increasing scale a/k in combination with the stabilising effect achieved by CDM and baryons. This result demonstrates that a possible enhanced growth of MaVaN perturbations can only take place on very small scales, i.e. it is a rather local phenomenon.

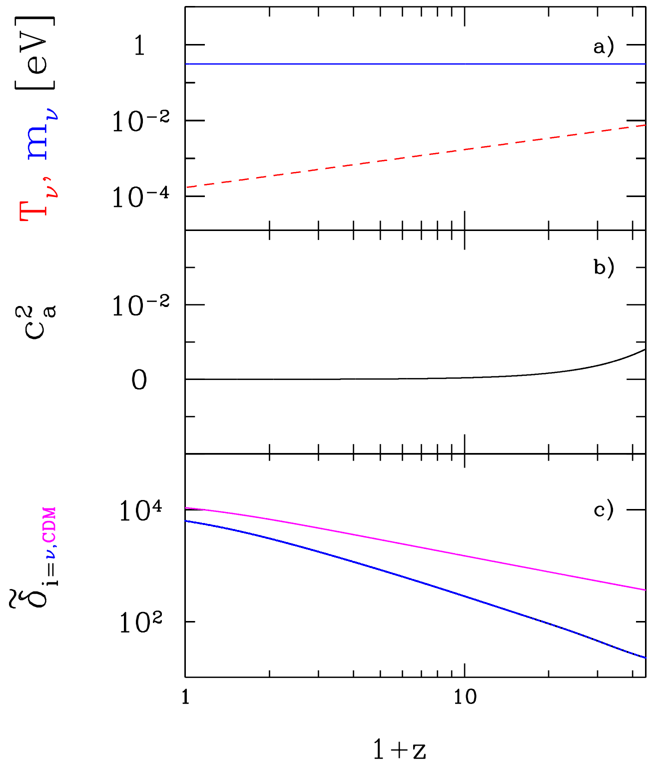

Figure 6. *a*) Neutrino mass m_ν (solid) and temperature T_ν (dotted) as a function of redshift. *b*) Neutrino sound speed squared c_ν^2 (dashed) and total dark energy sound speed squared c_a^2 (solid) as a function of redshift. *c*) Density contrast in neutrinos (solid) $\tilde{\delta}_\nu$, and density contrast in CDM $\tilde{\delta}_{\rm CDM}$ (dash-dotted) as a function of redshift on a scale $k = 0.1\,{\rm Mpc}^{-1}$. We have used $M = 0.0023195$ eV.

In conclusion, fig. 6 demonstrates that the adiabatic power-law model is characterised by a positive sound speed squared* and the neutrino density contrast on small scales is still in the linear regime today. Accordingly, the model can be viewed as stable until the present time. The slow growth of the neutrino perturbations was found to trace back to the behaviour of the neutrino mass which hardly evolves with time.

We would like to point out that in general according to Eq. 17 (cf. fig. 1) the restriction on the mass variation in a MaVaN model is alleviated at earlier times when finite temperature effects become more important. In other words, while the neutrino mass is only required to asymptote to an essentially constant value at late times, in the power-model (as a special case) it always stays nearly the same.

We conclude the subsection mentioning that the considered example constitutes a viable MaVaN model that resides permanently in the effective minimum and is characterised by a non-relativistic neutrino phase *and* a positive sound speed squared,

* We verified that models with larger values for n in Eq. (34) are also characterised by a positive total sound speed squared, while reproducing the standard cosmology. Furthermore, m_ϕ increases with n rendering the model more adiabatic.

a possibility that was not noticed in Ref. [63].

4.3. A no-go theorem for mass varying neutrinos?

In the following, we will comment on a no-go theorem in Ref. [63] which states that any realistic adiabatic MaVaN model with $m_\phi^2 > 0$ cannot be stable as soon as the non-relativistic neutrino regime starts. For its deduction the authors of Ref. [63] proceeded in the following way. They derived an expression for the total sound speed squared c_a^2 in the kinetic theory picture for $p_\nu \ll m_\nu$ assuming the perturbations to be plane waves. Independent of the choice of the scalar-neutrino coupling and the scalar potential which characterise a MaVaN model, c_a^2 turned out to be negative.

In the present work we have presented examples of models which demonstrate that a detailed analysis of the potential and coupling functions and an assessment of the influence of other important cosmic components, like CDM and baryons, are necessary in order to predict the growth of structure in neutrinos. In sec. 3 we found that the density contrast in neutrinos in the small scale limit only grows exponentially if the scalar-neutrino coupling is larger than all other relevant parameters, leading to negligible growth-slowing effects as provided by cosmic expansion and CDM gravitational drag.

In this case we verified numerically for the log-linear model of the last section that c_a^2 turns negative in agreement with the result of [63]. We would like to point out that finite temperature effects which can play a crucial role for the stability of a MaVaN model [70] were included in our calculation. However, as demonstrated by the result for the power-model, for a constant coupling $O(1)$ the evolution of the neutrino density contrast is not modified with respect to the uncoupled case in ordinary General Relativity. Accordingly, also on small scales $\sim m_\phi^{-1}$ the plane-wave solution did not apply and perturbations were driven by a positive sound speed squared. We checked numerically that the LHS of the stability criterion Eq. (17) derived in [70] gives a good indication for the sign of the sound speed squared.

Based on our analysis we conclude that viable adiabatic MaVaN models can be found which are stable until the present time. We indicate the relative smallness of the neutrino energy density as the main cause, since it enhances the stabilising influence exerted by CDM on the neutrino density contrast. Consequently, the dynamics in stable models are governed by CDM, largely independent of the sign of the sound speed squared, even in the highly non-relativistic regime.

However, for stable MaVaN models characterised by a positive sound speed squared until the present time (cf. the power-model) the allowed neutrino mass evolution at late times was found to be generically severely constrained. Note that in our stability analysis we have tested the growth of perturbations in the framework of linear perturbation theory valid on large length scales until the present time. Accordingly, we could get a feeling for the relevant physical effects leading to the possible clumping in neutrinos disentangled from any non-trivial non-linear effects inherent in small physical scales.

Furthermore, we have integrated the relevant equations using CMBFAST and

CAMB which work in the linear regime. Consequently, the mass of the scalar field had to be chosen small enough (however $\gg H$) to push the scales where possible instabilities could occur into the linear regime. As discussed in sec. 3 and verified by our numerical results for the power-model and $\beta = 100/M_{\rm pl}$, on scales larger than the range of the scalar field the attraction felt by neutrinos becomes considerably suppressed. Thus, the possible enhanced growth in neutrinos was found to be a rather local phenomenon. By increasing the scalar field mass and thus reducing the range of the scalar field, we would expect a local scalar field induced enhancement of the gravitational clustering of neutrinos in the non-linear regime (on scales, where neutrino free-streaming cannot inhibit the growth of perturbations) . Accordingly, resulting neutrino bound states would be interpreted as a contribution to the CDM small scale structure, which however, on average does not affect the equation of state of neutrino dark energy. Similarly, in chameleon cosmologies such an enhanced small scale growth of the CDM density contrast is predicted [84] due to the coupling to a scalar field with range $a/k = 250$ pc today. We thus refer to another interesting class of possibly stable MaVaN models characterised by a much larger scalar field mass. However, the detailed discussion of these models and their phenomenological implications lies beyond the scope of this paper.

5. Discussion

Models of neutrinos coupled to a light scalar field have been invoked to naturally explain the observed cosmic acceleration as well as the origin of dynamical neutrino masses. However, the class of MaVaN models characterised by an adiabatic evolution of perturbations in the non-relativistic regime may suffer from instabilities and as a result cease to act as dark energy. In this paper we analysed the stability issue in the framework of linear perturbation theory. For this purpose we derived the equation of motion of the density contrast in non-relativistic neutrinos in terms of the characteristic MaVaN model dependent functions, namely the scalar potential, the scalar-neutrino coupling, and the source terms provided by CDM and baryons. Furthermore, we modified both the CMBFAST [81] and CAMB [82] code to include a light scalar field coupled to neutrinos and numerically focused on two significant MaVaN models.

We found that the instabilities in the neutrino density contrast only occur if the influence of the scalar-neutrino coupling on the dynamics of the perturbations dominates over the growth-slowing effects (dragging) provided by CDM. More precisely, as long as the coupling $\beta < \sqrt{\frac{\Omega_{\rm CDM}-\Omega_\nu}{2M_{\rm pl}^2\Omega_\nu}}$ the gravitational drag felt by neutrinos towards the potential wells formed by CDM leads to a stabilisation of the perturbations until today, largely independent of the sign of the sound speed squared. As a result, the density contrast in MaVaNs grows moderately just as for uncoupled neutrinos in General Relativity, without any effects of the scalar field becoming apparent. We indicate the small contribution of the neutrinos to the total energy density as the reason for this behavior.

However, if the coupling is strong enough to render any growth-slowing effects negligible, the stability requires the model to exhibit a positive sound speed squared. This condition was found to strongly restrict the allowed neutrino mass variation at late times [70].

These results were obtained from considering representative limiting cases for the time dependence of the coupling. At first, we investigated MaVaN models characterised by a strong growth of the coupling and thus of the neutrino masses with time. In this case, at late times any growth-slowing effects on the perturbations provided by the expansion or the gravitational drag of CDM can be neglected. Consequently, independent of the choice of the scalar potential, the analytic equation for the evolution of the neutrino density contrast at late times involved a faster than exponentially growing solution. Our numerical results for such a model with logarithmic scalar potential illustrated that the onset of the instability is around the time when the neutrinos turn non-relativistic. This can be seen as the effect of the total sound speed squared becoming negative. Since the attraction between neutrinos increases rapidly, the sound speed changes sign as soon as the counterbalancing pressure forces in neutrinos have dropped sufficiently. As a result, the non-relativistic neutrino density contrast is inevitably driven into the non-linear regime and leads to the formation of neutrino nuggets [63].

This is in contrast to MaVaN models involving a constant coupling of moderate strength. In this case, the evolution of the neutrino density contrast at late times is described by a power law just as in the uncoupled case. We demonstrated numerically that the choice of a constant coupling and an inverse power law scalar potential leads to an adiabatic MaVaN model characterised by a positive sound speed squared. In addition, we found the neutrino density contrast to be still in the linear regime on scales where possible instabilities would grow fastest. Accordingly, the MaVaN model can be viewed as stable until the present time. However, the neutrino masses depend very weakly on changes in the scalar field and they hardly evolve at late times. This turns out to be a generic feature of stable MaVaN models characterised by a positive sound speed squared.

We would like to allude to another interesting class of possibly stable MaVaN models, whose quantitative discussion lies, however, beyond the scope of this paper. Due to the finite range of the scalar field, the enhanced growth of neutrino perturbations caused by the scalar-neutrino interaction was found to be a rather local phenomenon. Consequently, for MaVaN models characterised by a much smaller range of the scalar field, the possibly unstable regime of (non-relativistic) MaVaN perturbations can be shifted to much smaller length scales, where non-linear effects become important. In this case we expect a local contribution of the scalar-field induced clustering of neutrinos to the small scale structure of CDM (on scales where neutrino free-streaming does not inhibit the growth of perturbations). On average, however, it does not affect the equation of state of neutrino dark energy, which in this kind of models can thus still explain the apparent late time acceleration. This possibility was so far not noticed in the MaVaN literature. However, a similar reasoning can be found in models of chameleon cosmologies [84].

Acknowledgments

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